Abstract

I develop a model of venture capital (VC) intermediation that can quantitatively explain central empirical facts about VC activity and allows evaluating its impact on economic growth. The framework reveals how pro-cyclical VC investment dynamics reinforce themselves through a risk premium channel that rationalizes low funding standards in booms. VC investments’ aggregate impact is significantly larger than suggested by empirical magnitudes of VC commitments and exit valuations, even after accounting for large exposures to systematic and uninsurable idiosyncratic risks. The model is analytically tractable and yields exact solutions despite the presence of matching frictions, imperfect risk sharing, and endogenous growth.

Keywords: Venture Capital, Economic Growth, Innovation, Intermediation, Boom-Bust Cycles, Risk Premia, Idiosyncratic Risk, Imperfect Risk Sharing
1. Introduction

Innovation by new firms has long been viewed as central to economic growth and development (Schumpeter 1934). Over the past few decades, venture capital funds have emerged as prominent financial intermediaries facilitating the entry of young innovative firms. According to Kaplan and Lerner (2010), over 60 percent of initial public offerings (IPOs) have been VC-backed between 1999 and 2009, and four of the twenty companies with the highest market capitalization in the United States have been funded by venture capital. Furthermore, empirical studies find that venture capital has had significant influence on patented inventions.¹ Despite this remarkable record, the VC industry also has gone through dramatic boom-bust cycles that appear anomalous to many observers. Gupta (2000) and Gompers and Lerner (2003) argue for example that extreme movements in VC industry activity are a symptom of irrational overreaction, resulting in periods in which too many firms are funded, followed by ones in which not enough firms have access to capital.²

Despite the potential macroeconomic relevance of venture capital, the literature lacks to date a quantitative framework that can explain central empirical facts about the magnitude and cyclicality of VC activity, and allows evaluating VC investments’ contribution to economic growth. In this paper, I aim to take a step toward developing a unified macro-finance framework that fills this void. The model features three building blocks that are essential to achieving this goal: (1) A macroeconomic environment where innovations by new firms can generate endogenous contributions to economic growth. (2) A micro-founded representation of the VC industry that captures central features of the industry’s market structure and economic function, allowing the model to match empirical quantities of VC intermediation, such as VC commitments, VC manager compensation, and payoffs to entrepreneurs. (3) An environment with empirically plausible asset pricing implications that captures agents’ revealed attitudes toward growth and risk, and allows appropriately interpreting information encoded in market prices. The model accounts not only for the pricing of aggregate risk but also sheds light on the implications of imperfect risk sharing under VC contracts.

¹See, e.g., Kortum and Lerner (2000) and Samila and Sorenson (2011) for empirical evidence. ²See also Gompers and Lerner (2000, 2001) for evidence on lenient funding standards, high valuations, and high capital commitments during venture capital booms. Korteweg and Sorensen (2010) and Kaplan and Lerner (2010) also highlight the negative relationship between VC fund commitments and future returns.
The proposed framework can draw on a variety of empirical data on VC intermediation, asset prices, and macroeconomic dynamics to inform the model’s parameters. The calibrated model successfully matches salient dynamic properties of aggregate VC fund commitments, VC-backed IPOs and mergers and acquisitions (M&As), VC fund fee revenues, and payoffs to entrepreneurs, suggesting that these data are not necessarily evidence of investor irrationality. Accounting for the pricing of macroeconomic risks reveals that pro-cyclical entry by VC-backed firms endogenously reinforces itself, as it causes counter-cyclical post-IPO risk premia. The resulting lenient funding standards in booms help the model match extreme increases in VC investment, as observed during the internet boom of the late 1990s, as well as the negative empirical relation between VC fund inflows and future returns.

Within this framework, I quantitatively evaluate VC investments’ impact on agents’ consumption and expected lifetime utility. I express the corresponding value as a compensating variation in lifetime consumption, following classic approaches by Breeden (1984) and Lucas (1987). This type of compensating variation can be readily compared to aggregate VC exit valuations (IPOs and M&As), but it is more comprehensive. In particular, it also accounts for the part of VC investments’ impact on agents’ consumption that is not reflected in producer valuations. This compensating variation takes values between 1 and 2 percent of lifetime consumption, significantly exceeding aggregate VC fund commitments and exit valuations. Risk discounts embedded in these estimates — for both aggregate and idiosyncratic risk — are shown to be quantitatively important. Not only the strong cyclicality of VC investments’ growth contributions, but also imperfect risk sharing under VC contracts significantly dampens the value agents assign. Nonetheless, the overall economic magnitudes are significant — in particular relative to estimates of the costs of business cycles (see, e.g., Lucas 2003) — and suggest the potential relevance of policies affecting the VC industry.

Due to imperfect risk sharing, idiosyncratic risk can greatly affect the ex ante value entrepreneurs and VC managers assign to performance-sensitive payoffs from VC contracts. The

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3 The general concept of a compensating variation goes back to Hicks (1939).
4 In addition, whereas market values merely reflect an asset’s marginal impact on agents’ consumption dynamics, the considered compensating variation values non-marginal effects on consumption.
5 The VC industry was able to attract material capital commitments only after regulatory reforms and tax changes were implemented in the late 1970s and early 1980s (see, e.g., Kroszner and Strahan 2014).
extreme risks typical for venture capital imply that risk averse agents severely discount these risky contractual payoffs, consistent with the findings by Hall and Woodward (2007, 2010). Nonetheless, agents assign significant value to aggregate VC activity, in particular due to its positive impact on aggregate technological progress, and since diversified investors hold significant fractions of ventures’ financial claims.

The paper proposes a highly tractable general equilibrium environment with matching frictions, imperfect risk sharing, and endogenous growth that could provide a useful stepping stone for future studies demanding such features. The methodology provides exact solutions, without requiring computationally-intensive methods. In addition, as the model captures both risk-based and non risk-based factors affecting average returns to VC investments, it provides a cautionary note to empirical studies estimating risk-based asset pricing models, adding to other well-known challenges with VC data.

In the model, entrepreneurs generate venture ideas with heterogeneous success prospects. VC managers with industry connections and expertise can match VC funds with the most promising entrepreneurs and support their ventures. Heterogeneous access to high-quality entrepreneurs implies an imperfectly competitive market structure where VC funds can generate economic profits (“alpha”) that VC managers extract via fees. However, depending on venture productivity and macroeconomic conditions, rents from VC fund operation can be insufficient to cover the compensation required for fund managers’ human capital, causing breakdowns in intermediation and potential barriers to entry for new ventures in downturns.

Yet such barriers to entry also imply that persistent negative shocks to creative destruction and growth can be partially good news for ventures that obtained funding during a preceding boom and successfully entered the market. Although these firms’ dividend growth suffers when aggregate growth decreases, they benefit to a first-order degree from persistent declines in the funding and entry of new competitors. Venture investments thus can become hedges against macroeconomic

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Hochberg, Ljungqvist, and Lu (2007) document the importance of VC managers’ network connections for investment opportunity sets and access to information in venture capital markets. Kaplan and Strömb erg (2001, 2004) provide direct empirical evidence on VCs’ investment analyses, documenting the importance of VCs’ efforts to evaluate and screen entrepreneurs. Further, Gompers, Kovner, Lerner, and Scharfstein’s (2008) empirical findings highlight the importance of “industry-specific human capital,” including a “network of industry contacts to identify good investment opportunities as well as the know-how to manage and add value to these investments.”
low-frequency risks associated with persistent slow-downs in technological progress, especially in those industries where entry is highly pro-cyclical. Low discount rates and associated lenient funding standards in booms in turn endogenously reinforce the cyclicality of VC investment and entry.

**Related literature.** To my best knowledge, this is the first paper to develop a quantitative dynamic general equilibrium model that explains central empirical facts about the magnitude and cyclicality of venture capital activity and allows evaluating its impact on the macroeconomy. There is a large literature in macroeconomics and economic growth that addresses technological change but abstracts from venture capital intermediation and the pricing of risk. The most related papers in this literature are [Greenwood and Jovanovic (1999)](#) and [Hobijn and Jovanovic (2001)](#), which link expectations about future increases in the rate of creative destruction during technological revolutions to initial stock market declines. In my model, persistent variation in creative destruction also generates the corresponding reverse effect that expectations about future declines in entry amplify preceding booms. In particular, my model reveals that pro-cyclical variation in creative destruction materially reduces venture investments' risk premia, which is a result that would not be captured by a model that abstracts from the pricing of risk. For similar reasons, my paper deviates from the existing literature on finance and growth such as [King and Levine (1993b)](#) who consider project selection in a quality-ladder model with constant aggregate growth. This literature does not provide a model that quantitatively captures VC activity and does not address the key objects of interest of this study, such as cyclical variation in VC intermediation and funding standards, the impact of aggregate risk and risk pricing, and the performance-sensitive compensation of entrepreneurs and VC managers. Finally, there is also an influential literature analyzing the impact of (intermediary) leverage on macroeconomic activity and asset prices. This literature does not aim to address empirical facts about the venture capital industry and the mechanics of VC funds,

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which are unlevered institutions.

With regards to its implications for risk premia my paper is more closely related to the strand of literature that analyzes general equilibrium production-based models to examine the time-series and cross-sectional properties of returns (see, e.g., Gomes, Kogan, and Zhang 2003). Several papers in this literature address the asset pricing implications of technological innovation, although the papers in this literature do not consider venture capital intermediation and its effects on returns to investors and macroeconomic growth. In Garleanu, Kogan, and Panageas (2012) and Kogan, Papanikolaou, and Stoffman (2015) displacement risk arises as a priced risk factor in economies with imperfect risk sharing and can help explain empirical patterns in asset returns, such as the value premium. While I also analyze the effects of imperfect risk sharing, displacement risk in my setting affects discount rates primarily through its impact on firms’ exposures to low-frequency risk associated with persistent changes in aggregate growth. Kung and Schmid (2015) consider an expanding varieties model similar to Comin and Gertler (2006) to analyze the asset pricing implications of persistent movements in productivity in the presence of recursive preferences. The authors provide empirical evidence for the existence of innovation-driven low-frequency movements in aggregate growth rates. Several recent papers further analyze the implications of technology shocks that are embodied in new capital. Although the technology considered in my paper differs from the ones in these papers, similarities arise, as growth is affected by shocks impacting the development of new, higher-quality goods that displace older vintages of goods. Finally, Pastor and Veronesi (2009) highlight that the endogenous adoption of a revolutionary technology is initially associated with low discount rates and increasing expected cash flows, but over time, risk premia rise and eventually lead to falling asset prices as the nature of risk changes from idiosyncratic to systematic. In my model, declining discount rates in booms increase valuations and VC investment, consistent with empirical evidence that funding booms predict low returns (see footnote 2). Further, my paper speaks to the general equilibrium dynamics of IPO volume and is thus related to theories of IPO waves, such as Jovanovic and Rousseau (2001) and Pastor and Veronesi (2005).

Although the framework proposed in this paper captures a multitude of economic forces affecting venture capital activity and its linkages to the macroeconomy, it naturally also abstracts from a

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variety of channels that can create cyclicality, such as entrepreneurs’ self-fulfilling expectations about the implementation of innovations (Shleifer, 1986), rational herd behavior (Scharfstein and Stein, 1990), endogenous timing and learning (Gul and Lundholm, 1995), firm-specific learning-by-doing (Stein, 1997), high investment in new technologies due to incentives to learn about the curvature of the production function (Johnson, 2007), low expected returns and high investment in risky technologies in the presence of relative wealth concerns (DeMarzo, Kaniel, and Kremer, 2007), and complementarities in investors’ information production (Dow, Goldstein, and Guembel, 2011). Finally, my paper abstracts from behavioral frictions, which might also contribute to the cyclical behavior of VC activity.

