Insurance against Long-Run Volatility Risk: 
Demand, Supply, and Pricing

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Abstract

Despite its importance in many asset pricing and macroeconomic models, insurance against long-run volatility risk has received little empirical documentation regarding its demand, supply, or pricing. This paper bridges the gap. First, I show that households have directly purchased large quantities of insurance against long-run volatility risk through the minimum return guarantees available in variable annuities, a form of retail retirement and savings products offered by life insurance companies. Total net assets with such insurance grew from zero when the guarantees were first introduced in the early 2000s to over a trillion USD in 2015. To supply the guarantees to households, life insurance companies hedged themselves with significant amount of long-maturity options and variance swaps, which led to a sharp increase in the prices of these long-term volatility assets: the ten-year implied volatility increased from 20.0% before 2000 to 27.3% afterwards while the short-term counterpart (VIX) has remained unchanged (from 19.2% to 19.7%). Insurers’ hedging activities therefore explain the significant steepening of volatility term structure documented in the asset pricing literature such as Drechsler et al (2019). Relatively to the price they paid on hedging, however, the price that life insurance companies charged on the guarantees was much lower, at 15.0% in terms of implied volatility prior to the Great Recession. I show evidence of extensive product-cross subsidization – under-pricing of guarantees was used to drive sales and lock in customers that would be subject to over-priced base fees and mutual fund expenses.

Keywords — asset pricing, fund flow, long-run volatility risk, minimum return guarantee, put option, variable annuity

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1 Introduction

Do people care about long-run volatility risk? This is a fundamental question both for the pricing of long-lived assets (e.g., equities) and for the modelling of forward-looking agents in macroeconomic theories (e.g., firms’ investment behavior). If people care about long-run volatility risk, then there should be a large demand for insurance against the risk, and/or the price of the insurance should be high. However, there is limited empirical evidence on either front. There has been no documentation of real demand for long-run volatility insurance besides the illiquid and thinly-traded long-maturity derivatives market, and there are very few and contradictory empirical studies that analyze the price of long-run volatility risk.¹

This paper shows that there is actually a large retail market on insurance against long-run volatility risk, namely the minimum return guarantees offered in long-term variable annuities. Variable annuity is a retail retirement and savings product offered by life insurance companies where the policyholder can invest in a selected group of mutual funds with tax deferral benefits.² Beginning in the early 2000s, insurers began offering optional minimum return guarantees on the underlying mutual funds. These guarantees promise a floor on the investment return subject to significant restrictions on short-term liquidity, making them effectively long-dated in-the-money put options. Just like options, minimum return guarantees provide insurance against volatility risk, and, given the long horizon of variable annuities (on average over 15 years), they provide insurance against long-run volatility risk.

I address the following questions by studying the minimum return guarantees: How big is the demand for insurance against long-run volatility risk? Who supplies the insurance and ultimately bears the risk? What is the price of the insurance, and how is it affected by intermediation?

Households’ demand is large for the insurance. The variable annuity market has expanded substantially after the guarantees were introduced, with total net assets doubling from less than $1 billion in 2000 to more than $2 billion in 2015, accounting for 39% of life insurers’ liabilities and equaling to 21% of the open-end mutual fund industry. Section 5 shows that most of the aggregate sales can be attributed to the minimum return guarantees and Section 7 uses micro-level data and establishes a causal link between minimum return guarantees and higher sales.

To supply the minimum return guarantees to households, life insurance companies hedged themselves with large amount of long-term volatility derivatives including options and variance swaps. I gather data on insurers’ exposure to minimum return guarantees and their derivative trades using regulatory reports. Per thousand dollar of exposure to minimum return guarantees they wrote, insurers purchased $0.71 vega of S&P options and $0.54 vega notional of variance swaps with maturity longer than five years. These trades are all over-the-counter with investment banks as the counterparties. Therefore, the long-run volatility risk written in minimum return guarantees was born by life insurance companies together with banks.

Insurers’ hedging activities had a large impact on the prices of those long-term volatility assets. The purchases of long-maturity options and variance swaps (i.e., those with maturity longer than five years) by insurers were so large that they capture on average more than 20% of those long-term asset markets. Exactly when insurers started offering minimum return guarantees

²Section 3 provides more background on variable annuity, variable annuity mutual funds, and minimum return guarantees.
and started making large purchases of long-term volatility assets in the beginning of 2000s, a significant gap emerged between the price of long-run volatility and that of short-run volatility. The ten-year implied volatility (or variance swap rate) increased from 20.0% before 2000 to 27.3% afterwards, while the short-term counterpart (VIX) has remained unchanged. In other words, there was a significant steepening of the volatility term structure. Using simple OLS regression, I show that an average-sized monthly trade on long-term options and variance swaps by the life insurance industry, at around $3 million vega, was associated with 100 basis-point increase in the price of long-run volatility, measured by implied volatility or variance swap rate. Drechsler et al (2019) first document the structural break in the dynamics of volatility term structure in the beginning of 2000s, and this paper provides the explanation – the emergence of minimum return guarantees and hedging by life insurance companies.

I provide a partial equilibrium framework to rationalize the simultaneous increase in quantity and price of long-run volatility assets. There is a intermediary sector with an upward-sloping supply schedule of the asset. A positive exogenous demand shock leads to a movement along the supply curve to a new equilibrium, resulting in both higher quantity and higher price.

Insurers’ hedging of minimum return guarantees and the impact on asset prices provide novel insights to theories of asset pricing and financial intermediation. Insurers complete a missing part of the market by providing households with long-run volatility insurance through minimum return guarantees. In completing the market, insurers establish an intermediation between the households and the long-term volatility derivatives market by hedging the minimum return guarantees they wrote with long-maturity options and variance swaps. The intermediation modifies the relevant pricing kernel for those long-term volatility assets, which now reflects the large demand from households. In this new equilibrium with complete market, the price of long-run volatility insurance is high, which is consistent with the importance of long-run volatility risk underlying many consumption-based asset pricing models.

