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Market Microstructure Invariance: A Meta-Model

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Plan

- Review 1985 paper:
  - Kyle (1985)

Describe how my recent research builds on 1985 paper:

- A “metamodel” of market microstructure invariance: .
  - Kyle and Obizhaeva (2016*a*)
  - Kyle and Obizhaeva (2022)
- Smooth trading:
  - Kyle, Obizhaeva and Wang (2018)
- Flow trading:.
  - Budish et al. (2023)

Summary of 1985 paper

- Show how market liquidity or depth (reciprocal of market impact) results from a simple model of adverse selection.
- Liquidity results from demand for impatient trading in presence of private information.
- Informed trader trades strategically: recognizes own market impact. Game theory.
- Risk aversion is not part of the model.
- Model is simple. Optimal strategies are linear.

One-period Model in 1985 paper: Setup

- Informed trader observes fundamental value:

$$F = P_0 + \sigma_F \cdot Z_F, \qquad \text{with} \qquad Z_F \sim \mathrm{N}(0,1).$$

- Noise traders trade $Q_U = \sigma_U \, Z_U$, with $Z_U \sim \mathrm{NID}(0,1)$.
- Informed trader exercises market power, trades Q_I given by

$$Q_I(F) = \underset{x}{\operatorname{argmax}} \ \mathrm{E}\Big[\big(F - p\big(x, Q_U\big)\big) \cdot x \ \Big| \ F\Big],$$

- Market makers set price, after observing a pair of bets in random order, as symmetric function

$$p(Q_I, Q_U) = \ \mathrm{E}\left[F \mid \{Q_I, Q_U\}\right].$$

One-period Model in 1985 paper: Results

Simple derivation shows equilibrium given by

$$Q_I = \beta \cdot (F - P_0), \qquad p(Q_I, Q_U) = P_0 + \lambda \cdot (Q_I + Q_U),$$

with coefficients for trading intensity β and market impact λ given by

$$\beta = \frac{\sigma_U}{\sigma_F} \text{ and } \lambda = \frac{1}{2} \cdot \frac{\sigma_F}{\sigma_U}.$$

Model generates linear market impact proportional to the size of the order.

Continuous-time version

Assumptions:

- Noise trading assumed to follow Brownian motion.
- Informed trader (optimally) trades smoothly.

Results:

$$\Delta Q_I(t) = \beta(t) \cdot (F - P(t)) \cdot \mathrm{d}t \qquad \Delta P(t) = \lambda \cdot (\Delta Q_I(t) + \Delta Q_U(t)), \tag{1}$$

- Market impact λ is constant over time!
- Informed trader gradually reveals all private information.
- Prices follow a martingale from perspective of uninformed market makers.
- Therefore return volatility equals standard deviation of informed trader's signal.

Features of the Equilibrium

- Capture different roles for different players, reflecting 1980s markets (?): Informed trader, noise traders, market makers.
- Informed trader makes money at the expense of noise traders, who pay for liquidity. Market makers break even.
- Informed trader: In one-period model, exercises market power by trading half the quantity which would incorporate all information into prices. In continuous-time model, patiently smooths trading strategically to reduce price impact costs of trading, walking the residual supply schedule.
- Noise traders impatiently demand instantaneous liquidity without smoothing trading.
- Only market makers adjust quantities to prices instantanously.
- Market depth is ratio of standard deviation of noise trading to standard deviation of private information. More noise trading makes markets more liquid but does not affect volatility. More private information makes markets less liquid and more volatile.
- Prices are set in a “batch auction” in which all traders pay the same price.

Critique of 1985 model

- Distinction between market makers and other traders less important in today's more symmetrical all-to-all platforms.
  - Addressed by "smooth trading" model and "invariance" model.
- Noise traders could reduce losses in half by smoothing trading over a very short period of time.
  - Addressed by "smooth trading" model.
- Trading volume is infinite in continuous-time model (total variation of a diffusion).
  - Addressed by "smooth trading" model and 'invariance" model.
- Not clear how to "convert model" time into "business time" in actual markets so that model can be mapped into "clock time". Difficult to apply model empirically.
  - Addressed by market microstructure "invariance model".
- Informed and noise traders required to place "market orders" but might prefer to place "limit orders":
  - Addressed by "smooth trading" model and "flow trading" mechanism.
- Not clear how to apply "batch auction" model to a market in which many different assets are traded.
  - Addressed by "flow trading" mechanism.

Market Microstructure Invariances Deals with Related Problems

- How to map theoretical models of market liquidity based on adverse selection into data?
  Example: 1985 model has two parameters: σ_V and σ_U, which measure standard deviations of "information" and "noise trading", respectively.
  - Parameter σ_V maps into return volatility, but over what time period. One minute? One year?
  - Parameter σ_U maps into trading volume. One trade? Many trades summed together?
- If liquidity (market depth) is explained empirically by empirically observable volume and observable volatility, what is the role of the theoretical construct "adverse selection"?
- How can "liquidity" be measured empirically?

Idea: Metamodel of market microstructure invariance

Map 1985 model into empirically oriented mechanical metamodel of trading generating "market microstructure invariance". Metamodel provides empirical interpretation to theoretical parameters (σ_U and σ_V), thus connecting theory of adverse selection to empirical model of .

- "Metamodel" is a set of reduced-form log-linear equations which define relate liquidity variables.
- Divide liquidity variables into three categories: (1) observable (easy), (2) unobservable but with huge asset-specific variability (hard), (3) unobservable but calibrated to be approximately constant (invariant) across assets (easy enough).
- Solve for difficult variables in terms of easy variables and invariant variables.
- Result is "market microstructure invariance", a "mechanical model" of trading and liquidity, with a nice empirically operational measure of liquidity L but no concept of adverse selection. Multiplicative model enforces dimensional consistency for units of measurement.
- Can also add additional assumptions about "pricing accuracy" and "resilience" using invariant parameter measuring informativeness of signals, to illustrate the role of adverse selection dynamically.

Market Microstructure

Basic idea: Price changes result from price impact of “bets” (block trades).

We want to measure liquidity variables such as price impact.

To do so, we need to measure variables pertaining to number and sizes of bets.

Notation for Linear Price Impact: consistent with 1985 model

Finance theorists like linear models of price impact:

$$\text{Price Impact} = \Delta P = \lambda \cdot Q = 0.04 \text{ dollars/share}$$

Price impact cost in dollars:

$$\text{Dollar Price Impact Cost} = \Delta P \cdot Q = \lambda \cdot Q^2 = 400 \text{ dollars}$$

Price impact cost G as fraction of value traded:

$$G = \frac{\Delta P}{P} = \frac{\lambda \cdot Q^2}{P \cdot Q} = \frac{\lambda \cdot Q}{P} = 10 \text{ basis points}$$

Note: λ has units of dollars per share-squared, Q has units of shares, P has units of dollars per share, so G is dimensionless.

Model allows nonlinear price impact governed by parameter β, which we assume equal to on3. Suppose power function price impact for a bet Q:

$$\Delta P = \lambda \cdot Q^{\beta}.$$

Empirical Motivation for Invariance

- Emprical Problem: Parameters like bet arrival rate γ and bet size $\mathrm{E}[|Q|]$ are hard to measure or estimate.
- Can they be replaced with a parameter that is either easier to estimate or does not vary much across assets?
- Empirical strategy: Introduce a parameter C (dollars), which does not vary (much) across assets and time.
- Use “invariant” parameter C to replace parameter which are hard-to-measure and varying across assets, such as γ or $\mathrm{E}[|Q|]$.
- Assume “transactions cost invariance”: Ex ante expected dollar cost of a bet is constant (almost?)

$$C = \mathrm{E}[|Q \cdot \Delta P|] = \lambda \cdot \mathrm{E}\left[|Q|^{1+\beta}\right].$$

Now have more equations but also more variables.