2. The Economy

2.1. Agents

The economy is populated by infinitely-lived heterogenous agents with distinct skills, referred to as entrepreneurs, network VCs, white-collar labor, and blue-collar labor. Subscripts $S \in \{E, V, W, B\}$ index these four skill types. Entrepreneurs generate venture ideas. Network VCs are needed to match VC funds with the most productive entrepreneurs. White-collar labor can be employed by firms to implement venture ideas, or by VC funds to support intermediation. Blue-collar labor is required to produce intermediate goods based on existing patents. There is a double-continuum of agents with each skill type, which ensures that each agent is atomistic relative to a given industry.\footnote{Industries are formally introduced in Section 2.4} An agent with a skill $S$ is uniquely identified by a double index $(i'_S, i''_S) \in [0, 1]^2$. The set of all agents in the economy is given by the Cartesian product $\Phi = \{E, V, W, B\} \times [0, 1]^2$. At date 0, agents own identical shares of existing firms and are endowed with their skill.
2.2. Preferences

The presented setup can be thought of as the continuous-time counterpart of a discrete-time economy, where a period lasts for an interval of $\Delta t$, and where I consider the limiting case $\Delta t \to 0$. In this limit, agents’ preferences are described by stochastic differential utility (Duffie and Epstein, 1992a,b), which is a continuous-time version of the recursive preferences of Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). These preferences allow specifying risk aversion and intertemporal elasticity of substitution separately, which is key to matching asset pricing moments that are important for the analysis. The utility index over a consumption process $C$ is defined as:

$$J_t = \mathbb{E}_t \left[ \int_t^\infty m(C_\tau, J_\tau) \, d\tau \right].$$  \hspace{1cm} (1)

Here the function $m(C, J)$ is a normalized aggregator of current consumption and continuation utility that takes the form

$$m(C, J) = \frac{\beta}{\rho} \left( ((1 - \gamma)J)^{1-\frac{1}{1-\gamma}} C^\rho - (1 - \gamma)J \right),$$  \hspace{1cm} (2)

with $\rho \equiv 1 - \frac{1}{\psi}$, where $\beta > 0$ is the rate of time preference, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the elasticity of intertemporal substitution.

2.3. Labor Markets

Blue-collar labor and white-collar labor are supplied inelastically. Each agent with these skills is endowed with one unit of labor, such that the aggregate supply of each type of labor is one. Blue-collar and white-collar labor obtain distinct, potentially time-varying, equilibrium wage rates, denoted by $w_B$ and $w_W$ respectively. The market for entrepreneurs and network VCs will be described in Sections 2.4 and 2.5.

2.4. Production and Development of Intermediate Goods

The production of a final consumption good uses a continuum of intermediate goods. In this section, I discuss the production of intermediate goods based on existing patents, and how ventures’
prospects to generate new innovations vary across heterogeneous entrepreneurs. In Section 2.5 I describe VC funds’ role in facilitating the implementation of the best entrepreneurs’ ideas. In Section 2.6 I describe final good production.

**Intermediate good production.** Intermediate good varieties, also referred to as industries or product lines, are indexed by \( v \in \Psi = [0, 1] \). A firm that owns a patent for an intermediate good needs to employ one unit of blue-collar labor to manufacture one unit of this intermediate good.

**Quality ladder.** The description of innovative activity in the model builds on the quality-ladder technology of Schumpeterian growth models \([\text{Grossman and Helpman} 1991]\). This environment is a fitting starting point, as it captures the basic empirical regularity that innovative activity by new firms leads to creative destruction and endogenous economic growth.\(^{13}\) Process innovations can lead to quality improvements for intermediate goods. Let \( q(v, t) \) denote the quality of the best intermediate good available in industry \( v \) at time \( t \), and let \( M(v, t) \) be a counting process that denotes the number of innovations that occurred in industry \( v \) between time 0 and time \( t \). A quality ladder determines the evolution of the best available intermediate good in each industry:

\[
q(v, t) = \kappa^{M(v, t)} q(v, 0), \quad \forall \ v \text{ and } t, \tag{3}
\]

where \( \kappa > 1 \) and \( q(v, 0) \in \mathbb{R}_+ \). The quality ladder implies that each innovation leads to a proportional quality increase by a factor \( \kappa \). If a firm innovates, it obtains a perpetual patent for the new intermediate good. Yet, as standard in quality-ladder models, the patent system allows other firms to use this know-how to develop new intermediate goods of even higher quality. Further, the economy features the standard restriction that firms owning patents of different qualities cannot contract to share the higher monopoly profits that could be earned through collusion.\(^{14}\)

**Inputs required for innovative activity.** Two inputs are required to undertake a regular venture aimed at developing an intermediate good of higher quality: (1) a venture idea generated

\(^{13}\)See Greenwood and Jovanovic \(1999\), Jovanovic and Rousseau \(2003\), and Jovanovic and Rousseau \(2005\) for empirical evidence on the importance of entry by young firms and creative destruction in times of rapid technological change, such as the Electrification and IT eras.

\(^{14}\)This restriction is also customary in the patent-race literature (see, e.g., Tirole \(1988\) Reinganum \(1989\)).
by an entrepreneur or an existing firm, and (2) one unit of white-collar labor that implements this venture idea. Entrepreneurs and existing firms can generate any number of venture ideas but are heterogenous in their abilities to generate high-quality ideas. In particular, entrepreneurs can obtain temporary productivity shocks allowing them to generate venture ideas in a specific industry that have better prospects of yielding innovations than those generated by all other entrepreneurs or firms. Yet, to realize these superior venture ideas, additional complementing inputs are required: the most productive entrepreneur has to be matched with a VC fund and supported by it, as described below in Section 2.5.

Nature determines the most productive entrepreneurs as follows: at the beginning of each period \([t, t + dt]\), a random slice of the double continuum of entrepreneurs is matched to the single continuum of industries. In particular, in each period, nature draws a uniform \((0, 1)\) i.i.d. random variable \(E_t\). The entrepreneur becoming the most productive in industry \(v\) in that period is then identified by the double index:

\[(i'_{E}, i''_{E}) = (E_t, v).\]  

(4)

The random variable \(E_t\) determines which slice of the double continuum of entrepreneurs obtains a positive shock (that is, it pins down most productive entrepreneurs’ first index element, \(i'_{E}\)). The second index element then determines how these entrepreneurs are exactly associated with specific industries \(v\) — entrepreneurs obtaining positive shocks in industry \(v\) always have the second index element \(i''_{E} = v\). This matching process implies that, with positive probability, an entrepreneur obtains positive productivity shocks repeatedly over time in the same industry \(v\). In addition, in every industry \(v\), there is indeed always exactly one entrepreneur that is the most productive.\(^{15}\)

**Innovation arrivals at the industry level.** Before specifying the success prospects of individual venture ideas it is useful to define the arrival rate of innovations at the industry level. Suppose that a total measure \(n\) of efficiency-adjusted venture ideas are implemented in a given period, where \(n\) will be defined explicitly below. The Poisson arrival rate of a new innovation in this industry is

\(^{15}\)This matching process also ensures that the setup remains tractable when risk sharing is imperfect, which is considered in Section 5.
given by:
\[ h(v, Z, n) = \theta(v, Z) n^\eta, \]  
where \(0 < \eta < 1\), and where \(\theta(v, Z)\) is an exogenous industry-specific productivity process that is governed by the aggregate state variable \(Z\). The state \(Z\) follows a time-homogeneous continuous time Markov chain taking values in the set \(\Omega = \{1, ..., l\}\). The transition rates between states are denoted by \(\lambda_{ZZ'}\) and are collected in the generator matrix \(\Lambda\). I also define \(l \cdot (l - 1)\) counting processes \(N_t(Z, Z')\) that keep track of the number of Markov chain switches between all states that have occurred between time 0 and time \(t\).

Persistent stochastic variation in productivity across industries is essential to the paper’s objective to analyze cyclical behavior. The functional specification for \(h\) features diminishing returns to scale at the industry level, reflecting the notion that ventures undertaken in the same industry are effectively competing for similar innovations. The decreasing returns to scale parameter \(\eta\) will crucially affect the rents VCs obtain from intermediating funds to the most productive entrepreneurs.

**Heterogeneous venture success prospects.** Suppose that a measure \(n_1\) of the most productive entrepreneur’s superior venture ideas is undertaken. The Poisson intensity with which one of these ideas succeeds is given by:
\[ h_1(v, Z, n_1) = \theta(v, Z) n_1^\eta. \]  
In contrast, if a measure \(n_2\) of regular venture ideas — generated by any other entrepreneur or firm — is implemented, one of these ventures succeeds with Poisson intensity:
\[ h_2(v, Z, n_1, n_2) = h(v, Z, n_1 + \phi n_2) - h_1(v, Z, n_1), \]  
where \(\phi \in (0, 1)\) represents an efficiency discount. In case an innovation arrival generated by (7) occurs, ventures in the measure \(n_2\) have a uniform conditional probability of being the winner of the new patent. I define the total efficiency-adjusted measure of ventures as \(n \equiv n_1 + \phi n_2\).

The venture success prospects defined in equations (6) and (7) differ in two ways. First, whereas the success prospects of the most productive entrepreneur, \(h_1\), depend only on the quantity of
his own ideas that are implemented, \( n_1 \), regular ideas’ success prospects, \( h_2 \), depend on both \( n_1 \) and \( n_2 \). This difference can be interpreted as an intra-period first-mover advantage for the most productive entrepreneur: due to decreasing marginal returns at the industry level, the most productive entrepreneur can “skim off” venture ideas with the highest marginal success rates, leaving the remaining less productive ideas to regular entrepreneurs and firms. The second difference is the efficiency discount \( \phi \) that is applied to regular projects. This discount implies that a given measure of superior ideas generated by the most productive entrepreneur is always more productive than an identical measure of regular venture ideas. Moreover, as described in the next subsection, an overall productivity advantage will continue to exist after accounting for the supporting efforts that VC funds have to provide to implement these superior ideas. Thus, the best entrepreneur does not just have a first-mover advantage, but she is also indeed fundamentally more productive. As explained in Section 4.1.2, I will use empirical estimates from the existing literature (e.g., from Kortum and Lerner [2000]) to discipline parameters such as the efficiency discount \( \phi \) and the decreasing returns to scale parameter \( \eta \).

2.5. VC Funds

Consistent with the findings of the empirical literature, VC funds in the model can obtain access to “proprietary deal flow” by hiring VC managers with industry connections and expertise.\(^{[16]}\)

**VC fund technology.** As stated in Section 2.4, a most productive entrepreneur’s venture ideas cannot be implemented unless she is supported by a VC fund. In particular, a VC fund has to hire two types of human capital to be able to match with the most productive entrepreneur in a given industry and vintage and to support her venture’s implementation: (1) a network VC that currently has connections in the industry and (2) \( c_i \) units of white-collar labor that provide additional “generalist” support to this entrepreneur.\(^{[17]}\)

Let \( \iota_1(v, t) \in \{0, 1\} \) denote an indicator variable that takes the value 1 if, in equilibrium, a VC

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\(^{[16]}\) See footnote 6 for papers documenting these characteristic features of the VC industry.