The final part of the paper circles back to the household market and points out the stark price difference between the household market and the derivatives market. Despite the high price that they paid in the derivatives market (on long-maturity options and variance swaps), life insurance companies charged a much lower price on the minimum return guarantees they sold to households until the financial crisis. While implied volatility of long-term options and variance swaps was on average 25%, implied volatility on the guarantees was only 15%. The implied volatility of guarantees is derived under an option pricing framework and as that which equates present value of the fees and that of the payoffs.

Why were there different prices on long-run volatility insurance? Why were the more sophisticated insurers paying the higher price in the derivatives market while under-charging the less sophisticated households in the retail market? I show extensive evidence of cross-product subsidization: under-pricing of guarantees was used to induce flows to affiliated mutual funds and also lock in customers that would be subject to over-priced base fees and mutual fund expenses. By comparing the base fees and the mutual fund expenses of variable annuities that offer guarantees versus those that do not, I show that the former is much higher especially after the introduction of guarantees. Under a difference-in-difference framework, I further show that insurers garnered higher sales and higher flows to affiliated mutual funds by offering minimum return guarantees in variable annuities.
1.1 Related Literature

This paper is related to the recent debate on the price of long-run volatility risk and the term structure of volatility. On one hand, many asset pricing and macroeconomic theories assume that agents would pay a positive price to hedge against shocks to uncertainty, which would in turn generate an upward sloping term structure of volatility. For example, under Epstein–Zin preferences, periods of high expected future consumption volatility are periods of low lifetime utility and hence periods of high marginal utility, and investors would therefore desire to hedge news about future consumption volatility. The asset pricing models include Bansal and Yaron (2004), Drechsler and Yaron (2011), Drechsler (2013), Wachter (2013), Bansal, Kiku, Shaliastovich, and Yaron (2014), Campbell, Giglio, Polk, and Turley (2018), Seo and Wachter (2018). The macroeconomic models include Bloom (2009), Fernandez-Villaverde, Guerron, Rubio-Ramirez, and Uribe (2011), Gourio (2012), Gourio (2013), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014), and Christiano, Motto, and Rostagno (2014).

On the other hand, recent empirical papers including Dew-Becker, Gigliob, Le, and Rodriguez (2017) and van Binsbergen and Koijen (2017) suggest the opposite. Dew-Becker et al (2017) use data on variance swaps and long-term options and show that the term structure of forward variance prices is flat on average beyond the one-quarter horizon. They find that Sharpe ratios of claims on long-term forward variance are zero or even weakly positive, meaning that investors do not hedge or even pay to get exposed to volatility shocks. Van Binsbergen and Koijen (2017) find similar results using straddles. Drechsler, Drechsler, Kilic, and Yaron (2019) provide counter-arguments to the above. They argue that Sharpe ratios on long-term forward variance claims should be measured by the corresponding long-horizon return and volatility. In addition, they suggest that there was a structural break in the term structure around 2003, after which the term structure became significantly more upward sloping. Other empirical papers on the price of volatility risk include Andersen and Bondarenko (2007, Carr and Wu (2009), Egloff, Leippold, and Wu (2010), Bollerslev and Todorov (2011), Ait-Sahalia, Karaman, and Mancini (2014), and Segal, Shaliastovich, and Yaron (2015).

This paper contributes to this debate by documenting the institutional details behind the dynamics of the long-run volatility market. I show that there is a large and real demand for insurance against long-run volatility risk directly from households, through the minimum return guarantees in variable annuities. I also show that the emergence of minimum return guarantees is the reason behind the structural break identified in Drechsler et al (2019).

This paper also contributes to the growing preferred habitat literature that documents the large impact of institutional investors such as pensions and insurers on asset prices. Greenwood and Vayanos (2010), Domanski, Shin, and Sushko (2017) and Klingler and Sundaresan (2018) focus on the long-term interest rate market, and show that demand for duration hedging by long-term asset managers such as pension funds and insurers can affect long-term interest rate and swap rate. In contrast, this paper focuses on the long-term volatility market, and shows that insurers’ hedging of minimum return guarantees led to increase of long-run volatility price and a significant steepening of the volatility term structure.

This paper also adds to the study of minimum return guarantees in variable annuities. Earlier literature, including Milevsky and Posner (2001) and Milevsky and Salisbury (2005), focuses on the actuarial pricing of guarantees, and show that base fees for death benefits are over-priced while fees for optional life benefits are under-priced. This paper shows similar results using more granular data and also using comparisons between variable annuities without the optional life benefits. Koijen and Yogo (2018) develops an I/O type equilibrium model on
the demand and supply of minimum return guarantees and studies the implication of financial frictions and market power. In contrast, I document the size and growth of the demand for the guarantees, analyze insurers’ hedging behavior associated with supplying the guarantees, examines the asset pricing implication of insurers’ hedging activities, and study the pricing of guarantees in an risk-neutral option pricing framework. Sen (2019) also studies the hedging of variable annuity guarantees. Whereas she focuses on interest rate rate and the first moment of equity risk, I focus on the long-run volatility risk, which is a second moment.

This paper also relates to studies of mutual funds and financial advisors. Related literature includes Berk and van Binsbergen (2015), Pastor, Stambaugh, and Taylor (2015), and Massa and Yadav (2016).

This paper is broadly related to household finance, in particular the pricing of long-term financial products such as life insurance and retirement savings (TBD). Gottlieb and Smetters (2016) show that insurers sell front-loaded policies, make money on lapsers, and lose money on non-lapsers.

1.2 Roadmap

The rest of the paper is organized as follows. Section 2 provides background on variable annuities and minimum return guarantees offered and describes the various datasets I use. Section 3 documents the large demand for insurance against long-run volatility risk and insurers’ supply of the insurance by hedging themselves using long-term derivative assets on volatility. Section 4 studies the impact of insurers’ trading on asset prices, specifically the term structure of volatility. Section 5 documents the under-pricing of minimum return guarantees in the retail market and points to cross-product subsidization as an explanation for the anomaly. Section 6 concludes.