Easy to observe variables

Easy-to-observe quantities include price, volume, and volatility:

$$\begin{aligned}
\text{Price} &= P = 40.00 \text{ dollars/share} \\
\text{Trading Volume} &= V = 1.00 \text{ million shares/day} \\
\text{Returns Volatility} &= \sigma = 0.02/\text{day}^{1/2}
\end{aligned}$$

Difficult-to-observe variables

Hard-to-measure quantities that vary greatly across assets and time include bet size, number of bets, and the price impact coefficient:

$$
\begin{aligned}
\text{Size of Bet} &= Qt = 10\,000 \text{ shares} \\
\text{Number of Bets} &= \gamma = 100/\text{day} \\
\text{Execution Horizon} &= H = 1 \text{ day} \\
\text{Price Change per Bet} &= \Delta P = 0.04 = \text{ dollars/share} \\
\text{Price Impact Coefficient} &= \lambda = 5 \times 10^{-5} \text{dollars/share}^2 \\
\text{Price Error Variance} &= \Sigma^{1/2} = \text{ var}^{1/2}\left[\log\left(\frac{F}{P}\right)\right] = \log(2) \text{ (dimensionless)} \\
\text{Price Resiliency} &= \rho = 0.0040/\text{day}
\end{aligned}
$$

Meta-Model Equations

Add transactions cost invariance to obtain four equations:

$$V = \gamma \cdot \mathrm{E}\left[|Q|\right] \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \quad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[(\Delta P)^2\right] = \lambda^2 \cdot \mathrm{E}\left[|Q|^{2\beta}\right] \qquad \text{(Price Impact of one bet),}$$

$$C = \lambda\, \mathrm{E}\left[|Q|^{1+\beta}\right] \qquad \text{(Dollar impact cost of a bet).}$$

Need two invariant moment ratios, which define two new equation (ratios equivalent when $\beta = 1$.

$$m := \frac{\mathrm{E}\left[|Q|\right] \cdot \sqrt{\mathrm{E}\left[|Q|^{2\beta}\right]}}{\mathrm{E}\left[|Q|^{\beta+1}\right]}, \qquad m_\beta := \frac{(\mathrm{E}\left[|Q|\right])^{\beta+1}}{\mathrm{E}\left[|Q|^{\beta+1}\right]}.$$

Now have six equations.

Six parameters are easy to measure or almost constant:

$$P, \quad V, \quad \sigma, \quad C, \quad m, \quad m_\beta.$$

Six variables are hard-to-measure:

$$\gamma, \quad \lambda, \quad \mathrm{E}\left[\Delta P^2\right], \quad \mathrm{E}\left[|Q|\right], \quad \mathrm{E}\left[|Q|^{1+\beta}\right], \quad \mathrm{E}\left[|Q|^{2\beta}\right].$$

Now can solve six equations for six difficult variables in terms of six observable or invariant ones.

Solution with Liquidity L

Define observable "illiquidity" $1/L$ as volume-weighted expected price-impact cost of a bet:

$$\frac{1}{L} := \frac{C}{\mathrm{E}\left[|P \cdot Q|\right]} = \left(\frac{\sigma^2 \cdot C}{m^2 \cdot P \cdot V}\right)^{1/3}.$$

L is defined in terms of observable variables P, V, σ, and invariant parameters C, m.
Easy closed-form solution for equation system in terms of L:

$$\mathrm{E}\left[P \cdot |Q|\right] = C \cdot L, \tag{2}$$

$$\gamma = \frac{1}{m^2} \cdot \sigma^2 \cdot L^2, \tag{3}$$

$$\lambda = \left(\frac{1}{L}\right)^{1+\beta} \cdot C^{-\beta} m_\beta \cdot P^{\beta+1} = m_\beta \frac{P}{L}\left(\frac{P}{CL}\right)^{\beta}, \tag{4}$$

$$\mathrm{E}\left[\Delta P^2\right] = m^2 \cdot \frac{P^2}{L^2}. \tag{5}$$

Plus two more less interesting equations for two moment ratios.
There is one difficult variable on left side of each equation.
Therefore all variables on right side are either observable or invariant.

Bet size and trading costs

Then expected bet size and number of bets are given by

$$\mathrm{E}\left[|P \cdot Q|\right] = C \cdot L, \qquad \gamma = \frac{1}{m^2} \cdot \sigma^2 \cdot L^2.$$

Price impact is

$$\frac{\Delta P}{P} = \frac{1}{L} \cdot m_\beta \cdot |Z|^\beta, \qquad \text{where} \qquad Z := \frac{Q}{\mathrm{E}\left[|Q|\right]} = \frac{P \cdot Q}{C \cdot L},$$

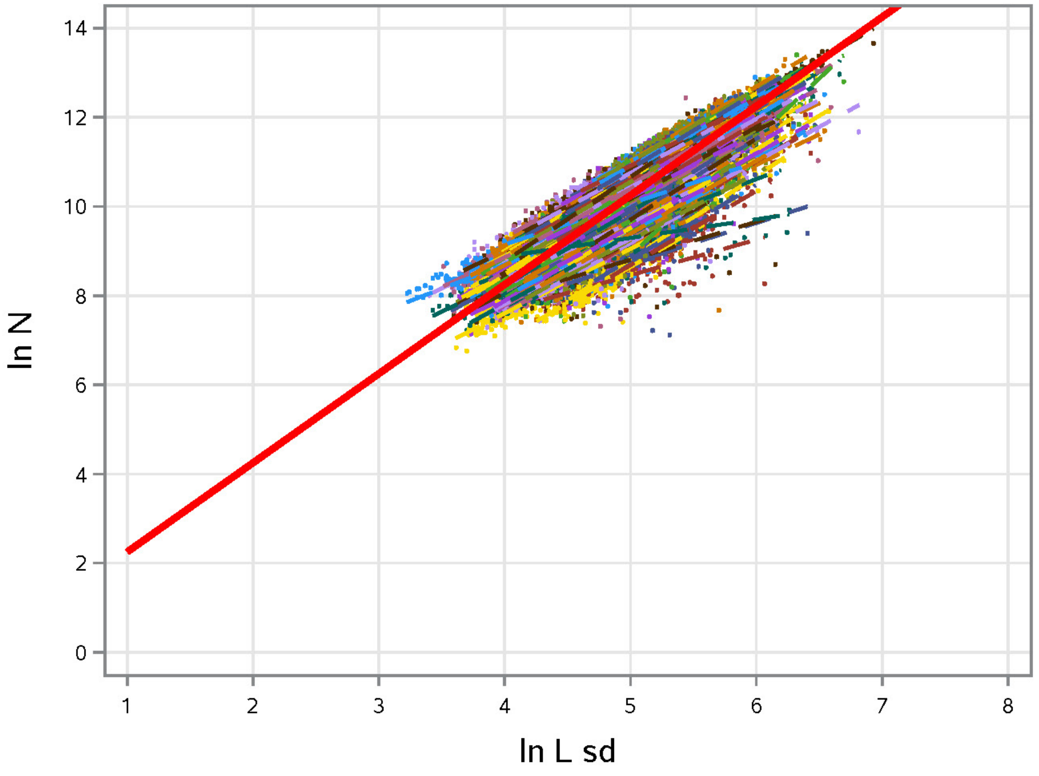
Market microstructure invariance is the hypothesis that C, m, and m_β are invariant across assets. This hypothesis implies a universal market impact formula and universal formula for size and number of bets, which requires estimation of only these three parameters and β!