\(^{[17]}\) Fund managers with white collar human capital are “generalists” in the sense that their human capital could also be used in other industries and in other functions. In contrast, a network VC’s connection is specific to a particular industry.
fund has this human capital to match with the best entrepreneur of the date-t vintage in industry \( v \). As agents require compensation commensurate with their outside options in the labor market, it is possible that no VC fund optimally hires the necessary human capital needed in a particular industry and period. Thus, a network VC’s connections and a best entrepreneur’s ideas may stay unused in equilibrium (then \( t_1(v, t) = 0 \)).

**Network VCs’ connections.** Network VCs’ connections to industries are governed by a matching process that is similar to the one that determines the most productive entrepreneurs. At the beginning of each period \([t, t + dt)\), nature determines a random slice of the double continuum of network VCs that has useful connections in the single continuum of industries. In particular, nature draws a uniform \((0, 1)\) i.i.d. random variable \( \mathfrak{U}_t \). The network VC that has useful connections in industry \( v \) in that period is then identified by the double index:

\[
(i_V', i_V'') = (\mathfrak{U}_t, v).
\]

(8)

As in the case of the process determining the best entrepreneurs, network VCs obtain useful connections repeatedly over time in the same industry with positive probability, and at each point in time, one network VC has useful connections in a given industry.\(^{18}\)

**VC fund objective and compensation.** VC funds raise capital from agents to finance ventures of a particular industry and vintage, where a vintage refers to the set of ventures of a specific period. Ventures that succeed are sold by a VC fund through an IPO or an M&A transaction.\(^{19}\) The proceeds from these transactions are paid out to VC fund share holders. A VC fund maximizes the market value of expected proceeds from IPOs and acquisitions, net of fund managers’ and entrepreneurs’ compensation and venture investment costs. There is free entry for new VC funds in each period.

Competition among VC funds for talent implies that the compensation for connected network VCs and the most productive entrepreneurs is bid up until these agents can extract the private

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\(^{18}\)This specification also ensures a tractable analysis of the model when risk sharing is imperfect (see Section 5).

\(^{19}\)Ventures and their new patent may be acquired by other existing firms, provided that each of these firms has only a finite number of patents corresponding to a zero measure of industries.
surplus created from funding the most productive entrepreneur. The allocation of this surplus, if available in equilibrium, is determined by Nash (1950) bargaining between the connected network VC and the most productive entrepreneur in a given industry. A network VCs’ bargaining power is given by \( \varphi \in [0, 1] \). Network VCs and entrepreneurs are compensated with venture equity claims.20

Other innovative activity. All entities that fund regular venture ideas (e.g., existing firms) have identical investment opportunities, act competitively, and fund venture ideas as long as they can make at least zero profits. Thus, these entities do not internalize external diminishing returns of regular venture ideas specified in equation (7). Yet, consistent with the interpretation of (6) and (7) as reflecting an intra-period first-mover advantage for the most productive entrepreneur, entities funding regular venture ideas can condition their decisions on the period’s realization of \( n_1 \).21

Contrary to these entities, a VC fund that matched with the most productive entrepreneur in an industry has an exclusive relationship with that entrepreneur, and internalizes diminishing returns to scale among her ideas (see the definition of \( h_1 \) in equation (6)). Ceteris paribus, this effect reduces the quantity of venture ideas a VC firm funds. On the other hand, a most productive entrepreneur’s venture ideas are more productive. To streamline the analysis, I will maintain the assumption that \( \phi < \eta \), which ensures that the productivity channel dominates, that is: holding market prices of patents fixed, a VC fund that matched with a best entrepreneur funds strictly more efficiency-adjusted venture ideas than normal competitive firms with access to regular venture ideas would.

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20 Since ventures either fail or succeed, and since the value of a venture conditional on a success is known at the beginning of a each period, both equity and debt contracts can achieve the same payoff profile, yielding either zero or a fraction of the successful venture’s value. In practice, VCs and entrepreneurs typically hold a substantial fraction of start-ups’ securities (see, e.g., Barry, Muscarella, Peavy, and Vetsuypens 1990; Korteweg and Sorensen 2010).

21 This assumption is relevant only in cases when VC funds optimally choose to operate probabilistically in a given industry (i.e., use mixed strategies). Yet this will not be an optimal behavior under the considered calibration. In the case of pure strategies, the equilibrium value of \( n_1 \) can be perfectly anticipated at the beginning of a period.
2.6. Final Good Production

The final good is produced by competitive firms according to the Cobb-Douglas production function

\[ Y_t = A_t \cdot \exp \left( \int \Psi \log [q(v,t) \cdot x(v,t|q)] \, dv \right), \tag{9} \]

where \( x(v,t|q) \) is the quantity of the intermediate good in industry \( v \) of quality \( q \) used in the production process.\(^{22}\) As standard in Schumpeterian growth models, there is no storage technology for intermediate and final goods — produced intermediate goods are used in final good production in the same period, and the output flow of the final good \( Y_t \) equals the flow of aggregate consumption \( C_t \) in equilibrium. The representation of the final good production function \(^9\) already indicates that, in equilibrium, the best intermediate good in each industry, which is of quality \( q(v,t) \), will be used in the production of the final good. The factor \( A_t \) in equation \(^9\) follows the stochastic differential equation

\[ \frac{dA_t}{A_t} = \delta(Z_t) \, dt + \sigma(Z_t) \, dB_t, \quad \text{with } A(0) > 0, \tag{10} \]

where \( B_t \) is a standard Brownian motion, and where the local drift \( \delta \) and the local risk exposure \( \sigma \) depend on the Markov state \( Z_t \). The factor \( A_t \) allows the model to account for sources of aggregate growth and uncertainty other than creative destruction, which is essential for an empirically plausible calibration of the model.

2.7. Risk Sharing

In the baseline model, agents are free to trade financial claims in a frictionless Walrasian market where Arrow Debreu securities are in zero net supply. In Section \(^5\) I relax this assumption and analyze the model’s implications when risks associated with the performance-sensitive components of venture capital compensation cannot be shared by agents via financial side contracts.

\(^{22}\) Apart from the factor \( A_t \), the specification for the final good production function \(^9\) is standard in the Schumpeterian growth literature. See, e.g., François and Lloyd-Ellis \((2003)\) and Klette and Kortum \((2004)\).
3. Decentralized Equilibrium

In the following, I analyze symmetric perfect Bayesian Markov equilibria of the decentralized economy. When structural parameters differ across industries, a symmetric equilibrium refers to symmetry among all industries with identical parameters. Standard definitions of an allocation and a decentralized equilibrium are relegated to Appendix A.2.

Aggregate state variables in the economy are the Markov state $Z$ and the level of aggregate output $Y$. Due to the iso-elastic properties of the setup, the economy scales by $Y$. Throughout, I will take advantage of the economy’s scaling property and directly consider prices and compensation rates scaled by output $Y$. I mark these scaled variables by a tilde. For example, the blue-collar and white-collar wage-to-output ratios will be denoted by $\tilde{w}_B(Z)$ and $\tilde{w}_W(Z)$, respectively.

Below, I first analyze industry-level equilibrium conditions taking as given the drift and volatility of aggregate consumption, denoted by $\mu(Z)$ and $\sigma(Z)$ respectively, the stochastic discount factor, denoted by $\xi_t$, and the blue-collar and white-collar wage-to-output ratios. Thereafter, I characterize the equilibrium conditions determining these endogenous aggregate variables.

3.1. Industry-level Quantities and Prices

3.1.1. Production

Producers of the final good behave competitively and take the prices of intermediate goods of various available qualities in each industry as given. The intermediate good producer with the highest-quality patent in an industry is the incumbent firm. Prices for intermediate goods, $p^x(v, t|q)$, are determined in the monopolistically competitive intermediate goods market. A final good producer’s first-order condition yields the unit elastic intermediate good demand:

$$x(v, t|q) = \frac{Y_t}{p^x(v, t|q)}.$$  \hspace{1cm} (11)

Given this intermediate good demand, the incumbent firm in an industry limits prices at the marginal cost of the previous incumbent who could enter using his inferior patent. If the previous
incumbent’s patent quality is inferior by a factor $\kappa$, then the profit-maximizing monopoly price for the highest quality intermediate good is given by:

$$p^x(v,t|q) = \kappa \cdot w_B(Y_t, Z_t).$$  \hspace{1cm} (12)

Thus, the incumbent firm with the highest-quality patent in industry $v$ earns the following monopoly profits:

$$\pi_M(v,t) = (p^x(v,t|q) - w_B(Y_t, Z_t)) \cdot x(v,t|q) = \left(1 - \frac{1}{\kappa}\right) Y_t. \hspace{1cm} (13)$$

3.1.2. Venture Funding

Due to the above-mentioned scaling property of the economy, strategies determining the measures of ventures receiving funding ($n_1$ and $n_2$) do not depend on the current level of aggregate output $Y$. Yet venture investment denominated in final good consumption units will depend on $Y$.

**Funding by VC funds.** First, consider the maximization problem of a VC fund that successfully matched with the best entrepreneur in industry $v$, such that $\iota_1(v,t) = 1$. The maximum surplus this VC fund can attain by funding the most productive entrepreneur’s venture ideas is given by:

$$\tilde{\pi}_1(v,Z) = \max_{n_1 \in \mathbb{R}^+_0} \left\{ h_1(v,Z,n_1) \tilde{P}(v,Z) - (n_1 + c_i) \tilde{w}_W(Z) \right\}. \hspace{1cm} (14)$$

The VC fund takes the market price of a successful venture, $\tilde{P}$, and the white-collar compensation rate, $\tilde{w}_W$, as given and chooses the mass of ventures it funds, $n_1$. When a venture succeeds, which happens with Poisson intensity $h_1$, a VC fund obtains revenues from selling the firm in an IPO or M&A transaction at the market price $\tilde{P}$. For expositional convenience, I refer to the expected market value of these transactions as **VC-backed IPO volume**, and introduce the corresponding variable $\tilde{ipo} \equiv h_1 \tilde{P}$.

The VC fund requires capital commitments from investors, which I define as $\tilde{com} \equiv (n_1 + c_i) \tilde{w}_W$. These commitments are used to make venture investments, $n_1 \tilde{w}_W$, and to pay the part of VC
fund managers’ compensation that rewards white-collar human capital, $c_t \tilde{w}_W$. This performance-insensitive part of fund manager compensation is akin to the “management fee” paid in practice. For states where $\tilde{c}om > 0$, I define $f^{man} \equiv \frac{c_t \tilde{w}_W}{\tilde{c}om}$ as the fraction of capital commitments that is used for this type of compensation.

The first-order condition associated with the maximization problem (14) yields the optimal scale of ventures a VC fund finances:

$$n_1(v, Z) = \left( \eta \theta(v, Z) \frac{\tilde{P}(v, Z)}{\tilde{w}_W(Z)} \right)^{\frac{1}{1-\eta}}. \quad (15)$$

If no VC fund matches with the most productive entrepreneur in a given industry (that is, if $\iota_1(v, t) = 0$), the entrepreneur’s venture ideas cannot be implemented, implying that $n_1 = 0$.