2 Background and Data

2.1 Variable Annuities and Minimum Return Guarantees

Variable annuity is a retail retirement and savings product offered by life insurance companies where households can invest in a portfolio of selected mutual funds. Compared to investing directly in open-end mutual funds, variable annuities have tax deferral benefit and provide life insurance. However, both benefits are limited.\footnote{Tax deferral benefit means that capital gains on the investment can be deferred and accumulate tax free if the first withdrawal occurs after the retirement age of 59.5. All deferred earnings are taxed at a higher ordinary income tax rate and therefore the tax advantage is limited (see Brown and Poterba, 2006, Table 5.2). Tax deferral benefit is included in any retirement product such as Roth IRA.}

The 2000 dot-com bust led to significant investment losses on many variable annuity accounts. Capitalizing on this event and to boost sales, life insurers started offering minimum return guarantees in variable annuities. The guarantees typically ensure policyholders a certain minimum rate of return, e.g. 5%, regardless of the performance of the underlying mutual funds. Given the

\footnote{The life insurance provided in variable annuities is >than the greater of account value and total premium to the beneficiary when the policyholder dies. Milevsky and Posner (2001) show that the actuarial value of the insurance is around 10bp whereas insurers usually charge over 50bp on the insurance.}
long horizon of variable annuities, minimum return guarantees are long-term as well. Koijen and Yogo (2019) and Sen (2019) provide detailed illustration and examples on different broad types of guarantees.

We focus on the minimum return guarantees known as the living benefits. Technically, the life insurance inherent in all variable annuities, called death benefits, is also a form of minimum return guarantee. It returns the greater of account value and total premium to the beneficiary when the policyholder dies. We consider death benefits (including enhanced death benefits) separately because they are distinct from the minimum return guarantees that emerged in the early 2000s because the two have separate fee structures, affect different objectives (bequest motive vs policyholder’s own utility), and incur different risks to the life insurance companies.

2.2 Mutual Funds Available in Variable Annuities

Because of regulatory requirement, the mutual fund options available in variable annuities are not the regular open-end mutual funds, but rather clones of popular existing open-end mutual funds. For example, a commonly offered S&P 500 index option in variable annuities is the Fidelity VIP (variable insurance product) Index 500 Portfolio, a clone of Fidelity Index 500 Fund which is available for purchase directly as an open-end mutual fund. In many cases, the options are created and managed by the insurance company itself (through its affiliated companies). For example, in most MetLife variable annuity contracts, the only S&P 500 index option is the MetLife Stock Index Portfolio, managed by a MetLife subsidiary MetLife Investors Series Trust. Section 5 documents the high expense ratio hence high profit margin of these mutual fund options in variable annuities compared to their counterparts in the open-end mutual fund space.

2.3 Example – MetLife GMIB

Consider MetLife Financial Freedom Select Variable Annuity issued by Metropolitan Life Insurance Company on May 1, 2003. Policyholders are allowed to allocate their investment among 32 mutual funds, including S&P 500 Index (through an affiliated mutual fund MetLife Stock Index Portfolio), money market mutual funds, value or growth portfolios, and etc. There are three major components on the recurring expenses: 1) annual separate account charge at 1.55%; 2) annual mutual fund expenses; 3) optional annual minimum return guarantee charge at 0.35% that will be explained in more detailed in the next paragraph. There is significant liquidity restriction – the penalty is as much as 7% on withdrawals in the first 7 years, on top of the Federal tax penalty for withdrawals before the age of 59.5.

With additional annual fees, the investor can insure the account with a minimum return guarantee called the Guaranteed Minimum Income Benefit, or GMIB, that would allow the investor to access the maximum of 1) the combined value of invested mutual funds (the “account value”) and 2) a guaranteed base that grows at a minimum “roll-up rate” of 6% per year. There are two catches. The first is that the guaranteed base can only be accessed by conversion to a life annuity at a pre-determined low conversion rate: per dollar of the guaranteed base, a 65-year-old

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5METROPOLITAN LIFE SEPARATE ACCOUNT E, FORM N-4, REGISTRATION NOS. 333-83716/811-4001.
6There are small annual contract fees that can be waived with sufficient investment size, which usually is $25,000 or $50,000.
male investor would get a life annuity that pays $0.421 per month, which is much lower than the $0.662 life annuity rate that MetLife was selling simultaneously on the competitive market according to Annuity Shopper (2003, P25). The other catch is the put optionality embedded in the minimum return guarantee fees: the seemingly low fees of 0.35% per year operates on the higher of the account value and the guaranteed base. In summary, investors pay an annual fee with in-the-money puts to the insurance company in return for a guaranteed life income with an in-the-money put.

Despite the complex restrictions and conditions, the significantly in-the-money put option still seems worth more than 0.35% a year. Indeed, Section 7 shows that the implied volatility, a measure of how expensive an option is, is only 15%, which is significantly lower than the expensive put options that MetLife bought in the derivatives market, at 25%. I show that the minimum return guarantees are underpriced to boost sales and attract flows to affiliated mutual funds.

Figure A.1 further illustrates the operation of this contract assuming that a 55-year-old policyholder bought the variable annuity in 2005 and invested in a S&P 500 index fund (e.g. MetLife Stock Index Portfolio). The dark red line tracks the guaranteed base while the blue line tracks the actual trajectory of S&P 500. The policyholder pays annual base fees and rider fees, denoted by the gray and black bars, during the entire accumulation period, assumed to be 10 years. Under this scenario, the embedded put option became deep in-the-money as the stock market collapsed during the 2008 financial crisis, and the guaranteed base was ultimately larger than the mutual fund value in 2015, at the end of the accumulation period. The policyholder could elect to access the guaranteed base and convert it into units of life annuities, and the red bars show income from life annuities as a result of the gap between the guaranteed base and the mutual fund value, i.e. the income because of the GMIB guarantee.