(Preliminary) Calibration: $C = \$2,000$; if $\beta = 1$, then $m \approx 0.25$ and $m_\beta = m^2$. If $\beta = 1/2$, then $m_\beta = m \approx 0.40$.

Are invariant parameters really empirically invariant?

Our previous research shows that the “invariant” parameters are indeed relatively constant across assets.

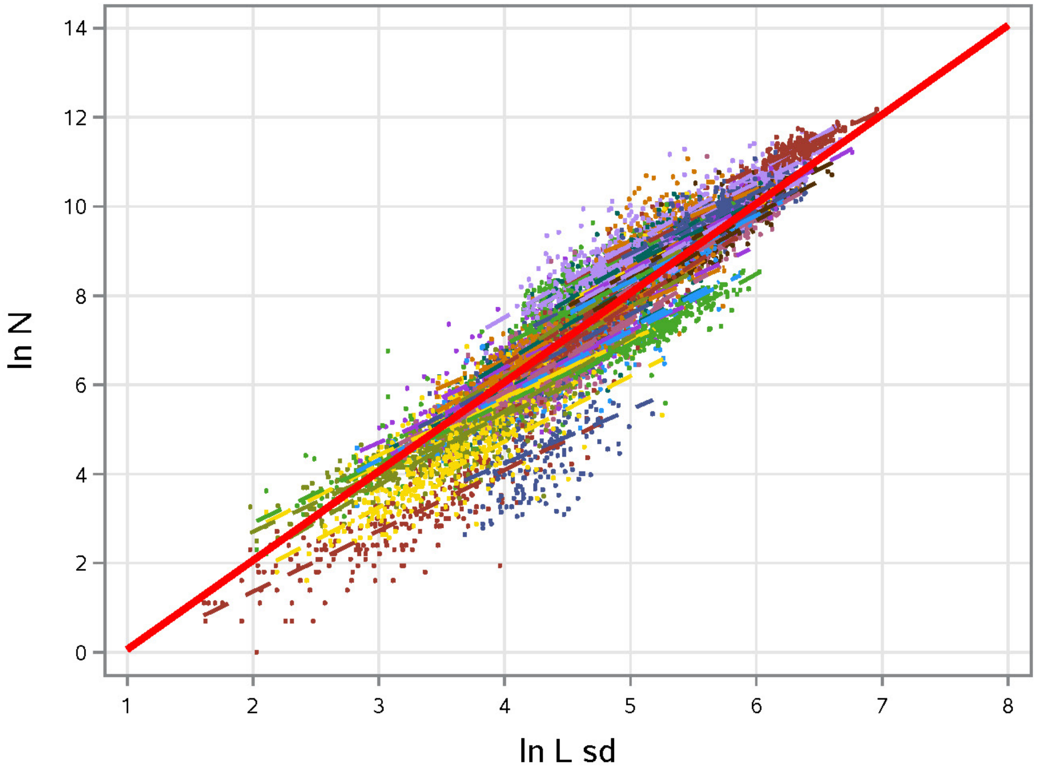
Number of Trades $\ln N$ vs $\ln(L\sigma)$: U.S. market



One-minute data from the Trades and Quotes (TAQ) dataset for January–December 2015 for 500 U.S. stocks in the S&P 500 index. The slope is close to 2, as predicted.

$$\log(N) = 1.005 + 1.842 \cdot \log(\sigma \cdot L)$$

Number of Trades $\ln N$ vs $\ln(L\sigma)$: Russian market

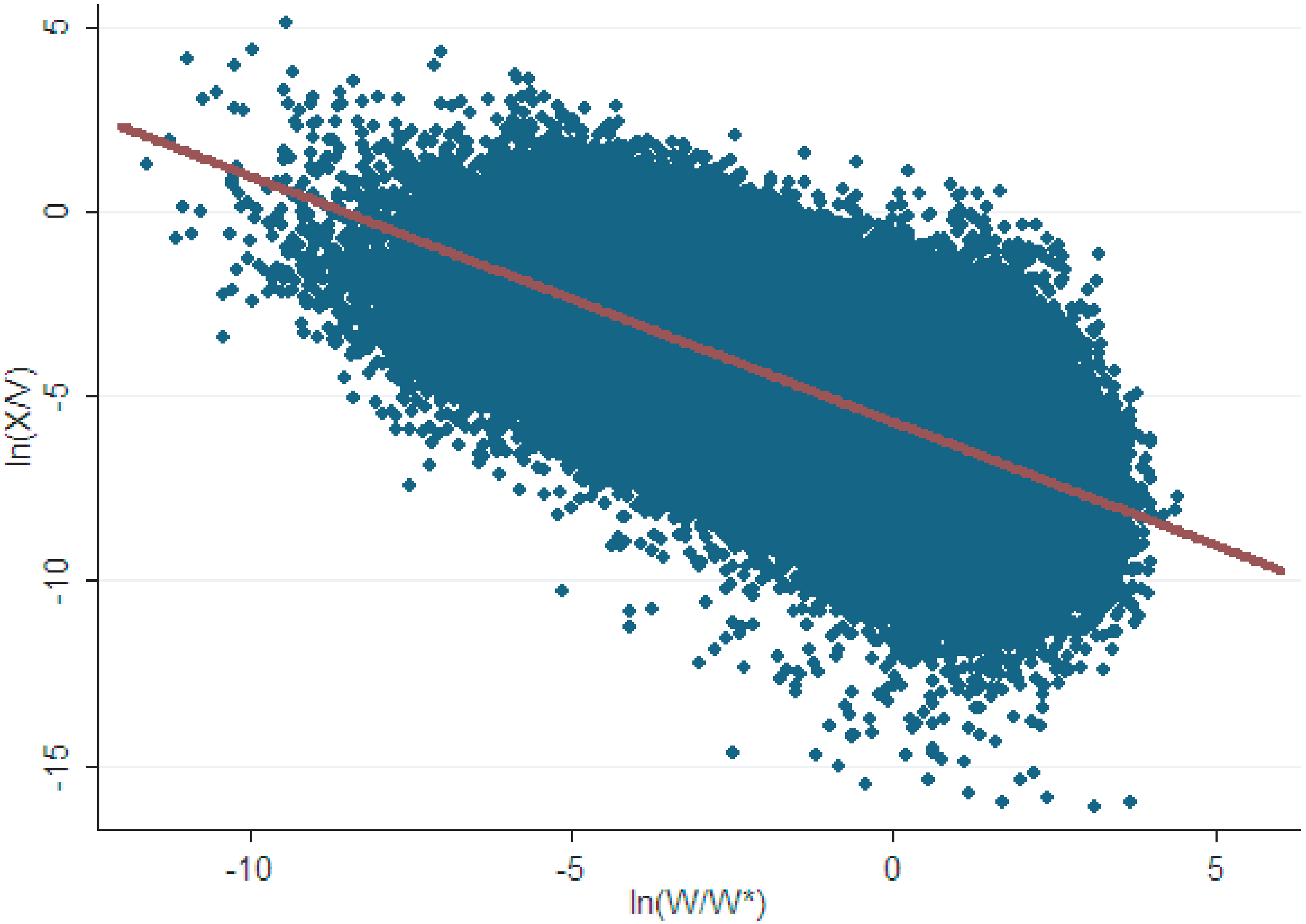


One-minute data from the Moscow Exchange for January–December 2015 for 50 Russian stocks in the RTS index. The slope is close to 2, as predicted.