Matching of VCs with entrepreneurs. A connected network VC chooses the probability with which she accepts the best available compensation offer from a VC fund and facilitates the fund’s matching with a best entrepreneur. I denote the corresponding Markov strategy by $\tilde{\iota}_1(v, Z_t) \equiv \text{Pr}[\iota_1(v, t) = 1|Z_t]$. Competition between VC funds implies that any positive surplus $\tilde{\pi}_1 > 0$, if it exists, will be appropriated by the connected network VC and the most productive entrepreneur, both of which are needed to implement the most productive entrepreneur’s ideas. These two agents jointly obtain a corresponding venture ownership share $\tilde{\pi}_1 \tilde{ipo}$, whenever $\tilde{ipo} > 0$. The connected network VC and the most productive entrepreneur Nash bargain over this stake and obtain fractions $\varrho$ and $(1 - \varrho)$ of it, respectively. The stake a network VC obtains is akin to a performance-sensitive “carry” fee observed in practice, which yields a payoff only in case a venture succeeds.$^{23}$ For states where capital commitments are positive, let $f^{car} \equiv \frac{\varrho \tilde{\pi}_1}{\tilde{com}}$ denote the expected value of this performance-sensitive carry fee as a fraction of commitments.

A connected network VC will facilitate the matching process only if a VC fund can generate a weakly positive value $\tilde{\pi}_1 \geq 0$, and can thus offer the network VC a stake with weakly positive value $\varrho \tilde{\pi}_1 \geq 0$. Formally, the probability of matching $\tilde{\iota}_1(v, Z)$ satisfies the following equilibrium

---

$^{23}$ Consistent with carry fees in practice, this fee is paid only when fund investors obtain a positive return on their investment.
conditions:
\[
\bar{\iota}_1(v, Z) = 0 \quad \text{if } \varrho \tilde{\pi}_1(v, Z) < 0,
\]
\[
\bar{\iota}_1(v, Z) \in [0, 1] \quad \text{if } \varrho \tilde{\pi}_1(v, Z) = 0,
\]
\[
\bar{\iota}_1(v, Z) = 1 \quad \text{if } \varrho \tilde{\pi}_1(v, Z) > 0.
\] (16)

Abnormal returns from VC investments. As VC funds can generate economic profits from their investments, it is useful to clarify to which extent VC fund investors obtain “abnormal” returns. Let \( npv \) denote the net present value of a VC fund’s investments before compensating VC fund managers, that is,

\[
\tilde{npv} \equiv \tilde{ipo} - (n_1 \tilde{w}_W + (1 - \varrho)\tilde{\pi}_1).
\] (17)

A VC fund’s \( npv \) is given by the market value of funded ventures minus the sum of venture investment cost and the market value of the entrepreneur’s equity stake. I define \( \alpha \) as the ratio of this \( npv \) to capital commitments, that is, \( \alpha \equiv \frac{\tilde{npv}}{com} \). This ratio may be viewed as a measure of abnormal returns from VC investments in that it does not represent a compensation for exposures to aggregate risk.\(^{24}\) The above results imply that, in equilibrium, the total value of management and carry fees equals the value of VC funds’ economic profits. Thus, net of fees, fund investors obtain zero abnormal returns on their investments:

\[
\alpha - f_{\text{man}} - f_{\text{car}} = 0.
\] (18)

As in Berk and Green’s (2004) partial equilibrium model of the mutual fund industry, competition for VC fund managers’ talent ensures that fund investors make zero “net-alpha” in equilibrium. That is, they obtain an after-fee risk-return profile that is as good as the one attainable in competitive financial markets. This view is consistent with several empirical studies in the VC literature finding that returns to investors net of fees are approximately zero.\(^{25}\)

\(^{24}\)As this measure capitalizes economic profits as a net present value, it is closely related to the generalized market equivalent measure proposed by Korteweg and Nagel (2014).

\(^{25}\)See, e.g., Hall and Woodward (2007) and Korteweg and Nagel (2014).
Funding of regular venture ideas. The measure of regular venture ideas implemented by new or existing firms is determined by the free-entry condition:

\[ n_2(v, Z, n_1) = \max \left\{ n_2 \in \mathbb{R}^+_0 : \frac{h_2(v, Z, n_1, n_2)}{n_2} \hat{P}(v, Z) - \hat{w}_W(Z) \geq 0 \right\} . \tag{19} \]

Condition (19) reflects that regular venture ideas are funded up to the point at which firms break even in expectation — the market value of the average regular venture equals the cost of venture idea implementation. The success intensity per regular venture is given by \( \frac{h_2}{n_2} \). Each venture incurs cost \( \hat{w}_W \) from compensating one unit of white-collar labor, which is needed to implement the venture idea. Whereas these agents obtain compensation commensurate with their outside options in the labor market, entrepreneurs do not obtain an extra surplus for generating regular venture ideas, as these ideas can be generated by any entrepreneur or firm, and are thus not scarce.

Following the optimal strategy (15), a VC fund extends funding to the most productive entrepreneur until the marginal benefit of implementing superior ideas is equal to the cost of implementation. Since regular venture ideas are strictly inferior to the most productive entrepreneur’s marginal venture idea, plugging (15) into condition (19) implies that no additional regular venture ideas will be funded \((n_2 = 0)\) in periods when the most productive entrepreneur already obtained funding \((\iota_1 = 1)\). Regular venture ideas are therefore funded only when matching with the best entrepreneur did not occur \((\iota_1 = 0)\). In this case, condition (19) yields:

\[ n_2(v, Z, 0) = \frac{1}{\phi} \left( \frac{\phi \theta(v, Z_t) \hat{P}(v, Z_t)}{\hat{w}_W(Z_t)} \right)^{\frac{1}{1 - \eta}} . \tag{20} \]

Funding by existing firms. Firms solve a different optimization problem when deciding on the funding of ideas in product lines \( v \) where they already have the best patent. An incumbent takes into account that it cannibalizes its own existing product. In contrast, firms that do not have the best product can steal business from the existing incumbent, implying that they have stronger private incentives to innovate. Thus, Arrow’s replacement effect, a typical feature of quality ladder models, applies — an incumbent firm optimally does not fund additional ventures in product lines where it
already has the best patent (see a formal proof in Appendix A.3) An incumbent can, however, fund ventures to enter new product lines and steal business from other firms, as in the model of Klette and Kortum (2004). In this case, incumbents solve the same maximization problem as all other entities funding regular ideas — the free entry condition (19) applies. Since it is not uniquely determined whether the measure \( n_2 \) of regular venture ideas is funded by established or new firms, and since the funding of regular ideas does not involve active VC intermediation, I will not interpret \( n_2 \) as VC-backed innovative activity going forward.

The above analysis leads to the following Proposition characterizing the equilibrium rate of creative destruction at the industry level.

**PROPOSITION 1** (Rate of creative destruction). The expected rate of creative destruction in industry \( v \) is given by:

\[
\bar{h}(v, Z) = \bar{\iota}_1(v, Z) h(v, n_1(v, Z)) + (1 - \bar{\iota}_1(v, Z)) h(v, Z, \phi n_2(v, Z, 0)),
\]

where the Markov strategies \( \bar{\iota}_1(v, Z_t) \), \( n_1(v, Z_t) \), and \( n_2(v, Z_t, 0) \) are characterized in equations (16), (15), and (20), respectively.

As VC firms’ endogenous intermediation decisions alter barriers to entry for the most productive entrepreneurs, they also affect the rate of creative destruction. The arrival rate \( \bar{h}(v, Z) \) represents both the rate of new entry by firms into industries and the rate of exit by incumbents that are getting displaced by these new entrants. As entry poses a competitive threat, the arrival rate \( \bar{h}(v, Z) \) has important effects on firms’ post-IPO prices and returns. I discuss these effects in greater detail in the next subsection.

### 3.1.3. Valuations in Competitive Financial Markets

Building on the results developed so far, I now characterize the market valuations of ventures that succeed in developing a new patent.

\[^{26}\text{Greenwood and Jovanovic (1999) further argue that incumbents are at a relative disadvantage in implementing new technologies during times of technological revolutions.}\]

\[^{27}\text{Formally, this interpretation is feasible as long as each incumbent firm is restricted to have the best patents in a finite number of product lines.}\]
**PROPOSITION 2** (Market valuations). The market price of a firm that holds the patent to the highest-quality intermediate good in industry $v$ is:

$$P (v, Y_t, Z_t) = \mathbb{E}_t \left[ \int_t^{\tau^*} \frac{\xi_t}{\xi_t} \pi_M (v, \tau) d\tau \right] = \tilde{P} (v, Z_t) \cdot Y_t,$$

(22)

where $\tau^* \equiv \sup \{ \tau : M (v, \tau) = M (v, t) \}$, and where the profit flow rate $\pi_M (v, t)$ is given in equation (13). The Hamilton Jacobi Bellman (HJB) equation associated with (22) implies that the function $\tilde{P} (v, Z)$ solves the following system of equations for all $Z \in \Omega$:

$$0 = 1 - \frac{1}{\kappa} - (r_f (Z) + rp^D (v, Z) + rp^J (v, Z) + \bar{h} (v, Z) - \mu (Z)) \tilde{P} (v, Z)$$

$$+ \sum_{Z' \neq Z} \lambda_{ZZ'} \left( \tilde{P} (v, Z') - \tilde{P} (v, Z) \right).$$

(23)

The expressions for the risk free rate $r_f$, the diffusion-risk premium $rp^D$, and the jump-risk premium $rp^J$ are provided in Appendices A.3 and A.5.

**Proof.** See Appendices A.3 and A.5.

Proposition 2 highlights that the industry-specific rates of creative destruction, $\bar{h}(v, Z)$, have important implications for ventures’ post-IPO market valuations. A firm has a higher propensity of losing its new incumbent position to yet another entrant when the rate of entry is expected to stay persistently high. Since the rates of creative destruction also vary with the aggregate state $Z$, they further affect exposures to aggregate macroeconomic conditions and thus risk premia, in particular the jump-risk premium $rp^J$. As the VC industry facilitates entry of new firms, its equilibrium behavior affects these risk exposures. A firm’s competitive expected return post IPO is given by the sum of the risk-free rate $r_f$ and the diffusion and jump risk premia, $rp^D$ and $rp^J$.

### 3.2. Aggregate Quantities and Prices

**Output.** The following proposition characterizes the dynamics of aggregate output $Y$, which equals aggregate consumption $C$ in equilibrium.
PROPOSITION 3 (Aggregate output). Aggregate output follows the stochastic differential equation
\[
\frac{dY_t}{Y_t} = \mu(Z_t) \, dt + \sigma(Z_t) \, dB_t,
\]
where the local drift $\mu$ takes the following form:
\[
\mu(Z_t) = \delta(Z_t) + \log [\kappa] \int \bar{h}(v, Z_t) \, dv.
\]

Proof. See Appendix A.4.

Proposition 3 shows that the local drift of aggregate output, $\mu(Z_t)$, can be decomposed into the exogenous component $\delta(Z_t)$ and an endogenous component that depends on the equilibrium rates of creative destruction across industries, $\bar{h}(v, Z_t)$.

Stochastic discount factor. In the baseline model, agents can diversify idiosyncratic risks in equilibrium. The following Proposition characterizes the corresponding dynamics of the stochastic discount factor $\xi_t$.

PROPOSITION 4 (Stochastic discount factor). The stochastic discount factor follows a Markov-modulated jump-diffusion,
\[
\frac{d\xi_t}{\xi_{t-}} = -r_f(Z_{t-}) \, dt - \gamma\sigma(Z_{t-}) \, dB_t + \sum_{Z' \neq Z_{t-}} (e^{\zeta(Z_{t-}, Z')} - 1) (dN_t(Z_{t-}, Z') - \lambda_{Z_{t-}, Z'} dt),
\]
where the function $\zeta(Z, Z')$ is determined by agents’ HJB equation provided in Appendix A.5.