2.4 Data

I used a variety of datasets for different parts of this paper which I describe in detail below.

Insurance companies file quarterly statutory reports with National Association of Insurance Commissioners (NAIC) that contain detailed information on their financial information. In particular, the reports contain balance sheet items, exposure to minimum return guarantees (General Interrogatories), and transactions in derivative instruments (Schedule DB). I gather data on the information above from reports organized by SNL Financial. Using insurers’ own classification and regular expression technique on the descriptions, I restrict to transactions on equity options and volatility/variance swaps.

I gather prospectuses from EDGAR’s database on variable annuities, focusing on Form-497 (material changes). The prospectuses have detailed information on contract terms including fees and details on optional minimum return guarantees. The prospectuses also contain information on historical returns and fees on the underlying mutual funds.

Morningstar Direct has quarterly data on historical total net assets and estimated net flows for a large sample of variable annuities and their subaccounts. The data are available from 1999Q1 to present.

Drechsler et al (2019) provide me with data on historical variance swap rates with maturities

7Both life annuities have 10 years guaranteed, to be comparable.
ranging from one month to ten years.

3 Demand and Supply of Long-Run Vol Insurance

3.1 The Large Demand for Long-Run Vol Insurance

This section documents the large and growing demand directly from households for long-run volatility insurance through minimum return guarantees. Figure 1 shows aggregate total net assets of variable annuities, which doubled from less than $1 billion in 2000 to over $2 billion as of 2015. It also shows that the majority of variable annuities is insured with a minimum return guarantee and the share of insured variable annuities has been increasing. Since the guarantees were not introduced until the early 2000s, the total net assets insured with a minimum return guarantee grew from zero then to over $1 trillion as of 2015. The size and growth of the market is even bigger if including indexed annuities, a close cousin of variable annuities that has been growing at 7.6% on average over the past decade.\(^8\)

As discussed in Section 2, all variable annuities include the standard death benefits, which amount to a put option exercised by death. Therefore, the basic variable annuities already contain some level of insurance against volatility risk, even though the insurance is small.

Apart from aggregate time series, more evidence can be drawn in the cross section on demand for minimum return guarantees. I separate variable annuities into those that have ever offered guarantees and those that have not, and compare their paths of total net assets which are scaled to be one in Q3 of 2003, the average time that minimum return guarantees were introduced. The result is shown in Figure 2. As evident in the graph, the two groups had similar growth dynamics before the introduction of minimum return guarantees. After the introduction of guarantees, variable annuities that offer guarantees grew visibly faster than the variable annuities without guarantees.

Section 5 shows more evidence in a formal difference-in-difference framework. It establishes causal evidence that the introduction of minimum return guarantees led higher sales and flows to the guarantee-offering variable annuities, insurance companies, and mutual funds.

\(^8\)The value of an indexed annuity grows according to the return of a pre-determined index subject to caps and floors (the floor constitutes a minimum return guarantee), whereas the value a variable annuity is that of the underlying mutual funds. Indexed annuities are direct liabilities of the life insurance companies (general account liabilities), whereas for variable annuities, the mutual funds are held in separate accounts – meaning that they are not liabilities of the insurance companies – and only the optional guarantees are on the general account.
Figure 1: Total Net Assets of Variable Annuities

This figure shows aggregate total net assets of variable annuities, those insured with minimum return guarantees (data not available until 2005), aggregate reserves for indexed annuities (data not available until 2010), and the share of variable annuities of life insurers’ total liabilities.

Figure 2: Scaled Total Net Assets, With vs Without Guarantees

This figure plots the total net assets of the treatment group and the control group, both scaled to be one in Q3 of 2003, the average time that treatment started. Treatment refers to first offering of any minimum return guarantees in variable annuities. The treatment group consists of insurers that offer any guarantees in any of their variable annuities, and the control group consists of the rest of the insurers.
3.2 Supply and Hedging of Long-Run Vol Insurance

How did life insurance companies manage the large quantities of long-run volatility risk they incurred from selling minimum return guarantees? They hedged with financial derivatives such as long-maturity options and variance swaps. This section documents insurers’ intensive trading activities on these long-term volatility assets, and how these activities represent hedging motives and correspond to insurers’ levels of exposure to the minimum return guarantees.

Figure 3 plots the aggregate net trades on long-maturity options and variance swaps by all life insurance companies. I focus on net measures because insurers not only long options but also write options to partially offset the hedging cost (e.g. collars). In addition, insurers frequently terminate derivative contracts before maturity, which amounts to short positions. The sharp increase in net trades of these derivatives mirrors the growth of minimum return guarantees in the early 2000s. In recent years, insurers have stopped purchasing these long-term hedging assets. I will show that this is mainly because of the disappearance of long-run vol insurance from minimum return guarantees in the last part of this section.

Figure 4 confirms that these derivatives positions reflect hedging. Here I look at the cross section of insurers and plot their net trade positions against their exposures to minimum return guarantees. Exposures are measured by Total Relative Account Value which are total net assets of accounts with minimum return guarantees. On average, per thousand dollar of exposure to minimum return guarantee insurers hedged with $0.71 vega of options and $0.54 vega of variance swaps that have maturity longer than five years.

3.3 Fragility and Disappearance of Long-Run Vol Insurance

The 2008 financial crisis saw unprecedented levels of volatility. The minimum return guarantees gave households the great protection they needed but also rendered many insurers in significant financial distress. Hartford and Lincoln were bailed out by the government through TARP, and many insurers exited the market of writing minimum return guarantees.

Furthermore, for the insurers that remain in the market, they have altered the minimum return guarantees so much that they no longer provide much insurance against volatility risk. For example, many variable annuities now require dynamic allocation to equities, and automatically transfer investment to safe assets when there is heightened volatility risk (higher implied volatility). Similarly, many variable annuities only include target volatility funds as investment options. These target volatility funds perform the dynamic asset allocation described above. As another example, many variable annuities now charge a floating fee on the guarantees that is directly indexed to VIX. This amounts to charging households the short-term variance premium.