$$\log(N) = -3.085 + 2.239 \cdot \log(\sigma \cdot L)$$

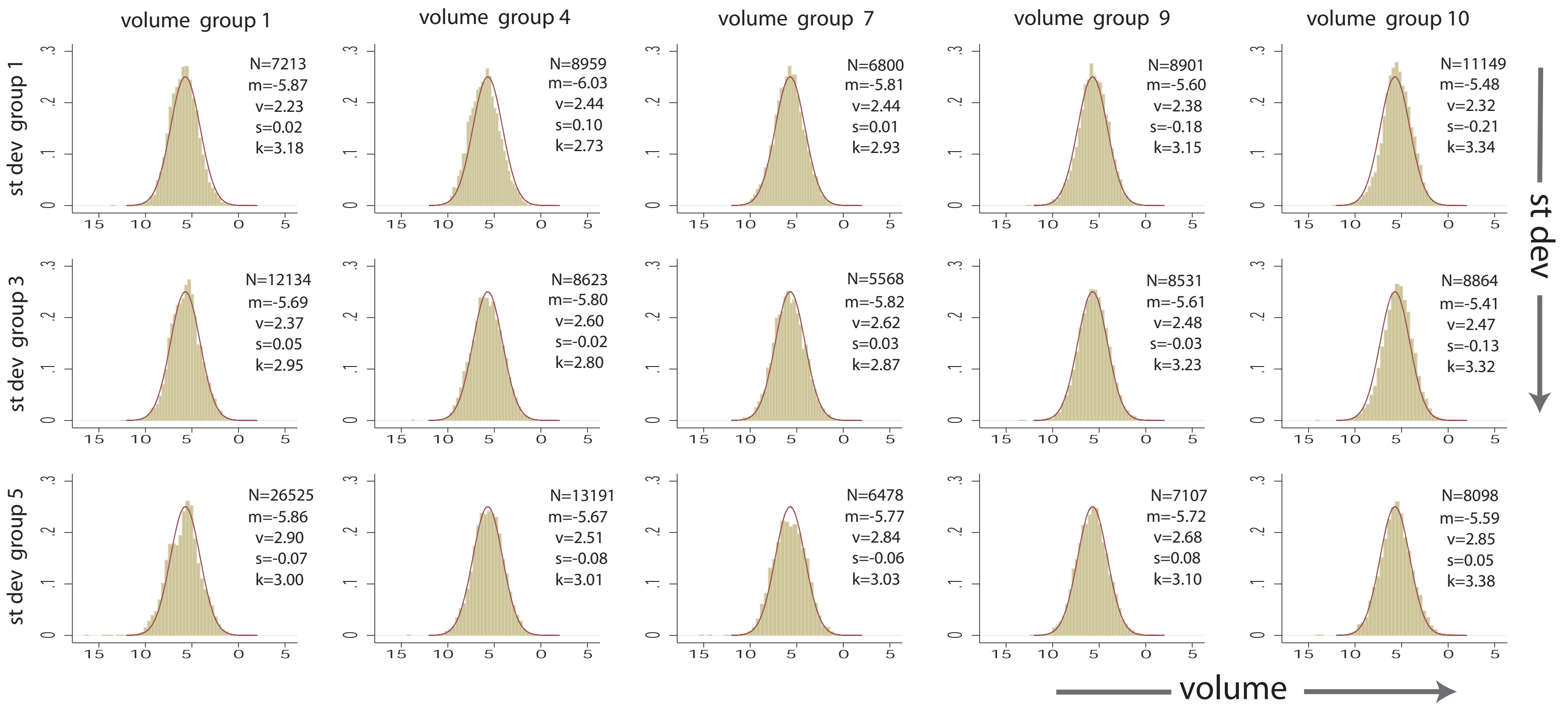
Portfolio Transition Order Size

Figure from Kyle and Obizhaeva (2016*a*)



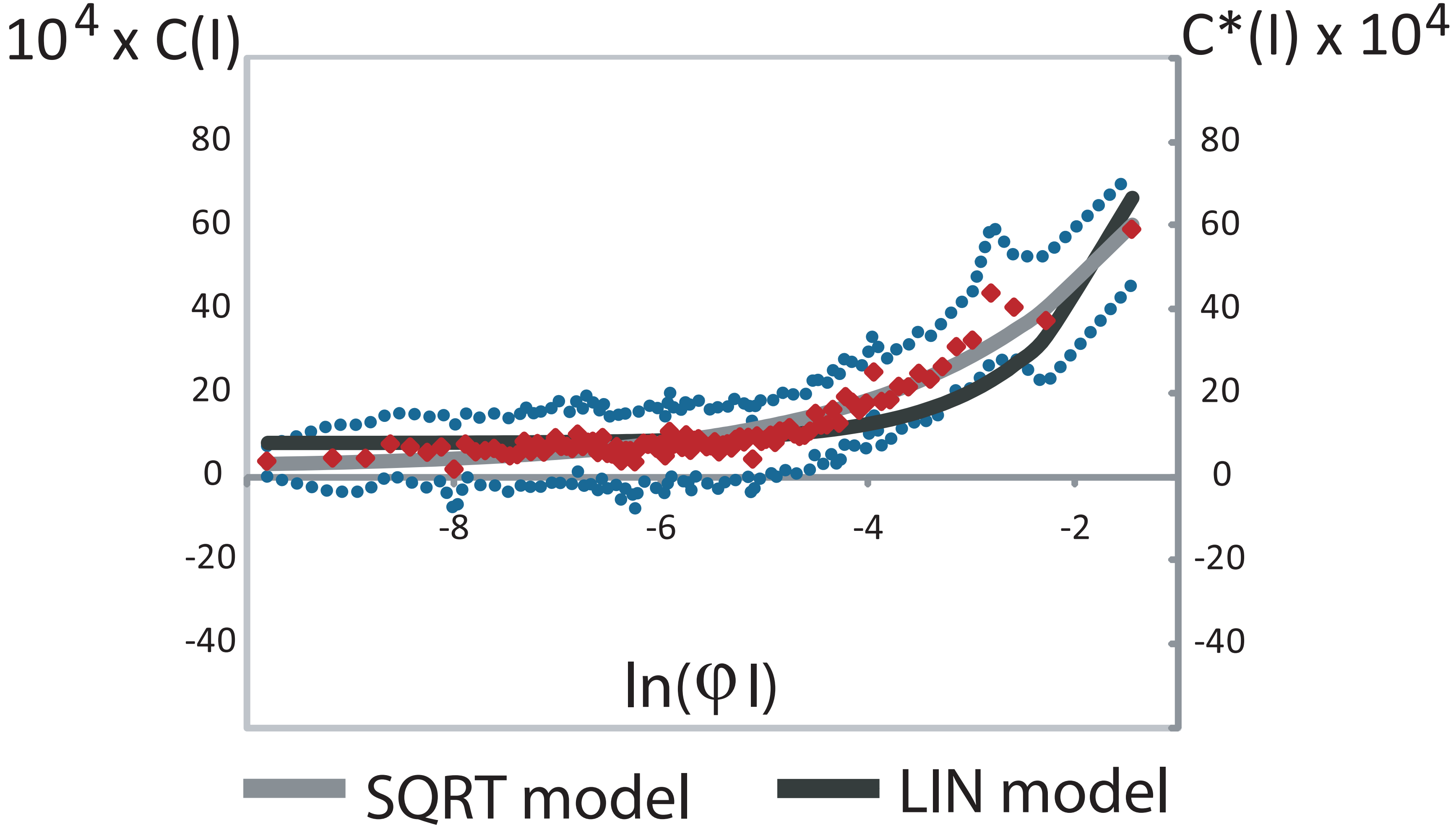
Distribution of Portfolio Transitions Orders

Figure from Kyle and Obizhaeva (2016*a*)



Square Root Model in Portfolio Transition Orders

Figure from Kyle and Obizhaeva (2016*a*)



Where is Adverse Selection?