Proof. See Appendix A.5.

In Section 5, I will analyze the model under the assumption of imperfect risk sharing.

Labor markets and wages. Combining the optimal intermediate good demand (11) and intermediate goods prices (12) with the fact that one unit of blue-collar labor produces one unit of the
final good leads to the following market-clearing condition for blue-collar labor:

\[ \int_{\Psi} x(v, t|q) \, dv = \frac{1}{\kappa \cdot \bar{w}_B(Z_t)} = 1. \] (27)

It follows that the blue-collar wage-to-output ratio is constant and given by \( \bar{w}_B = 1/\kappa \). The white-collar wage-to-output ratio \( \bar{w}_W(Z_t) \) solves the market clearing condition:

\[ \int_{\Psi} \left[ \tilde{\iota}_1(v, Z) (n_1(v, Z) + c) + (1 - \tilde{\iota}_1(v, Z)) \, n_2(v, Z, 0) \right] \, dv = 1. \] (28)

The Markov strategies \( \tilde{\iota}_1(v, Z), n_1(v, Z) \), and \( n_2(v, Z, 0) \) were characterized in equations (15), (16), and (20) as functions of the white-collar wage rate \( \bar{w}_W \) and the market price \( \bar{P} \). Thus, \( \bar{w}_W \) varies across aggregate states \( Z \), as the demand for white-collar labor from VC funds and innovative firms responds to forward-looking information encoded in market prices, \( \bar{P} \).

4. Calibration and Evaluation

In this section, I calibrate the model and evaluate its predictions. The calibration aims to match the dynamics of a set of key variables relevant for macroeconomic dynamics, asset prices, and VC activity in U.S. data, such as consumption growth, risk free rates, risk prices, and the magnitude and cyclicality of VC intermediation.

4.1. Choosing Parameters

Below, I first discuss preference parameters and aggregate consumption dynamics before addressing parameter choices relating to VC activity and endogenous growth.

4.1.1. Consumption Dynamics and Preference Parameters

I parameterize the model based on a nine-state Markov chain for the aggregate state \( Z \). The exogenous drift component of aggregate output growth, \( \delta \), and the local volatility, \( \sigma \), are chosen so that the resulting equilibrium consumption process matches the one estimated in [Chen (2010)].
state by state. Throughout, I present state-dependent parameter values and equilibrium outcomes in three-by-three matrices that sort the nine Markov states along two dimensions: expected consumption growth, $\mu$, and local volatility, $\sigma$ (each categorized into low, med, and high). Table 4 in Appendix A.1 reports the values of $\mu$ and $\sigma$, and Table 5 lists the unconditional probabilities of all Markov states. It is worth noting that every Markov state $Z$ is associated with a distinct drift $\mu(Z)$ — for example, there is still variation in the drift across states categorized as “high $\mu$” in the tables. Table 6 in Appendix A.1 provides the values of the exogenous and endogenous components of the aggregate growth drifts.

In addition, I choose the same preference parameters ($\beta$, $\psi$, and $\gamma$) as in Chen (2010) (see Table 1). Together, these choices discipline consumption dynamics and asset pricing moments of the economy and ensure reasonable aggregate risk premia and risk free rate dynamics, thus providing a good laboratory for studying macroeconomic conditions and venture capital activity.

### 4.1.2. VC Activity and Endogenous Growth

The remaining parameters of the model relate to VC activity and the endogenous component of growth. VC activity is empirically concentrated in specific industries such as IT and Pharma, and has historically been highly volatile and cyclical. To capture these empirical regularities, I consider a parsimonious parameterization of the economy with two subsets of industries, denoted by $\Psi'$ and $\Psi''$. Across these two sets of industries, structural parameters and productivity dynamics differ. I designate the set of industries $\Psi'$ as one in which VC intermediation may occur at times, and the set $\Psi''$ as one in which VC intermediation does not obtain. The second set aims to represent industries that are atypical candidates for VC activity, as for example the steel industry. This specification

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28 This process approximates the discrete-time model calibration in Bansal and Yaron (2004). I thank Hui Chen for providing the estimated Markov chain generator matrix.
29 Panel (a) of Table 7 in Appendix A.1 reports the risk-free rate in the 9 Markov states. Moreover, Chen (2010) provides a detailed description of the asset pricing implications under this specification for preferences and consumption dynamics. In particular, Chen (2010) reports the properties of risk premia and price dividend ratios of a levered claim to aggregate consumption, a standard approach in the asset pricing literature (see, e.g., Abel 1999, Campbell 2003, Bansal and Yaron 2004).
30 Kortum and Lerner (2000) find for example that the top three industries that obtain venture capital are drugs, office & computing, and communication equipment, constituting 54 percent of VC investments.
32 Cross-sectional allocations of resources would be naturally symmetric across all industries if all industries faced the exact same technological conditions at any point in time.
with two sets of industries keeps the analysis with industry heterogeneity parsimonious, while still capturing the above-mentioned stylized facts. In particular, I assume that the parameter value of \( c_i \) is prohibitively high in industries \( \Psi'' \), reflecting the notion that VC funds’ efforts to support the best entrepreneurs’ in the context of a start-up provides insufficient net-benefits here; in these industries, innovative activity is more profitable if undertaken by other types of (existing) firms.\(^{33}\) Apart from the parameter values of \( c_i \), the two sets of industries differ also in terms of their productivity dynamics, a necessary feature to capture the above-mentioned strong cyclicality of VC activity.

\[\text{Table 1} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of time preference</td>
<td>( \beta )</td>
<td>0.015</td>
</tr>
<tr>
<td>Elasticity of intertemporal substitution</td>
<td>( \psi )</td>
<td>1.500</td>
</tr>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>( \gamma )</td>
<td>7.500</td>
</tr>
<tr>
<td>Efficiency discount of regular venture ideas</td>
<td>( \phi )</td>
<td>0.323</td>
</tr>
<tr>
<td>Jump size of the quality index per innovation</td>
<td>( \kappa )</td>
<td>1.148</td>
</tr>
<tr>
<td>Decreasing returns to scale parameter</td>
<td>( \eta )</td>
<td>0.585</td>
</tr>
<tr>
<td>White-collar labor required for VC fund operation (( v \in \Psi' ))</td>
<td>( c_i )</td>
<td>0.027</td>
</tr>
<tr>
<td>Network VC bargaining power</td>
<td>( \varrho )</td>
<td>0.346</td>
</tr>
<tr>
<td>Mass of industries ( \Psi' )</td>
<td>( \int_{\Psi'} dv )</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Table 1 reports the values of all state-invariant parameters of the model. In the following, I sequentially discuss how various parameter choices are disciplined.

**Measure of industries \( \Psi' \).** The total mass of industries that potentially exhibit active VC intermediation, \( \int_{\Psi'} dv \), is set to match the historical average ratio of capital commitments to the VC industry relative to aggregate consumption, which I estimate to be 0.3 percent based on data reported by the National Venture Capital Association (2012).\(^{34}\)\(^{34}\) Alternatively, the net-benefits of VC activity are reduced if the best entrepreneurs have only marginally better ideas than regular entrepreneurs and firms. The exact channels rendering VC funds’ operation suboptimal in industries \( \Psi'' \) is not essential to the analysis, as this paper does not perform counterfactual analyses to ask under which conditions the industries \( \Psi'' \) might attract VC activity.

\(^{34}\)My estimate is based on capital commitments to VC funds from 1990 to 2010 (National Venture Capital Association, 2012). I scale these numbers by U.S. final consumption expenditure in each year (The World Bank, 2012). To estimate the expected value, I compute the average of this time-series. The calibrated model matches this number,
Innovation step size $\kappa$. The parameter $\kappa$ is chosen to match the innovation step size estimate in Acemoglu, Akcigit, Bloom, and Kerr (2013), which implies choosing $\kappa = 1.148$. This parameter choice is also consistent with empirical evidence on IT sector markups estimated in Ward (2013). \[^{35}\]

Efficiency discount $\phi$. I use Kortum and Lerner’s (2000) patent production function estimates to choose the parameter $\phi$, which measures the relative efficiency of regular and VC-backed innovative activity, and thus affects the benefits of VC intermediation.  \[^{36}\]

White-collar human capital of VC funds $c_i$. The parameter $c_i$ applying to industries $\Psi'$ influences the magnitude of performance-insensitive VC management fees. I set this parameter so that average management fees amount to 3 percent of invested capital, targeting empirical findings by Hall and Woodward (2007). \[^{37}\] Higher values of $c_i$ effectively increase the strike price of the option to provide active VC intermediation, thus making it a less likely equilibrium outcome.

Decreasing returns to scale parameter $\eta$. In the model, the combined payoffs of VC managers and entrepreneurs are tightly linked to the decreasing returns to scale parameter $\eta$, as this parameter influences the magnitude of rents from active VC intermediation. Hall and Woodward (2007) (p. 21) find that general partners’ total earnings (management plus carry fees) are 26 percent of funds invested. Hall and Woodward (2007) further estimate that whereas entrepreneurs historically earned an average of $9.2 million (in 2006 dollars) from each company that attracted VC funding, VC managers received an average of $5.5 million in fee revenue from each company they backed. I set the parameter $\eta$ so that in equilibrium the ratio of the sum of VC fees and entrepreneurial payoffs to capital commitments is consistent with these empirical estimates, which leads me to choose $\eta = 0.585$. \[^{38}\] This parameter value also lies within one standard deviation of the point that is, $E\{f_{\text{man}} \mid \bar{c}\text{om} > 0\} = 0.03$ in industries $v \in \Psi'$.

\[^{35}\] In the model the parameter $\kappa$ pins down the profits-to-sales ratio of an incumbent firm, which is given by $(1 - 1/\kappa)$. Ward (2013) estimates average aggregate IT sector markups from 1972 – 2012 and finds that $1/(1 - \frac{\text{EBITDA}}{\text{Sales}}) - 1 = 0.142$. In my model, this markup is obtained by choosing $\kappa = 1.142$.

\[^{36}\] Kortum and Lerner (2000) estimate a patent production function $P = (R + bV)^\alpha$, where in their specification $P$ refers to patenting, $R$ measures R&D expenditures not funded by VC firms, and $V$ is R&D funded by VC firms. Based on a variety of empirical tests, the authors obtain an average estimate of the parameter $b$ of 3.1 (see footnote 26 in Kortum and Lerner (2000)). This estimate corresponds to a value of 1/3.1 for the parameter $\phi$ in my model.

\[^{37}\] In the calibrated model $E[f_{\text{man}} \mid \bar{c}\text{om} > 0] = 0.03$ in industries $v \in \Psi'$.

\[^{38}\] The total payoffs to VC managers and entrepreneurs as a fraction of capital commitments is thus $0.26 \cdot (1 + \frac{9.2}{5.5}) \approx 0.69$. The calibrated model yields: $E[f_{\text{man}} + f_{\text{car}} + \frac{(1-\phi)\bar{c}\text{om}}{\bar{c}\text{om}} \mid \bar{c}\text{om} > 0] = 0.69$. 27
estimate Kortum and Lerner (2000) obtain for the decreasing returns to scale parameter of their patent production function.\footnote{When using an instrumental variable approach, Kortum and Lerner 2000 estimate a decreasing returns to scale parameter of about 0.52 (see Panel B of Table 4 in Kortum and Lerner 2000).}