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10 Ideally exposures should be measured by vega (change of value with respect to volatility) or present value. Gross Amount of Reserve measures the present value but after is net of any hedging.
Figure 3: Insurers’ Net Trades on Long-Maturity (> 5Y) Vol Derivatives

Figure 4: Holdings of Vol Derivatives vs Exposure to Guarantees
4 Insurers’ Impact on the Price of Long-Run Vol

This section studies the impact of insurers’ hedging activities on the price of long-term volatility assets. I show that the sharp increase in the purchase of long-term volatility assets by insurers to hedge minimum return guarantees coincide with a significant steepening of the volatility term structure. The correlation is likely causal, given the timing and the large quantities of those purchases. I sketch a simple framework to rationalize the simultaneous increase in quantity and price of these long-term volatility assets.

4.1 Insurers capture a large share of the long-run vol market

Although life insurers are a relatively small player in the short-term derivatives market, they capture a significant share of the trading activities of long-maturity options and variance swaps. Figure 5 compares long-maturity options held by life insurance companies versus those sold by all U.S. banks and dealers, as reported by the Federal Reserve to Bank of International Settlement. It shows that the life insurance industry commands a large portion of the outstanding positions and is a key driver of the dynamics of market. Note that I only look at S&P options for insurers, whereas the market estimate includes all options with different benchmarks (e.g. Nikkei), and therefore insurers’ share might be significantly under-estimated here.

Similarly, Figure 9 compares variance swap trades by all life insurance companies versus aggregate trades in the market as reported by Dew Becker et al (2017, Table 1). Again it shows that hedging activities by life insurers have a large footprint in the long-maturity derivatives market (over 80% of the trades of variance swaps with maturity over five years). Given their dominant presence, insurers should have a large impact on the prices of these long-term volatility assets.

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11The BIS data does not directly provide numbers for long-maturity options by U.S. banks and dealers. Instead, it provides data on outstanding amount for all options of different maturities and separately all options by broker dealers of different countries. I infer the intersection by assuming that U.S. represent a constant share of the market in every maturity bucket.
Figure 5: Aggregate Long-Maturity Option Holdings – Insurers vs Market

Figure 6: Aggregate Var Swap Trades by Maturity (in months) – Insurers vs Market
4.2 Insurers’ hedging steepened the volatility term structure

Figure 9 shows historical prices of long-run and short-run volatility, measured by variance swap rates. The spread between the long-run and the short-run variance swap rates reflect the relative price long-run vol vs short-run vol, i.e., the slope of the volatility term structure. As can be seen in the graph, there is a visible structural break in the dynamics of the prices around 2003, when the long-run vol price begun to “de-couple” from the short-run vol price. In other words, the term structure of volatility became significantly more upward sloping. This phenomenon is also documented in Drechsler et al (2019).

Figure 9 also shows insurers’ trades on long-run volatility assets (options and variance swaps) and the long and short-term volatility rate. As shown before, the trades became significantly bigger after the introduction of minimum return guarantees at the beginning of 2000s. Not surprisingly, the sharp increase of insurers’ trade coincide with the steepening of volatility term structure.

The correlation between insurers’ trades and the increase of long-run vol price is likely causal, for the following reasons. First, as shown in the previous sub-section, insurers command a large share of the market of long-term options and variance swaps. It is likely that the sharp increase in quantity demanded by insurers had price impact, and the direction of the impact should be an increase in price, as what’s happened in reality. Secondly, the timing is very precise. The “de-coupling” of long-run price from short-run one started right when insurers began to make large quantities of trades to hedge their guarantee exposures. Furthermore, there are extensive anecdotal evidence on price impact, e.g. Santander (2012).

Lastly, Figure 10 plots prices paid by insurers on the long-term option and variance swap trades, and these prices are invariably marginally above the current market prices.

To quantify the impact of insurers’ trades on the price of long-run volatility, we run the following regression on the monthly level:

\[ E_t^Q[\sigma_{t,T}^2] = \alpha + \beta \text{Insurers'Trade} + \gamma E_t^Q[\sigma_{t,t+1}^2], \quad (1) \]

where variance swap rates of 10-year (1-month, i.e. the VIX) are used to measure the price of volatility of 10-year (1-month). The regression results below show that the 10-year variance swap rate is strongly associated with insurers’ trades. Specifically, an average sized monthly trade, $3.045 million vega, implies 1.084 increase in 10-year variance swap rate.
Figure 7: Impact of Insurers’ Trades on Long-Run Volatility Price

Figure 8: Vol Prices Paid by Insurers
Table 1: Impact of Insurers’ Trades on Long-Term Volatility Price

This table reports OLS regression results using monthly observations from 1995 to 2017. The LHS variable is the price of long-term volatility measured by 10-year variance swap rate. The RHS variable of interest is aggregate monthly trade on long-term SP options and/or variance swaps (maturity of 10 year or more) by the life insurance industry, measured by vega, notional, and net cost, respectively. Standard errors in parentheses use GMM to correct for heteroskedasticity and serial correlation.

<table>
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<th>Option &amp; VS Vega $ million</th>
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<th>Notional $ billion</th>
<th>Net Cost $ billion</th>
<th>Var Swaps Vega $ million</th>
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<td>0.558*** (0.097)</td>
<td>2.912*** (0.797)</td>
<td>31.369*** (4.697)</td>
<td>0.381*** (0.095)</td>
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</tbody>
</table>

Table 2: Impact of Insurers’ Trades on Long-Term Volatility Price (Robustness)

This table reports regression results on the same specification as Table 1 except for using PC1 instead of VIX as the control variable. PC1 is the first principal component of the volatility term structure, i.e. the seven times series of variance swap rates with maturities of 1 month, 3 month, 6 month, 1 year, 2 year, 5 year and 10 year.