- Meta-model derives an empirical formula for liquidity L without an underlying model of adverse selection.
  - This is consistent with mechanical aspects of trading experienced in markets, where information is invisible.
- We need theories based in economics to link meta-model to adverse selection.
- This enables further link to pricing accuracy, probability of informed trading, and precision of signals.

We next show that the meta-model and dimensional approach are consistent with the 1985 paper.

1985 paper: Implied Meta-Model

Equilibrium maps into a linear version of the meta-model ($\beta = 1$) with two bets, $Q = Q_I$ and $Q = Q_U$:

$$V = \gamma \cdot \mathrm{E}\left[|Q|\right] \qquad \text{(Definition of volume)}$$

$$\sigma^2 = \gamma \cdot \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] \qquad \text{(Bets generate all volatility),}$$

$$\mathrm{E}\left[\Delta P^2\right] = \lambda^2 \cdot \mathrm{E}\left[Q^2\right] \qquad \text{(Market impact of one bet),}$$

$$C = \lambda \cdot \mathrm{E}\left[Q^2\right] \qquad \text{(Dollar impact cost of a bet.)}$$

One-Period 1985 Model: Invariance

Theoretical parameters map empirically to L as

$$\sigma_F := \frac{2 \cdot m \cdot P}{L} = \frac{2 \cdot P\sigma}{\sqrt{\gamma}}, \qquad \text{and} \qquad \sigma_U := \frac{C \cdot L}{m \cdot P} = \frac{\mathrm{E}\left[|Q|\right]}{m}.$$

Solution to model implies

$$\text{Dollar Trading Profit} = \mathrm{E}\left[(F - p(Q_I, Q_U))Q_I\right] = \frac{\sigma_F \cdot \sigma_U}{2} = C$$

Market microstructure invariance is consistent with assumption that informed trader pays C for a private signal and breaks even.

Model operates over different horizons for securities with different liquidity (in this case measured by $\gamma = \sigma^2 \cdot L^2 / m^2$).

Meta-model and Adverse Selection

- The meta-model describes how orders affect prices, but does not relate prices to information or adverse selection. Adverse selection can be added to meta-model in a manner consistent with economic models.
- Liquidity L can be linked through information flow to pricing accuracy and resiliency.
- Dynamic model leads to rich results about informativeness and resiliency of prices.

Pricing Accuracy

Define error variance of prices like Black (1986):

$$\Sigma = \operatorname{var}\left[\log\left(\frac{F}{P}\right)\right] \approx \operatorname{var}\left[\frac{F-P}{P}\right].$$

Private signals have invariant precision τ, which, from perspective of informed trader, reduces error variance by fraction τ from Σ to $(1-\tau)\cdot\Sigma.$ Then each bet reduces price variance only by fraction $\theta^2\cdot\tau$ because market cannot distinguish informed bets from noise.

$$\frac{\sigma^2}{\gamma} := \mathrm{E}\left[\left(\frac{\Delta P}{P}\right)^2\right] = \frac{\lambda^2\cdot \mathrm{E}\left[Q^2\right]}{P^2} = \theta^2\cdot\tau\cdot\Sigma.$$

Pricing accuracy can be inferred from market liquidity L:

$$\Sigma = \frac{\sigma^2}{\theta^2\cdot\tau\gamma} = \frac{m^2}{\theta^2\tau\cdot L^2}.$$

These formulas are consistent with 1985 model.

Market Resiliency

Define “market resiliency” ρ as rate at which prices converge to changing fundamental value; also equal to rate at which noise shocks die out from prices:

$$\rho = \theta^2 \cdot \tau \cdot \gamma.$$

Resiliency ρ is function of invariance parameters $\frac{\theta^2 \cdot \tau}{m^2}$ and observable volatility σ and liquidity L:

$$\rho = \frac{\sigma^2}{\Sigma} = \frac{\theta^2 \tau}{m^2} \cdot \sigma^2 \cdot L^2.$$

By introducing the invariant quantity $\theta^2 \tau$ measuring the precision of a bet, we have mapped the meta-model into the concept of adverse selection.

We have obtained hard-to-measure values of pricing error variance Σ and market resiliency ρ as simple functions of easy-to-measure parameters and other invariant quantities.

Trading Liquidity, Funding Liquidity, and Time

Liquidity L is related to the rate at which bets arrive γ (business time):

$$L = \left(\frac{m^2 \cdot P \cdot V}{C \cdot \sigma^2} \right)^{1/3} = \frac{\gamma^{1/2} \cdot m}{\sigma}.$$

Riskiness of large bets in risky assets depends on horizon over which liquidation takes place:

- Margins and Repo Haircuts: Liquidation of defaulted collateral takes time.
- Bank Capital: Bank assets extremely illiquid. Selling bank assets is impractical. Raising new capital takes time.
- Bank Equity Issuance: Issuing equity takes time.
- Government Securities: Can be sold quickly.
- Stock Market Crashes: Can be caused by large sales over short periods of time. See Kyle and Obizhaeva (2016*b*) on stock market crashes.

This implies that margins, repo haircuts, and bank capital should be proportional to $1/L$, taking into account time dimension of liquidity of underlying assets.

Smooth Trading: The Model

A theoretical model of continuous-time informed trading among symmetrically informed traders.

- Symmetrical setup with one type of trader.
- Traders have a continuous flow of new private information.
- Traders are relatively overconfident: All traders inconsistently believe their private information is more informative than other traders believe it to be. Otherwise, traders are fully rational.
- All traders attempt to reduce price impact costs by strategically trading smoothly trading over time.
- Traders make speculative trades which are expected to be profitable but actually break even.
- Risk aversion makes traders reduce inventories when signals offer no profit opportunity.
- Traders submit continuously changing “flow-demand” schedules to trade at some rate (say one share per second) as a function of price.
  - Like model ofKyle (1989) with continuous trading.

Smooth Trading: Results

- Trading volume collapses to zero unless traders are “overconfident enough”.
- With enough overconfidence, there is an equilibrium with linear strategies, in which traders provide liquidity to one another symmetrically.
- The price averages together traders signals; trading occurs because of disagreement.
- From each trader’s perspective, there is both permanent and temporary price impact. “Permananet price impact” means the price is a function of a trader’s invenory (which is a result of past trading). “Temporary price impact” means that the price is also a function of the trader’s current rate of buying or selling (time derivative of inventory).
- Traders trade gradually towards a “target inventory”.
- Since information decays as a result of other traders receiving new information, urgency of trading trades off losses from the decay rate of signals against gains from slowing down trading to reduce temporary price impact.
- Traders believe that their temporary price impact incorporates about half of their current signal into prices. Due to overconfidence, all of the signal is incorporated into prices.
- Compared with 1985 model, smooth trading model is complicated (300 equations).