**VC bargaining power $\varrho$.** In the model, VC carry fees and entrepreneurial payoffs are bargained fractions $\varrho$ and $(1 - \varrho)$ of the rents $\tilde{\pi}_1$, respectively. I choose the parameter $\varrho$ so that the ratio of carry fees to entrepreneurial payoffs corresponds to the estimates by Hall and Woodward 2007 mentioned above.\footnote{The ratio of total VC fees to the value of payoffs received by entrepreneurs is 5.5/9.2. Since carry fees account for a fraction 0.23/0.26 of total VC fees, the ratio of carry fee payoffs to entrepreneurial payoffs is approximately (5.5 · 0.23/0.26)/9.2, which corresponds to the ratio of bargaining powers in the model, $\frac{\varrho}{1-\varrho}$, and yields $\varrho = 0.346$.} Overall, the calibration thus ensures that the model matches average aggregate VC carry fees, VC management fees, and entrepreneurial payoffs.\footnote{On average, carry fees represent 23 percent of provided capital, as estimated by Hall and Woodward 2007. In the calibrated model $E[f_{car} | \tilde{com}] > 0 = 0.23$ in industries $v \in \Psi'$.}

**Productivity $\theta$.** The only remaining state-dependent parameter of the model is productivity $\theta$. The productivity processes in the two sets of industries $\Psi'$ and $\Psi''$ have different means and are specified as differentially exposed to macroeconomic conditions, as measured by the state-contingent consumption growth drifts estimated in Chen 2010. For industries $\Psi'$, I specify a non-linear relation that ensures that the model matches the highly cyclical and skewed distribution of aggregate commitments to VC funds in the data. The caption of Table 2 reports this relation as well as the corresponding state-contingent values of $\theta$. The specification is characterized by three parameters relating to the mean, the volatility, and the skewness of the productivity distribution. Mean productivity is set so that industries $\Psi'$ exhibit an average rate of creative destruction that is representative of the types of sectors that receive VC funding. In particular, using estimates from Caballero and Jaffe 1993, I match an average rate of creative destruction of 8 percent.\footnote{This number is the average estimated rate of creative destruction across the sectors “Drugs,” “Computers and data processing,” “Electronic communications,” and “Medical” reported in Table 5 of Caballero and Jaffe 1993.} The remaining two parameters are set so that the volatility and the peak levels of aggregate capital commitments to VC funds in the model match their data counterparts.\footnote{Scaled commitments are measured as discussed in footnote 34. The standard deviation of these empirical values is 0.3 percent. Correspondingly, in the calibrated model $Std[\tilde{com}] = 0.3$ percent. The historical peak level of annual commitments was 1.3 percent of U.S. final consumption expenditure, and was reached in the year 2000. In the model this level is reached in the high-\(\mu\)/high-\(\sigma\) state (see Table 2 b).} Empirical peak levels were reached at the end of the internet boom, when aggregate commitments scaled by aggregate
consumption were more than three standard deviations above their average value. The proposed specification also replicates the strong correlation between VC activity and aggregate conditions, consistent with existing empirical evidence. Finally, as discussed below in Section 4.2.1 these dynamics for $\theta$ do not just ensure that the calibration matches the empirical value of the inputs used for VC investments (as reflected by VC commitments), but also the value of the output they generate.

For industries $\Psi''$, I consider a simple linear specification for $\theta$ that is characterized by two parameters determining average productivity and the exposure to macroeconomic conditions (see caption of Table 8 in Appendix A.1). I set the mean of $\theta$ in industries $\Psi''$ so that the average rates of creative destruction across all industries $\Psi$ matches the average rate of creative destruction in the U.S. economy of 4 percent per year, as estimated by Caballero and Jaffe (1993). The exposure parameter is set so that the standard deviation of white-collar wage-to-output ratios relative to their mean, $\frac{\text{Std} \left[ \tilde{w} \right]}{E \left[ \tilde{w} \right]}$, is consistent with empirical estimates obtained from income shares data for the top 5 percent of income earners in the United States. White-collar wage-to-output ratios are reported in Table 7.b. I associate income fluctuations for white-collar labor with those for top income earners, as agents from this group are likely to be recruited for innovation- and finance-related jobs in practice. An important feature captured by the calibration is the empirical fact that top earners’ income share has substantial positive comovement with the business cycle (see Piketty and Saez, 2003; Parker and Vissing-Jorgensen 2009, 2010). Consistent with this observation, white-collar human capital, which is essential for economic growth, can appropriate a larger fraction of aggregate output in states with high aggregate growth (see high $\mu$ states in Table 7.b).

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45 I use data provided by Alvaredo, Atkinson, Piketty, and Saez (2012). After de-trending income shares for the top 5 percent (incl. capital gains), I find that the volatility of income shares relative to their mean was 5.5 percent during the time period from 1990 to 2010. I focus on this time period, since top income and wage shares display a U-shaped pattern over the last century (see Piketty and Saez, 2003). Since the 1980s there has been an upward trend in the income share of the top 1 percent and the top 5 percent.

46 Naturally, there is no single percentile cut-off in the income space that provides a clear-cut match to income for white-collar labor in the model. In this regard, it is worth noting that empirically, the income share of the top 1 percent also drives much of the variation in the top 5 percent.
4.2. Results of the Calibration

In this section, I analyze the implications of the calibrated model. First, I discuss aggregate measures of VC intermediation and evaluate the impact of VC investment on the macroeconomy. Second, I analyze the payoffs and returns to various parties involved in the VC process. Tables 2 and 3 report outcomes in VC-backed industries $\Psi'$ in each of the nine Markov states.

4.2.1. The Aggregate Impact of VC Intermediation

Capital commitments. Table 2a indicates the states in which VC funds provide active intermediation. Table 2b reports the capital commitments the VC industry obtains as a fraction of aggregate consumption. In boom states (high $\mu$ states), the VC industry attracts larger commitments, with peak levels reaching magnitudes comparable to those observed during the internet boom. In contrast, in recessions (low $\mu$ states), VC funds are unable to attract capital, as rents from active intermediation are insufficient to cover management fees that compensate VC managers’ white-collar human capital. As a result, active VC intermediation breaks down in these states.

VC-backed IPO and M&A volume. Table 2d reports the aggregate market value of VC-backed IPOs and M&A transactions scaled by aggregate consumption.\(^{47}\) Consistent with the findings of the empirical literature, the model features large booms and busts in transaction volumes that are strongly correlated with VC fund commitments and aggregate conditions.\(^{48}\) The calibration provides a quite remarkable fit of empirical moments relating to IPO and M&A volume, which were not targeted in the calibration. On average, transactions amount to 0.5 percent of aggregate consumption in the model, whereas in the data the average is 0.6 percent.\(^{49}\) The standard deviation is 0.6 percent both in the data and in the model. Aggregate transactions peaked during the internet boom at 2.4 percent, and reach 2.1 percent in the model.

The close match of both VC capital commitments (i.e., the value of VC inputs) and VC exit

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\(^{47}\) As noted in Section 3.1.2, successful ventures can either undertake an IPO or be acquired by an existing firm.

\(^{48}\) See, e.g., Gompers and Lerner (1998) and Jeng and Wells (2000).

\(^{49}\) Estimates are based on data published by the National Venture Capital Association (2012) for the time period from 1990 to 2010. I add the “Post Offer Value” of venture-backed IPOs to prices of known transactions from venture-backed Mergers & Acquisitions. I scale the sum of these two valuations by U.S. final consumption expenditure (The World Bank, 2012) in each year.
Table 2
State-contingent Outcomes in VC-Backed Industries $\Psi'$

The panels report state-contingent outcomes in VC-backed industries $\Psi'$. Panel (a) indicates in which states VC firms provide active intermediation. Panel (b) reports capital commitments to VC funds scaled by aggregate consumption. Panels (c) and (d) list the rates of creative destruction and the aggregate value of VC-backed IPOs and acquisitions scaled by aggregate consumption. Panels (e) and (f) report productivity and the ratio of entry rates to productivity. The state-contingent productivity values follow from the calibrated relation:

$$\theta(v, Z) = a + b \cdot (\mu(Z) - \min_{\forall Z}\{\mu(Z)\})^c,$$

where, $a = 0.0063$, $b = 24.53$, $c = 1.85$, and $\mu(Z)$ are the aggregate growth drifts from Chen (2010).

<table>
<thead>
<tr>
<th>(a) Active VC intermediation: $\bar{\iota}_1(v, Z)$</th>
<th>(b) Capital commitments: $\int_{\Psi'} \tilde{c}om(v, Z)dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>med</td>
</tr>
<tr>
<td>high</td>
<td>1</td>
</tr>
<tr>
<td>$\mu$</td>
<td>med</td>
</tr>
<tr>
<td>low</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Rate of creative destruction: $\bar{h}(v, Z)$</th>
<th>(d) VC-backed IPOs: $\int_{\Psi'} \tilde{ipo}(v, Z)dv$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>med</td>
</tr>
<tr>
<td>high</td>
<td>0.223</td>
</tr>
<tr>
<td>$\mu$</td>
<td>med</td>
</tr>
<tr>
<td>low</td>
<td>0.001</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(e) Productivity: $\theta(v, Z)$</th>
<th>(f) Entry-to-productivity ratio: $\frac{\bar{h}(v, Z)}{\theta(v, Z)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>med</td>
</tr>
<tr>
<td>high</td>
<td>0.129</td>
</tr>
<tr>
<td>$\mu$</td>
<td>med</td>
</tr>
<tr>
<td>low</td>
<td>0.013</td>
</tr>
</tbody>
</table>

valuations (i.e., the value of VC output) is reassuring. In particular, it lends additional support to the calibrated specification for VC investments’ productivity.\footnote{Using market values as a measure of innovative output is a natural benchmark that also has been used in the context of established publicly listed firms (see, e.g., Kogan, Papanikolaou, Seru, and Stoffman [2017]). As discussed below in Section 4.2.3, start-up valuations in the model encode low expected returns, such that high IPO valuations are not merely the result of high expected cash flows.} Moreover, as the calibration
matches the payoffs to entrepreneurs and VCs, it also implies a good representation of the payoffs to VC investors, which are the third party receiving payoffs from IPOs and M&A transactions.

**Creative destruction.** Rates of creative destruction $\hat{h}$, reported in Table 2c, spike during booms when many entrants obtain finance from VC funds. As a result, even those firms that successfully undertook an IPO in a boom are exposed to high failure risk in the subsequent years, due to the possibility of displacement by newly arriving rivals. The Markov chain dynamics do, however, imply substantial mean reversion, such that annualized exit rates over longer horizons are substantially closer to the 8% average.51 The dynamics of creative destruction are tightly linked to those of IPO volume, consistent with the findings of Jovanovic and Rousseau (2003, 2005), who document spiking rates of entry and creative destruction as central characteristics of technological revolutions. The arrival rate of innovations $\tilde{h}$ is highly cyclical not only because of exogenous variation in $\theta$, but also because venture investment endogenously collapses in recession states, when new entrants do not receive funding from active VC funds (see Tables 2e and 2f).