<table>
<thead>
<tr>
<th></th>
<th>Option &amp; VS Vega $ million</th>
<th>Options Only Vega $ million</th>
<th>Notional $ billion</th>
<th>Net Cost $ billion</th>
<th>Var Swaps Vega $ million</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insurers’ Trade</td>
<td>0.197*** (0.039)</td>
<td>0.389*** (0.065)</td>
<td>2.132*** (0.543)</td>
<td>19.351*** (3.178)</td>
<td>0.190*** (0.056)</td>
</tr>
<tr>
<td>PC1</td>
<td>1.469*** (0.103)</td>
<td>1.378*** (0.092)</td>
<td>1.393*** (0.089)</td>
<td>1.430*** (0.093)</td>
<td>1.361*** (0.092)</td>
</tr>
<tr>
<td>Observations</td>
<td>261</td>
<td>261</td>
<td>261</td>
<td>261</td>
<td>261</td>
</tr>
<tr>
<td>R2</td>
<td>0.540</td>
<td>0.579</td>
<td>0.595</td>
<td>0.577</td>
<td>0.584</td>
</tr>
<tr>
<td>Mean Trade Size</td>
<td>3.045</td>
<td>2.005</td>
<td>0.275</td>
<td>0.033</td>
<td>1.041</td>
</tr>
<tr>
<td>Average Trade Impact</td>
<td>0.600</td>
<td>0.780</td>
<td>0.586</td>
<td>0.639</td>
<td>0.198</td>
</tr>
</tbody>
</table>
4.3 Intermediary Asset Pricing of Long-Run Vol

This section proposes a simple partial equilibrium framework that explains the joint dynamics of price and quantity of long-term volatility. The model has two periods and features a continuum of intermediaries that supply the long-run vol insurance. This intends to mimic the actual market structure of long-term put options where at each point in time the market was supplied by more than twenty investment banks. Each intermediary is a price taker and chooses its supply $s$ to maximize the weighted average of its first period payoff and its expected payoff in the second period:

$$\max_s \beta sP + E[s\tilde{R}] - \frac{\gamma}{2} \operatorname{Var}(s\tilde{R}),$$

where $\beta$ is relative weight between first and second periods, $P$ price of a unit of insurance, $\tilde{R}$ payoff of a unit of insurance in the second period, and $\gamma$ risk aversion. Assuming that $\mu = E[\tilde{R}]$ and $\sigma^2 = \operatorname{Var}(\tilde{R})$ are known to the intermediary, then optimal supply is given by:

$$s^* = \frac{P + \mu}{\frac{\gamma}{\sigma^2}},$$

which leads to a upward sloping supply schedule in the aggregate intermediary sector, i.e. $\partial S/\partial P > 0$, where $S = \int s^*_i \text{d}i$.

The demand of insurance is assumed to be exogenous. A positive demand shock would therefore lead to both a higher quantity and price of the insurance in equilibrium, just as what we see in the dynamics of long-term options and variance swaps.

![Figure 9: Upward Sloping Supply Schedule of Insurance by Intermediary](image)

Figure 9: Upward Sloping Supply Schedule of Insurance by Intermediary
5 Under-Pricing in the Household Market

The previous section shows that insurers paid on average 27.3% implied volatility for the long-term options and variance swaps to hedge exposures from minimum return guarantees. This section shows that they charged a much lower price for the minimum return guarantees before the 2008 financial crisis. The implied volatility that equates present value of the guarantee fees and that of the payoff is on average 15.0%, around the same level as the historical realized 10-year volatility.

Why were there different prices on long-term volatility insurance? Why were the more sophisticated insurers paying the higher price in the derivatives market and under-charging the less sophisticated households in the retail market? I show extensive evidence of cross-product subsidization. The minimum return guarantees, rather than the profit center themselves, are ways to boost sales of the underlying variable annuities and affiliated mutual funds which have high profit margin. I show that: 1) the base fees of variable annuities with guarantees are much higher than those without; 2) the mutual funds available in variable annuities (VAFs) are mostly affiliated with the insurance companies writing the variable annuities, and these VAFs charge on average 0.5% higher fees than other mutual funds with similar performance and characteristics; and 3) in a difference-in-difference framework, the introduction of minimum return guarantees increased flows to VAFs by $1 billion per year from 2000 to 2008.

5.1 Implied Volatility of Minimum Return Guarantees

5.1.1 Method

As described in Section 2, minimum return guarantee is similar to a put option. I therefore study it under an option pricing framework, and evaluate its price by the implied volatility. Specifically, the environment is a standard Black-Scholes Economy where the interest rate is constant at $r$ and the underlying account value $S_t$ follows a geometric Brownian motion:

$$dS_t = \mu S_t dt + \sigma S_t dW_t.$$

The goal is to find the volatility $\sigma$ (the implied volatility) such that present value of the fees equates that of the payoffs. Mathematically, present value of cash flow $X$ at time $t$ is given by:

$$PV(X) = e^{-rt}E^Q[X].$$

For example, if the guarantee is the same as an European put option, then there is an analytical solution to the present value of its payoff, which is monotonically increasing in volatility. The implied volatility can be uniquely pinned down by the observed price (i.e. present value of the fees). However, most of the guarantees have complex payoff structures. Therefore, I derive implied volatility using Monte Carlo simulations under the risk-neutral measure:

$$dS_t = (r - \delta)S_t dt + \sigma S_t dW^Q_t.$$

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where $\delta$ is the dividend yield, which is set to zero because dividends are by default reinvested in mutual funds in a variable annuity contract.

To match with the derivatives pricing data which are based on European options and do not go beyond ten years, I set the payoff of the guarantee to be a bullet payment in ten years whenever possible. Correspondingly, $r$ is set to be the ten-year U.S. Treasury yield.