Comparison of smooth trading with 1985 model

- Symmetrical smooth trading model mimics today's markets by letting all traders trade on information, demand liquidity to reduce inventories, and profit from providing liquidity to other traders.
- Impatience with which noise traders demand liquidity in 1985 model is replaced by more patient traders who smooth inventory changes out over time.
- Trading volume is a flow (not a diffusion), which implies trading volume is finite and well-defined.

“Flow Trading”

How to implement smooth trading with a practical set of institutions, given that traders want to trade both single assets and portfolios of assets?

1985 paper has “batch auctions”.

- To implement smooth trading, need batch auctions for flows. This implies persistent orders which trade patiently over time, such as one share per second for about 3 hours to trade 10000 shares.
- To implement trading in portfolios, need to allow traders to submit orders to trade flows of portfolios (linear combinations of assets) as a function of the portfolio’s price.
- Allowing prices and quantities to be continuous instead of discrete (pennies and round lots of 100 shares) makes market clearing prices easier to find.
- Piecewise linear demand schedules are computationally tractable.
- To approximate continuous trading, need frequent batch auctions, such as once per second.

Traders want to trade smoothly and want to trade portfolios

- Brokers “work orders”. Institutional investors build positions over time.
- VWAP algoritms try to implement smooth trading.
- Mutual funds and ETFs allow trade in portfolios. Markets sometimes trade lists or bundles of assets, such as stocks or bonds.

Customers incur costs associated with discreteness in today’s markets:

- High frequency traders pick off stale bids and offers and race for time priority.
- Traders need to place and cancel many orders to execute a large trade.
- Long-short arbitrage strategies require placing and canceling many trades.

Results

- Since demand schedules are locally linear, there is no bid ask spread (but there is market impact).
- Market clearing prices are result of solving a quadratic program based on imputing a utility function to orders and maximizing utility.
- Can find prices for 1000 assets with 1 million orders (outright, spread, portfolios) in about one second on typical workstation.
- Flow trading simplifies implementation of strategies to trade individual assets of portfolios.
- Flow trading should undermine ability of high frequency traders to profit from speed but facilitates market market-making or arbitrage strategies which profit from carrying inventories through time.

Conclusion

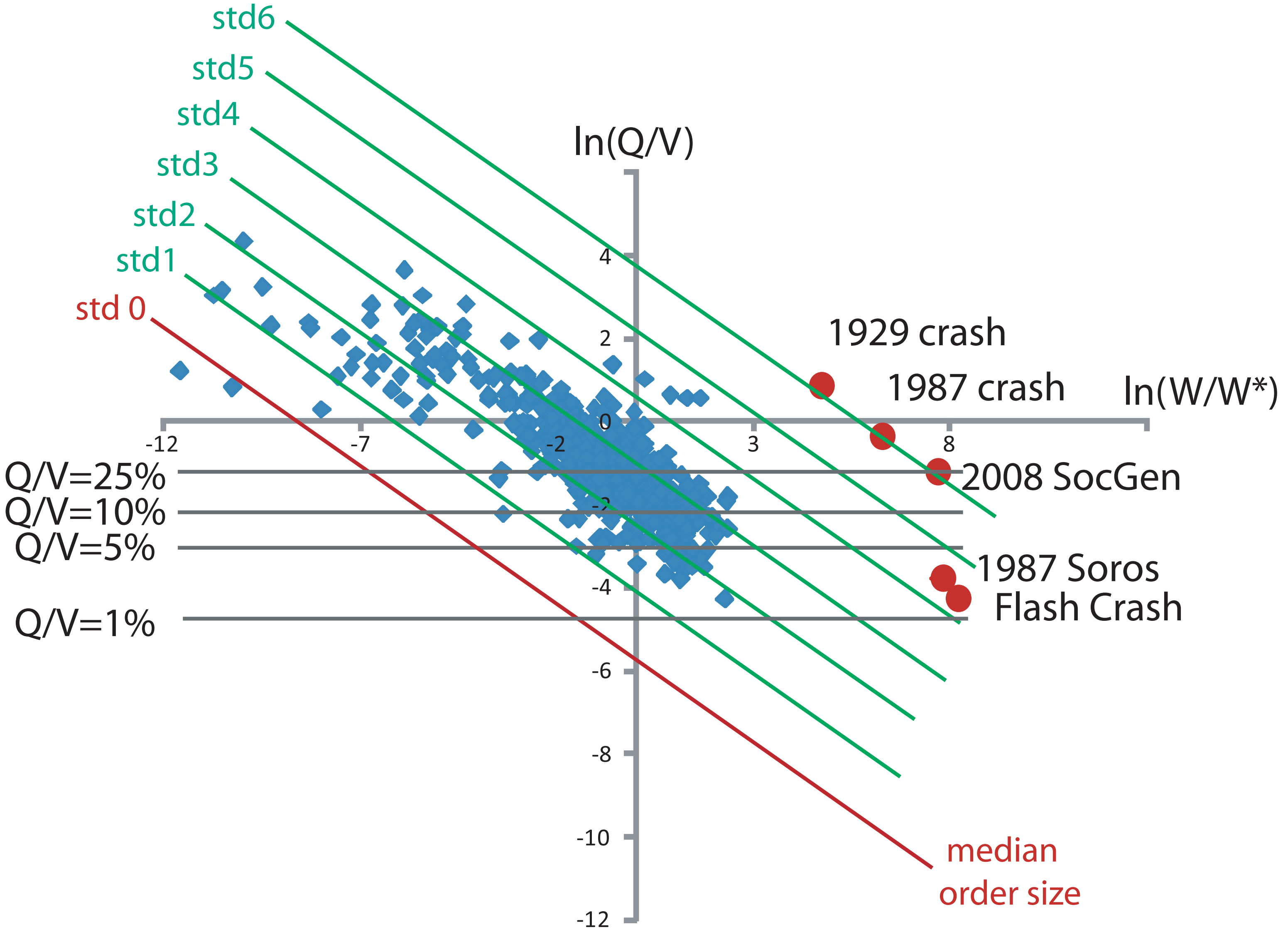
- Can model symmetric provision of liquidity with smooth trading model and implement with flow trading.
- Smooth trading equilibrium offers liquidity over time and makes temporary price impact important.
- Smooth trading model has a well-defined concept of trading volume, and invariance model connects volume with liquidity.
- Invariance model consistent with 1985 model when “business time” is measured in bets.
- Smooth trading model allows all traders to demand and supply liquidity. Flow trading mechanism implements this with persistent orders.
- Flow trading mechanism implements frequent “batch auctions” in portfolios.

APPENDIX: More Empirical Evidence

Working papers can be found on SSRN.