**The aggregate impact of VC investment.** The calibrated model is a natural laboratory to evaluate VC investments’ impact on the aggregate economy. In states with active VC intermediation, VC investments contribute an endogenous fraction of agents’ consumption growth. Proposition 3 implies that these state-contingent growth contributions are given by the product of three quantities: the arrival rates of innovations $\tilde{h}$ reported in Table 2c, quality growth per innovation, $\log[\kappa]$, and the mass of industries featuring VC intermediation, $\int \Psi' dv$. The corresponding average growth contribution is about 3 to 4 basis points per annum, although the fluctuations around this mean are large. We know from the existing macroeconomic literature that contributions to consumption growth are very valuable (Lucas, 1987). Indeed, the compensating variation valuing these contributions is between 1 and 2 percent of lifetime consumption.52 This value significantly exceeds average levels of VC fund commitments and exit valuations, which amount to about 0.3 percent and 0.6 percent in both the data and in the model.

51 The 5-year survival rate of a firm entering the stock market in the average high-$\mu$ state falls into the range of estimates empirical studies typically find for dot-com era entrants, which is between 40% and 50% (see, e.g., Goldfarb, Kirsch, and Miller 2007; Wagner and Cockburn 2010; Luo and Mann 2011).

52 This calculation is performed for the unconditional expectation of the value function, $\mathbb{E}[J]$, starting from an allocation where agents hold claims to constant fractions of aggregate consumption.
VC investments are more important than suggested by commitments and exit valuations, as a significant fraction of VC investments' impact is not reflected in the discounted value of start-ups' future cash flows. In particular, the intangible capital producers obtain from innovating (a patent) generates profits only for a limited time period, until rivals develop superior products. In contrast, innovations have a permanent positive effect on the economy, as other innovators can build on them in the future — as suggested by Newton, innovators stand “on the shoulders of giants.” Ceteris paribus, the more rapid creative destruction is, the shorter is the time period during which a producer receives payoffs from a given innovation. Yet, as highlighted below in Section 4.2.3, the model also reveals a counterbalancing force: the cyclical properties of creative destruction imply that producer payoffs from intangible capital command low discount rates, which increases their market values. Despite this mitigating effect, startups' exit valuations overall still only partially reflect VC investments' total impact on consumption.

The strong cyclicality of VC activity captured by the calibration has a large impact on the above estimates, as agents are averse to persistent fluctuations in growth. To determine the magnitude of this effect, I also compute a compensating variation under the supposition that VC investments create the same average growth contributions, but without any fluctuations. The compensating variation valuing these constant growth contributions is about 3 percent of lifetime consumption, indicating a very significant risk discount embedded in the baseline estimates. In Section 5, I will further analyze how the results are affected when entrepreneurs and VC managers cannot share idiosyncratic venture risks through financial side contracts.

The magnitudes of these estimates are also significant when compared to existing evaluations of the impact of other macroeconomic phenomena, such as the costs of business cycles (see Lucas (2003) for an overview). Alvarez and Jermann (2004) estimate for example that the benefit of eliminating consumption fluctuations corresponding to business cycle frequencies is at most 0.49 percent of lifetime consumption. These relative magnitudes suggest that VC investments, despite their relatively small scale, have relevant macroeconomic implications.
4.2.2. Payoffs to VC Managers and Entrepreneurs

In this section, I evaluate the model’s predictions for payoffs to VC managers and entrepreneurs.

Payoffs to VC managers. Panels (a) and (b) of Table 3 report management fees and expected carry fees as fractions of commitments. Whereas management fees are paid with certainty using fund commitments, carry fees are stochastic at the fund-level and are only paid when a VC fund generates profits from bringing ventures to the market. As discussed above, fees are set such that fund investors obtain zero “net alpha” in equilibrium (as in Berk and Green 2004) — thus, fees correspond to what is sometimes referred to as “gross alpha” in the literature. VC funds generate economic profits at the time when new ventures are selected and are still subject to purely idiosyncratic success risk. Total fee revenues are given by the product of commitments reported in Table 2b and the sum of the two fee rates \((f_{\text{man}} + f_{\text{car}})\). While the model is calibrated to match the average level of VC fee revenues relative to contributed capital, it predicts strongly pro-cyclical variation in total fee revenues, consistent with findings of the empirical literature. In addition, in booms, performance-sensitive carry fees represent a larger share of total fees.

Payoffs to entrepreneurs. High idiosyncratic risk and highly skewed return distributions are key features of venture capital data, as discussed in the literature. The model replicates this fact — the vast majority of ventures that obtain funding fail, while a few obtain large payoffs from IPOs or M&A transactions. Since entrepreneurs are compensated with venture ownership stakes, their payoffs are also exposed to this extreme idiosyncratic risk. While in the baseline model this risk can be shared via separate side contracts, I analyze in Section 5 the model’s implications when this type of risk sharing is infeasible. Nash bargaining weights determine the relative amounts of venture equity backing entrepreneurial compensation and VC carry fees. In particular, the calibrated VC bargaining power parameter \(\varrho = 0.346\) implies that aggregate entrepreneurial payoffs in each state are about twice as large as aggregate carry fees, which are given by \(\tilde{\text{com}} \cdot f_{\text{car}}\). Aggregate payoffs to entrepreneurs are thus also highly cyclical, just like aggregate carry fee revenues and VC-backed IPO volume.

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Panels (a) and (b) list VC management fees and expected carry fee revenues as fractions of VC fund commitments. Panels (c) and (d) report jump-risk premia and diffusion-risk premia in each of the nine Markov states.

<table>
<thead>
<tr>
<th>(a) VC management fees: $f^{man}(v, Z)$</th>
<th>(b) VC carry fees: $f^{car}(v, Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>low</td>
</tr>
<tr>
<td>high</td>
<td>0.010</td>
</tr>
<tr>
<td>$\mu$</td>
<td>med</td>
</tr>
<tr>
<td>low</td>
<td>0.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Jump-risk premium: $rp^J(v, Z)$</th>
<th>(d) Diffusion-risk premium: $rp^D(v, Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>low</td>
</tr>
<tr>
<td>high</td>
<td>$-0.056$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>med</td>
</tr>
<tr>
<td>low</td>
<td>$-0.016$</td>
</tr>
</tbody>
</table>

### 4.2.3. Expected Post-IPO Returns

Whereas venture success risk is purely idiosyncratic at the implementation stage, the cash flows that a venture generates conditional on a success are exposed to aggregate risk. Ventures that are sold in competitive markets, for example in an IPO, thus earn a risk premium. This risk premium is the sum of a jump-risk premium $rp^J$ and a diffusion-risk premium $rp^D$, which compensate investors for exposures to Markov state shocks $dZ$ and Brownian shocks $dB$, respectively. These two risk premia components are reported in Panels (c) and (d) of Table 3.

Consistent with the findings of the existing asset pricing literature, much of the action in total risk premia is due to compensation for risks associated with low-frequency movements in aggregate growth,\(^{54}\) which is captured by the jump-risk premia $rp^J$ in the model. In contrast, diffusion risk premia are relatively small in magnitude.\(^{55}\) Jump-risk premia are affected by the industry-specific dynamics of creative destruction, which depend on both productivity and venture funding. These

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\(^{54}\) See, e.g., Bansal and Yaron (2004), Parker and Julliard (2005), Hansen, Heaton, and Li (2008), and Chen (2010).

\(^{55}\) Incumbent firms have a beta of one with respect to the Brownian innovations $dB$. See Chen (2010) for an analysis of levered claims in an environment with the same SDF dynamics.
risk premia are strongly counter-cyclical and negative, pushing down expected post-IPO returns, which are given by the sum of the two risk premia components and the risk-free rate \( r_f \) reported in Table 7.a. Expected returns are particularly low during booms (high \( \mu \) states), when they range between minus 1.5 percent and minus 9.3 percent. Thus, whereas VC funds’ abnormal returns (which equal fees) spike in booms, reaching about 25% of committed capital, investors’ expected post-IPO returns become locally negative.

These low discount rates imply high IPO and acquisition valuations relative to expected future dividends, creating the appearance of a “bubble.” High valuations in turn direct large amounts of capital to VC funds. Endogenous declines in discount rates in booms thus play an essential role in rationalizing “lenient” funding standards, in the sense that marginal VC investments yield on average low future dividends. Time-varying funding standards and apparent valuation bubbles are central characteristics of the venture capital “boom-bust-cycle,” which has received significant attention in the literature.\(^{56}\) While some of the existing literature argues that the strong negative correlation between VC fund inflows and future returns indicates investor irrationality (e.g., Gompers and Lerner, 2003) or increased competition between investors in booms (e.g., Gompers and Lerner, 2000), the quantitative model presented here illustrates that this pattern also emerges in an environment with rational investors that obtain competitive returns after fees.

The fact that jump-risk premia are lowest in booms indicates that firms in VC-backed industries \( \Psi' \) are particularly good hedges against low-frequency risks (shocks \( dZ \)) in these times. In booms, households are worried about changes in the state \( Z \) that would lead to a persistent slow-down in aggregate growth. However, such shocks are partially good news for ventures that already attracted funding and successfully entered the market during a boom. Although their dividend growth suffers when aggregate growth declines, these incumbents benefit to a first-order degree from a decrease in entry by additional competitors. As venture funding for new firms dries up in downturns and rates of creative destruction plummet, incumbents face a lower threat of displacement, making them relatively safer.

\(^{56}\) See, e.g., Gompers and Lerner (1998), Jeng and Wells (2000), Gompers and Lerner (2003), and Korteweg and Sorensen (2010). Kaplan and Lerner (2010) conclude that “...there is a strong negative correlation between VC returns and the preceding years’ capital commitments and investments.” Poor post-IPO return performance is also consistent with the empirical phenomenon known as the “net issues puzzle” (Loughran and Ritter 1995).
This effect on risk premia is naturally strongest in exactly those industries that have the highest levels of IPOs in booms and exhibit the most extreme busts in entry in ensuing downturns, a pattern typical for young growth industries, such as the IT industry during the internet boom. Pro-cyclical entry is thus self-reinforcing through a risk premium channel — pro-cyclical entry lowers discount rates in booms and raises venture funding in these times, which in turn makes entry more pro-cyclical.

Consistent with this mechanism, IPO volume and VC funding in the data are strongly correlated and pro-cyclical\(^5\) and lower levels of entry and IPO volume indeed positively affect incumbents’ stock prices and operating performance\(^6\). Moreover, if one classifies VC-backed firms post IPO as “growth firms,” then empirical analysis of the existing literature also directly support the model’s prediction that these firms have lower exposures to low-frequency risks and therefore low risk premia\(^7\).

While these various empirical observations are consistent with the mechanism and the predictions of the model, I discuss in the next subsection several empirical challenges associated with estimating standard risk-based asset pricing models in an environment as the one proposed in this paper.

4.3. Econometric Challenges

The presented model conceptualizes at least four challenges econometricians face when applying standard risk-based asset pricing models in the context of venture capital, adding to well-known measurement and selection problems with VC data\(^8\).

First, a typical assumption of standard models, such as the CAPM or the consumption-based capital asset pricing model (CCAPM), are perfectly competitive financial markets. Yet, in the case of venture capital, investors exhibit strong heterogeneity with regards to their access to investment

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\(^6\) See, e.g., Hsu, Reed, and Rocholl (2010).

\(^7\) See empirical evidence provided by Parker and Julliard (2005), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton, and Li (2008), and Bansal, Kiku, and Yaron (2012).