Mortality and lapsation are two important risks that affect the payoffs of the guarantees but are outside the usual Black-Scholes framework. Both mortality and lapsation, if realized, terminate the contract and release the insurer from the guarantees (death benefit is a liability covered by the base fee). I use the one-year mortality rate of a 60-year-old male according to Social Security’s Actuarial Life Table, $\rho = 1\%$. This reflects the average mortality rate from the life insurer’s perspective. Lapsation is very hard to model and is usually ignored in financial analysis (e.g. Koijen and Yogo 2015). We also take this conservative approach and ignore lapsation in the baseline model, and show that only a lapsation rate of higher than 3% can eliminate the underpricing.

5.1.2 Illustration of Method using MetLife GMIB

I illustrate the method using the GMIB example described in Section 2. The guarantee costs an annual fee of:

$$0.35\% \cdot \max(S_t, S_0(1 + 6\%)^t),$$

where the annual fee of 0.35% operates on the higher of the account value and the guaranteed base that grows at an annual roll-up rate of 6%. The payoff is assumed to be a bullet annuitization in year ten, which is positive only if the account value fall far short of the guaranteed base:

$$\max(0, \beta S_0(1 + 6\%)^T - S_T),$$

where $T$ is set to be ten and $\beta$ corresponds to the discount associated with the pre-determined below-market annuitization rate.$^{12}$ I simulate 10,000 trajectories of the account value, $\{S_t\}_{t=0}^{10Y}$, which can be used to obtain risk neutral expectations of fees and payoffs, which then in turn can be discounted to obtain present value at time 0. A unique implied volatility can be found that equates present value of the fees with that of the payoffs.

5.1.3 Results

I calculate implied volatility for a representative sample of guarantees, including those offered by MetLife.

Figure 10 below shows the results. The price of guarantees was low before the crisis: the implied volatility of guarantees averaged 15% even though the implied volatility from derivatives of similar maturity and structure was above 25%. After the crisis, many of the guarantees have included asset allocation restrictions, effectively lowering or eliminating the insurance against long-run volatility risk.

$^{12}$The market annuitization rates are from Annuity Shopper.
5.2 Why were Guarantees Under-Priced?

The previous sub-section establishes that insurers charged on average only 15.0% implied volatility on the guarantees, which is in stark contrast with the 27.3% implied volatility they were paying in the derivatives market. Why were there different prices on long-run volatility insurance? Why were the more sophisticated insurers paying the higher price in the derivatives market while under-charging the less sophisticated households in the retail market? I show evidence of extensive cross-product subsidization as an answer.

First, although the guarantees themselves were under-priced, the base fees of a typical guarantee-offering variable annuity were over-priced by a wide margin. I compare the average base fee of variable annuities that offer guarantees with those that do not, and show that the difference is positive and large, especially after the introduction of optional guarantees.

Next, I examine the mutual funds available in variable annuities and show that they are mostly clones of the popular options in the open-end mutual fund market. However, these clones either charge higher fund expenses than their open-end counterparts or are shoddy knockoffs.
created by insurers that under-perform their open-end counterparts significantly. The over-charging and under-performing is significantly worse for funds associated with guarantee-offering variable annuities. Therefore, the mutual funds available in variable annuities are also over-priced relative to their performance.

Lastly, the inclusion of guarantees was associated with higher sales to the variable annuities. Using a difference-in-difference framework, I show that the association is likely causal.

In sum, under-priced minimum return guarantees were used as a bait to induce sales to an over-priced underlying product. The tactic did work, as evidenced by the additional sales due to the introduction of guarantees. The result provides novel evidence to cross-product subsidization and consumer myopia in the field of household finance.

It is important to note that the under-pricing phenomenon is specifically prior to the 2008 financial crisis, after which guarantees fees have increased substantially and vol insurance has diminished if not disappeared from the guarantees. The fee increase is addressed in Koijen and Yogo (2018) and reflects that insurers have under-estimated risk. Indeed, conversations with industry practitioners reveal that: 1) insurers ignored the moral hazard problem, where the accounts with minimum return guarantees took on more investment risks, as documented in Milevsky and Kyrychenko (2008); 2) the guarantee business has changed from a sales-oriented management to a risk-oriented management. Under-estimation of risk and excessive risk taking are definitely part of the reason why guarantees were under-priced. In contrast, I show reasons why insurers could have rationally and optimally under-priced the guarantees.

5.2.1 Over-Priced Base Fees

Policyholders pay three types of fees for a variable annuity: base fees, mutual fund expenses, and optional guarantee fees. Here I compare base fees on variable annuities with and without any optional minimum return guarantees, and show that the fees are higher for the former. Under the assumption that there is no difference in the products that the fees are supposed to provide, namely the death benefits, the difference in fees can be interpreted as higher profit margin.

As of right now, I only use MetLife (with guarantee) and TIAA (without guarantee) for this analysis. In 2007, MetLife was the largest writer of variable annuity with guarantees ($143 billion VA assets), while TIAA was the largest writer of variable annuity without guarantees ($298 billion VA assets). Ideally, with more data and time, I should compare all variable annuities by all insurers.

Profit margin can reflect market power independent of the offering of minimum return guarantees. To mitigate that, I should control for proxies of market power such as insurer size. Here, MetLife is smaller than TIAA in terms of VA total net assets, so its market power should be smaller in theory, and therefore my findings here would be an under-estimation of the over-pricing.

There is existing literature on the fair value of base fees. Milevsky and Posner (2001) use an option pricing framework and find that the base fees are over-priced relative to their actuarial value, which they claim to be 10bp. In contrast, I focus on the difference in base fees between variable annuities that offer option minimum return guarantees and those that do not. To the extent that the actuarial value measures the true cost, I shows that all variable annuities charge higher base fees than the cost, but the variable annuities with guarantees are able to sustain a
much higher profit margin than those without.

Figure 11 shows that variable annuities that offer guarantees charge a much higher base fees than those that do not, especially after the start of offering of guarantees.