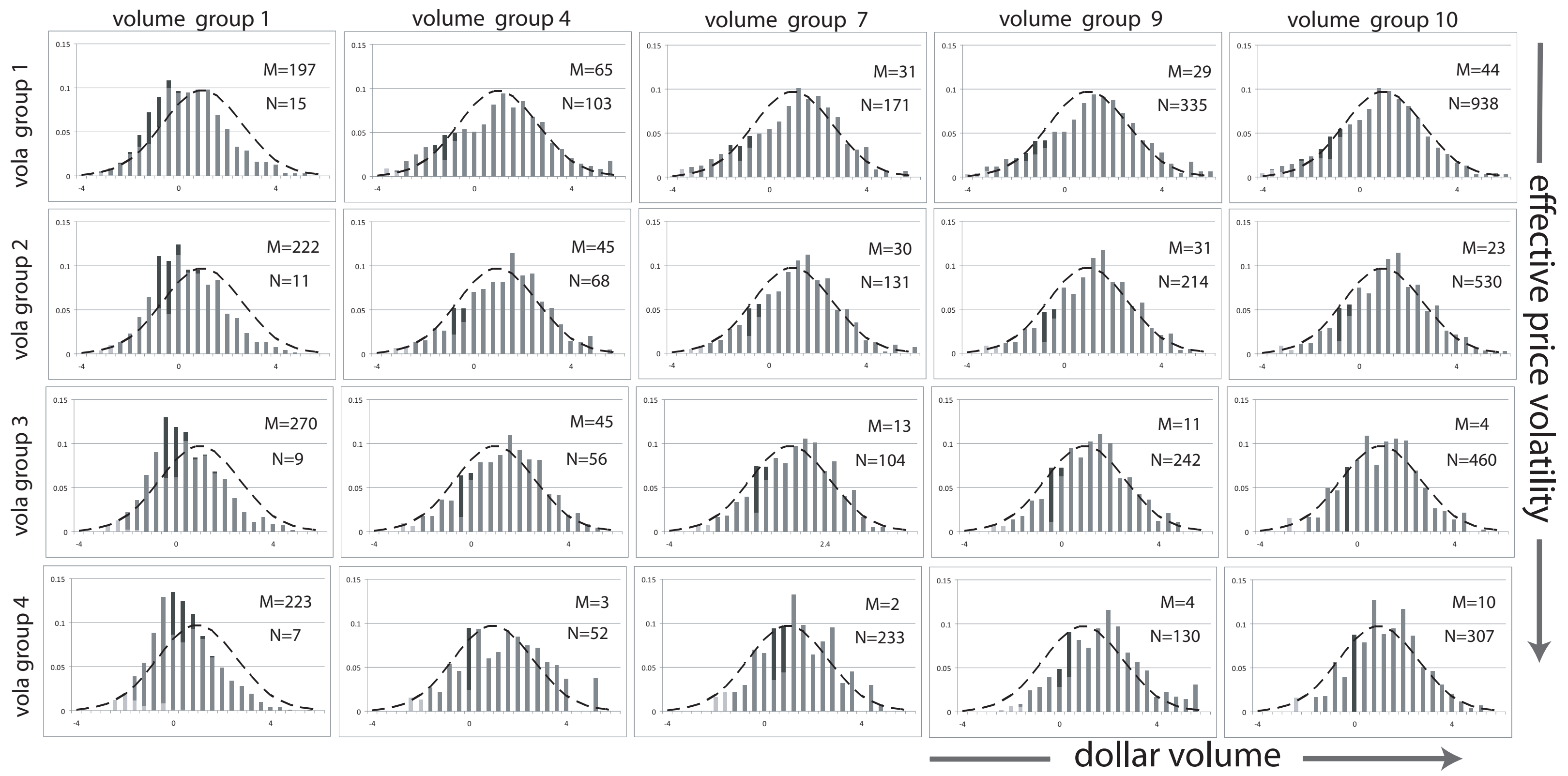
Large Bets and Stock Market Crashes

From Kyle and Obizhaeva (2016*b*): “Large Bets and Stock Market Crashes”



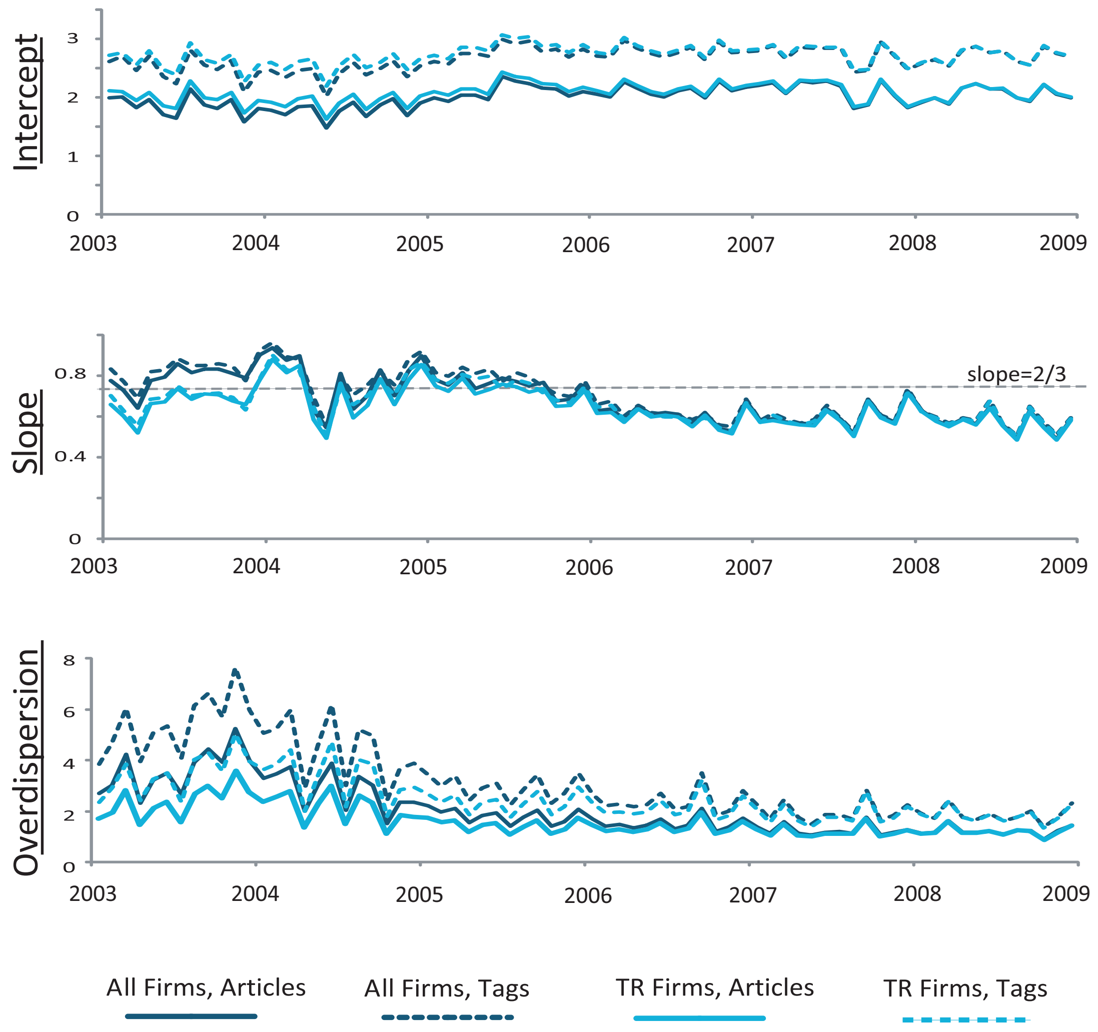
Volume-Weighted Distribution of NASDAQ Trades (1993)

Figure from Kyle, Obizhaeva and Tuzun (2017)



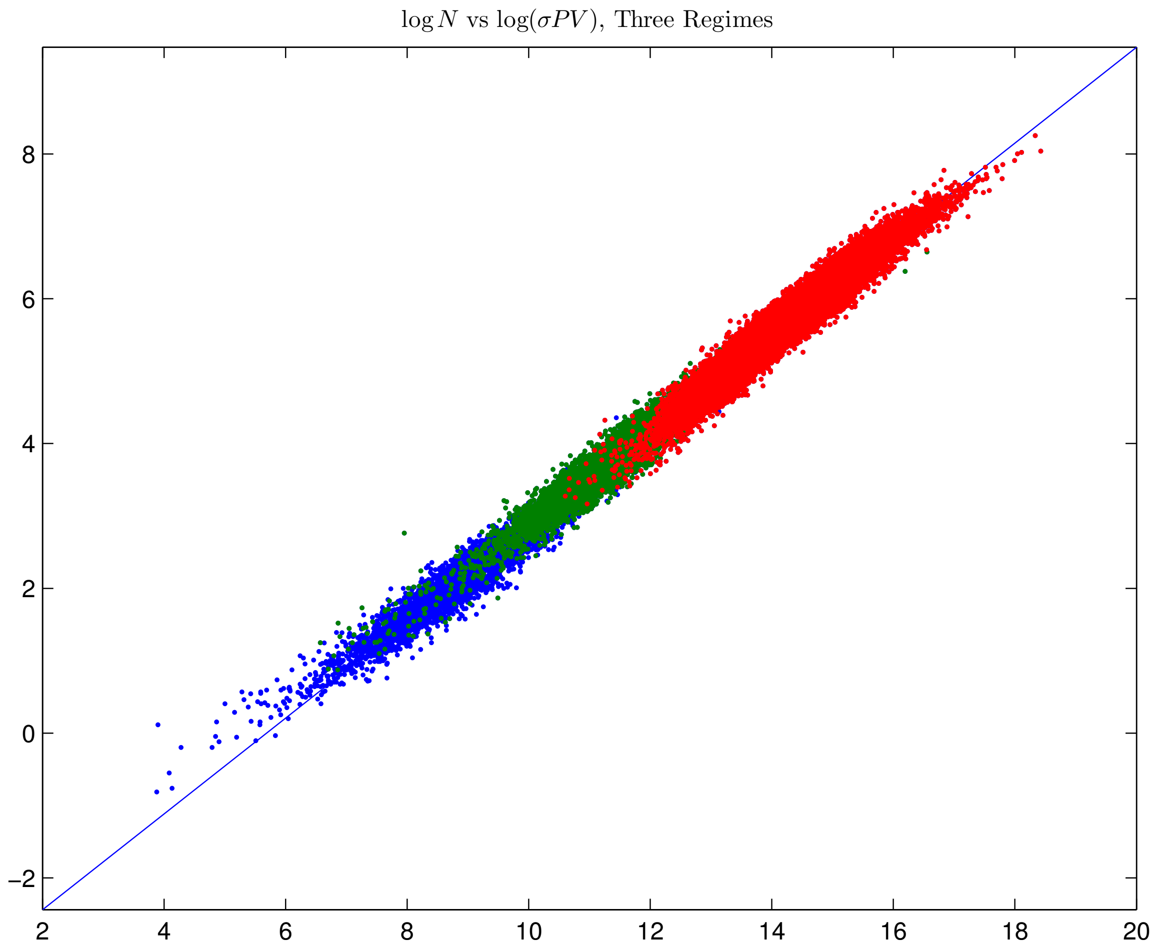
Reuters News Articles

Figure from Kyle et al. (2010)



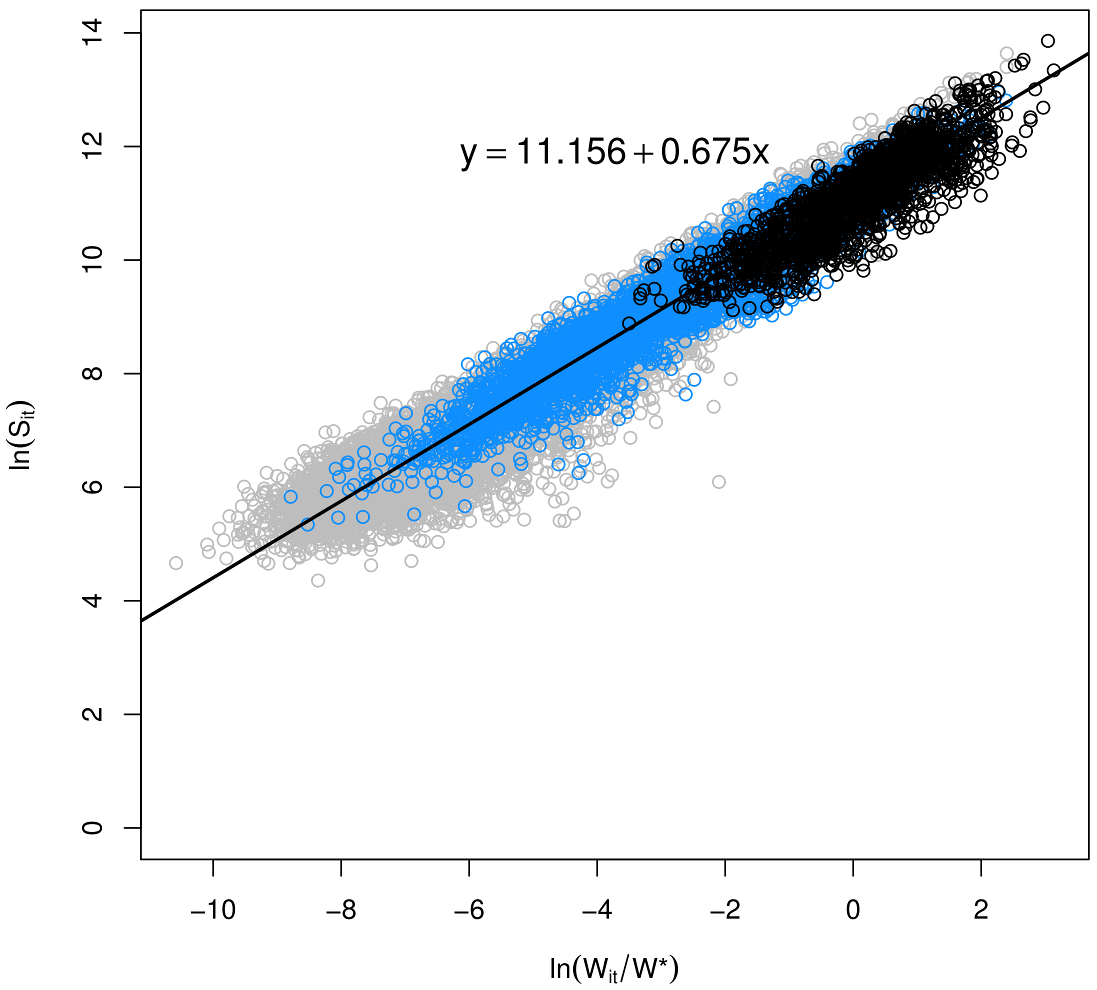
Trade Size in S&P 500 E-mini Futures Contracts

Figure from Andersen et al. (2015)



Switching Points on the Korea Exchange

Figure from Bae et al. (2016)



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