5.2.2 Over-Priced Mutual Fund Expenses

In a variable annuity contract the policyholder can allocate his investment among a menu of mutual funds. Due to regulatory requirements, rarely are these mutual funds the same funds available in the open-end market. Most of the mutual funds available in variable annuities are either 1) clones of popular open-end mutual funds managed or co-managed (as subadvisor) by managers of the original funds, or 2) knockoffs created by insurers themselves. For example, “JNL/AQR Large Cap Relaxed Constraint Equity Fund” is a replica of the popular open-end fund “AQR Large Cap Relaxed Constraint Equity Fund” but managed by a subsidiary of Jackson National Life Insurance Company (Jackson National Asset Management) and with AQR as the subadvisor. “MetLife Stock Index Portfolio” is a fund managed by a subsidiary of Metropolitan Life Insurance Company (MetLife Adviser, LLC) that tracks the S&P 500 index. In all MetLife variable annuities, this is the only option to track the index.

Recent mutual fund literature such as Berk and van Binsbergen (2015) argue that fees reflect investor skill. To avoid the contamination of skill difference, I focus on comparisons 1) between different clones of the fund; 2) between different index funds with the same target index. This allows me to narrow down to homogeneous products and rule out any skill difference.

Comparison between clones of the same fund. TBD.

Next, I look at index funds and group them by their target indices. I then compare the expense ratios of those included in variable annuities that offer guarantees with those that do not offer. For example, for index funds that track the S&P 500 index, I would compare expense ratios of funds such as MetLife Stock Index Portfolio, which is offered in MetLife’s variable annuities that all offer guarantees, with expense ratios of funds such as TIAA-CREF S&P 500 Index Fund, which is offered in TIAA variable annuities, all of which do not offer any optional minimum return guarantees. Figure 12 shows that there is a large difference in the average expense ratios (weighted by total net assets), and the funds included in variable annuities with guarantees have much higher fund expenses.

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13Some variable annuities only have one mutual fund option. This was rarely the case before the 2008 financial crisis.
Figure 11: Comparison of Base Fees, With vs Without Guarantees

This figure compares the average base fees (in %, weighted by total net assets) of variable annuities offered by MetLife (with guarantee) and TIAA (without guarantee).

Figure 12: Comparison of Fund Expense Ratios, With vs Without Guarantees

This figure compares the average expense ratios (in %, weighted by total net assets) of similar index funds by MetLife (MetLife Stock Index Portfolio, included in variable annuities with guarantees), TIAA (TIAA by TIAA-CREF S&P 500 Index Fund, included in variable annuities without guarantees), as well as Vanguard (Vanguard 500 Index Fund, open-end fund offered outside variable annuities). All three funds describe themselves as tracking the S&P 500 index in their prospectuses.
5.2.3 Higher Sales and Fund Flows Because of Minimum Return Guarantees

I have shown that variable annuities offering guarantees are over-priced on their base fees and mutual fund expenses, but if consumers are price-sensitive and switch to lower-cost alternatives, over-pricing would not lead to higher profit. This section shows that under-priced guarantees are used as a bait to maintain or even induce a higher inflow of capital.

I use a difference-in-difference framework using three levels of data granularity: variable annuity/quarter, insurer/quarter, and fund/quarter. “Treatment” here is defined as offering of any minimum return guarantee. Specifically, a variable annuity is “treated” if it ever offers any minimum return guarantees. Similarly, an insurer is “treated” if any of its variable annuities ever offer any guarantees, and a fund is “treated” if it is ever included in a “treated” variable annuity. The variable annuities, insurers, and funds that have never been associated with any guarantee serve as the control group. Treatment occurs at the first offering of any guarantee, and the periods before and after the treatment are defined as the “pre” periods and the “post” periods, respectively. The goal is to identify and measure the difference in sales between the treatment group and the control group, and then compare that difference in the pre periods versus the post periods.

Figure 2 in Section 3 illustrates the main intuition. Formally, I run the following regression:

\[
\text{Flow}_{i,t} / \text{TNA}_{i,t-1} = \text{Treated}_i \times (\text{Post}_{i,t} = 0) + \text{Treated}_i \times (\text{Post}_{i,t} = 1) + \text{Return}_{i,t-1} + FE_t + \epsilon_{i,t}
\]

I follow the mutual fund literature and measure flow as growth relative to size and include lagged return to capture the well-known flow-performance relationship.

Table 3 shows the results. It shows that the treatment group saw higher sales than the control group after the start of the treatment, while the difference is not significant in the pre periods. For interpretation, using the second column, treatment (introduction of minimum return guarantee) led to 2.2% higher quarterly growth rate on the insurer level. The results are insignificant on the variable annuity level, mainly because new guarantees are usually rolled out together with new variable annuity contracts.

Table 3: Effect of Minimum Return Guarantee on Sales

<table>
<thead>
<tr>
<th>Sample:</th>
<th>All variable annuities</th>
<th>All insurers offering VA</th>
<th>Funds that are sold both open-end and through VA</th>
<th>All open-end and VA funds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit of Observation:</td>
<td>VA/Quarter</td>
<td>Insurer/Quarter</td>
<td>Share Class/Quarter</td>
<td>Fund/Quarter</td>
</tr>
<tr>
<td>Treated * (Post = 0)</td>
<td>0.021 (0.015)</td>
<td>0.013* (0.007)</td>
<td>0.023 (0.017)</td>
<td></td>
</tr>
<tr>
<td>Treated * (Post = 1)</td>
<td>0.028 (0.025)</td>
<td>0.022** (0.010)</td>
<td>0.012*** (0.004)</td>
<td></td>
</tr>
<tr>
<td>Net Return (Lag)</td>
<td>1.037* (0.623)</td>
<td>0.895** (0.423)</td>
<td>0.358* (0.171)</td>
<td></td>
</tr>
<tr>
<td>Quarter FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>96215</td>
<td>6731</td>
<td>14235</td>
<td></td>
</tr>
<tr>
<td>R2</td>
<td>0.157</td>
<td>0.256</td>
<td>0.196</td>
<td></td>
</tr>
</tbody>
</table>
Reference