\(^8\) See, e.g., Hwang, Quigley, and Woodward (2005), Korteweg and Sorensen (2010), and Hall and Woodward (2007, 2010) for papers providing empirical approaches to address selection and measurement problems with VC data.
opportunities. This heterogeneity implies that VC firms may have the ability to make economic profits on their investments. It is generally difficult to cleanly empirically differentiate this profit component from the risk premium component, in particular since the two components have inverse cyclical patterns — whereas abnormal returns peak in booms, conditional risk premia plummet — and since available VC data typically does not provide reliable market valuations at standard frequencies.\footnote{VC firms have discretion over setting and reporting valuations over time, which also causes the above-mentioned selection problems.}

Second, valuations are affected not only by high frequency shocks ($dB$) but also by low-frequency shocks ($dZ$), implying that estimating a single market beta picks up an amalgamation of two risk exposures. In fact, Tables 3.c and 3.d indicate that the two risk premium components associated with high- and low-frequency shocks have opposite signs and opposite cyclical dynamics. As the betas are further correlated with variation in market risk premia and volatility, estimating unconditional betas also leads to biases.\footnote{See, e.g., \cite{Grant1977} and \cite{JagannathanWang1996}.}

Third, while precisely estimating exposures to low-frequency risks requires long time-series of data, VC data spans only a relatively short time period, since the industry started to attract significant commitments only in the last two to three decades.

Fourth, the model captures the fact that the stock market systematically mismeasures the total wealth portfolio, since it consists of incumbent firms only and misses the value of future entrants. Using a market index such as the S&P 500 as a proxy for the total wealth portfolio can thus be particularly problematic in times when a large part of future growth is expected to be generated by future entrants.

5. Imperfect Risk Sharing

The analysis thus far assumed that agents can perfectly share risks. In particular, entrepreneurs and network VCs were able to insure idiosyncratic risks associated with performance-sensitive compensation by selling state-contingent contractual claims in financial markets. In this section, I explore the model’s implications when financial markets are instead incomplete in that risk sharing
of this type is infeasible. Exposures to idiosyncratic risks can for example emerge as optimal contractual arrangements in the presence of agency problems\textsuperscript{63}, which are likely a concern in venture capital.

A convenient feature of the proposed setup is that the analysis of the general equilibrium economy remains tractable in the presence of this market incompleteness, and does not require additional simplifying assumptions. Entrepreneurs and network VCs cannot enter ex ante contracts that specify payments conditional on the future realizations of their own venture capital opportunities (network connections or superior ideas), and thus, stay exposed to uninsurable idiosyncratic income risks. I provide the details of the technical analysis in Appendix A.6 and describe the key economic insights in this section.

**Consumption and portfolio choices.** Due to the presence of undiversifiable idiosyncratic income shocks, the economy now features dispersion in consumption growth across agents. While solving economies of this type typically requires approximation techniques or computationally-intensive methods, analyzing the proposed model under incomplete risk-sharing remains tractable. This result is in part due to the extreme nature of idiosyncratic risk faced by VC managers and entrepreneurs in the model. Large idiosyncratic uncertainty, in particular for entrepreneurs, is in turn also an important empirical regularity, as emphasized by \cite{HallWoodward2010}. While most VC-backed entrepreneurs don’t receive any payoffs associated with IPOs or acquisitions, very few receive over a billion dollars. Because of this large idiosyncratic risk, \cite{HallWoodward2010} find that, “an entrepreneur with a coefficient of relative risk aversion of two places a certainty equivalent value only slightly greater than zero” on the interests in her own company. A very similar result obtains in the presented model, which effectively takes this type of idiosyncratic risk to the limit. The extreme discount applied to future risky payoffs from VC contracts, in turn, implies that the dynamic portfolio optimization problems of agents involved in the VC process become similar to those of agents that do not expect to receive performance-sensitive venture payoffs. Constrained optimal risk sharing then implies that at any point in time, all agents optimally hold the diversified market portfolio of tradable securities and choose the same consumption-to-tradable-wealth

\textsuperscript{63}See \cite{AdmatiPfleiderer1994} and \cite{HolmstromTirole1997}. See also \cite{Schmidt2003} and \cite{Casamatta2003} for papers that consider optimal security design in the context of venture capital.
As a result, the cross-sectional wealth distribution does not emerge as a state variable — the only aggregate state variables are still $Y$ and $Z$, just as in the baseline analysis.

**Consumption dynamics and asset prices.** I now discuss how consumption dynamics and asset prices differ from the baseline model with complete risk sharing. Let $\tilde{P}_T(t)$ denote the market price of the portfolio of all tradable assets that are in positive net-supply at time $t$, also referred to as the *market portfolio*. Recall that new entrants are created through the joint efforts of entrepreneurs, network VCs, and white-collar labor. Correspondingly, in every period, the total equilibrium compensation this group of agents obtains is exactly equal to the total market value of the new entrants. A time-varying, endogenous fraction of this total market value is granted as performance-sensitive compensation to entrepreneurs and network VCs, and claims to this compensation cannot be traded until the idiosyncratic success risk of ventures is realized and an IPO occurs. The remaining, complementary fraction of total entrant equity is granted to diversified investors in exchange for the upfront capital they provide, which is used to compensate white-collar labor.

Let $\tilde{\pi}_j(t)$ denote the share of the tradable market portfolio owned by agent $j = (S, i'_S, i''_S)$ at time $t$. The aggregate wealth gain obtained by idiosyncratically successful entrepreneurs and network VCs is given by:

$$\int \Psi \bar{\iota}_1(v, Z) \tilde{\pi}_1(v, Z) dv dt,$$

(30)

where $\bar{\iota}_1(v, Z)$ and $\tilde{\pi}_1(v, Z)$ were defined in Section 3. The concentration of these wealth gains among a subset of agents implies that, in relative terms, all other agents in the economy become poorer — almost all agents’ wealth shares decline, but a few agents obtain very large gains. As all agents optimally choose identical consumption-to-tradable-wealth ratios, each agent’s consumption growth can be written as the sum of aggregate output growth, $\frac{dY}{Y}$, and the growth in the agent’s ownership share, $\frac{d\bar{\pi}_j}{\bar{\pi}_j}$. Since only a measure zero of agents obtain large wealth gains from venture

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64 A similar result obtains in the asset pricing model of Garleanu, Panageas, Papanikolaou, and Yu (2015) who consider a setting with extreme idiosyncratic risks in order to study the effects of imperfect risk sharing on the value premium.
capital in each period, an agent’s consumption growth is almost surely given by:

\[
\frac{dC_j(t)}{C_j(t)} = \mu(Z_t)dt + \sigma(Z_t)dB - \int \Psi_1(v, Z_t) \pi_1(v, Z_t)dv - \int \Psi_1(v, Z_t) \pi_1(v, Z_t)dv dt.
\]

(31)

As shown in Appendix A.6, the equilibrium market prices of all assets that are tradable are set as if the dynamics for all agents’ marginal utility was governed by the consumption dynamics displayed in equation (31). Relative to the economy with perfect risk sharing these consumption dynamics are perturbed by a displacement term: almost all agents’ consumption growth is dampened, relative to perfect risk sharing, as a small subset of entrepreneurs and VCs obtain disproportionately large wealth gains. This displacement term is given by the ratio of the aggregate uninsurable wealth gains by VCs and entrepreneurs displayed in (30) to the value of the market portfolio of tradable assets, \( \tilde{P}_T \).

**Market prices and allocations.** To gauge the impact of imperfect risk sharing on the quantitative results discussed in the previous sections, I re-solve the model with the parameter values chosen in Section 4.1. A convenient feature of the model is that all industry-level equilibrium conditions presented in Section 3.1 and the aggregate output dynamics and labor market clearing conditions in Section 3.2 still hold in the setting with imperfect risk sharing. Solutions to these equilibrium conditions are, however, affected by the fact that consumption dynamics and the corresponding state pricing process \( \xi \) are different under incomplete risk sharing. In particular, as discussed above, the SDF dynamics relevant for tradable assets are effectively governed by the perturbed consumption growth dynamics shown in equation (31). I find that the perturbations due to the highlighted displacement term are quantitatively small, since the value of venture equity obtained by entrepreneurs and VCs is small relative to the value of the stock of all tradable assets.

---

65 Several recent asset pricing papers feature related concepts of displacement risk in endowment economies with market incompleteness and highlight that this type of risk can help address asset pricing phenomena such as the value premium (see Garleanu, Kogan, and Panageas, 2012; Kogan, Papanikolaou, and Stoffman, 2015; Garleanu, Panageas, Papanikolaou, and Yu, 2015). Kogan and Papanikolaou (2014) show that preferences with “keeping up with the Joneses” motives can amplify the asset pricing implications of uninsurable wealth gains.

66 This displacement effect implies that the aggregate wealth share of the group of entrepreneurs and network VCs converges to one in the long run. The wealth distribution is, however, inconsequential for the equilibrium dynamics of the variables of interest in this study. Therefore, I refrain from introducing additional model features, such as overlapping generations.

67 Consistent with the calibration in Section 4.1 the parameters \( \delta(Z) \) are chosen such that aggregate consumption growth still matches the values in Chen (2010) state by state.
assets in the economy. The average value of the displacement term is 0.25 basis points and the maximum value is less than 1 basis point. As the total wealth gains of VCs and entrepreneurs are pro-cyclical, imperfect risk sharing then has a very minor dampening effect on the cyclicality of almost all agents’ consumption growth. The dynamics of the state pricing process therefore also remain very similar when compared to the baseline analysis. Facing almost identical market prices, agents and firms also behave almost the same way. Consequently, the model with imperfect risk sharing is still consistent with the moments targeted in Section 4.1 and generates quantitatively similar predictions as discussed in Section 4.2.

Utility from VC compensation and aggregate VC investment. Despite these similarities in aggregate outcomes, entrepreneurs and venture capitalists lose significantly, in utility terms, due to the increased riskiness of their consumption profile relative to the perfect risk sharing benchmark, consistent with the findings of [Hall and Woodward (2007, 2010)]. As network VCs and entrepreneurs have to be exposed to idiosyncratic risk to take advantage of their venture capital opportunities — for example, due to moral hazard frictions and the corresponding need for high-powered incentives — they cannot deviate to a less risky consumption profile when pursuing these opportunities.

While imperfect risk sharing strongly affects agents’ utility from performance-sensitive VC contracts, it also materially affects the value assigned to VC investments’ aggregate impact. Relative to the estimates obtained under perfect risk sharing in Section 4.2.1, the compensating variation valuing VC investments’ contributions to consumption drops by 8 percent.\textsuperscript{68} Despite the extreme nature of idiosyncratic risks affecting performance-sensitive VC contracts, substantial value remains (92% of the initial estimate) for several reasons. First, a sizable fraction of idiosyncratic venture risk is held by investors that hold diversified portfolios. Second, not all compensation in venture capital is performance-sensitive and exposed to idiosyncratic risk.\textsuperscript{69} Third, as discussed in Section 4.2.1, VC-backed innovations have a positive impact on technological progress and aggregate growth that is not reflected in startups’ market values (yet performance-sensitive VC contracts are

\textsuperscript{68}This calculation does not evaluate an agent’s loss from moving from an economy with perfect risk sharing to one with imperfect risk sharing. Instead, it compares compensating variations in two distinct types of economies; one with and one without perfect risk sharing. The last paragraph of Appendix A.6 provides additional details.

\textsuperscript{69}In the model, VC managers also collect performance-insensitive management fees. In addition, agents hired by startups to implement ideas also obtain white-collar compensation. In practice, VC managers obtain additional diversification benefits from investing in multiple ventures (see Hall and Woodward (2007)).
claims to these market values only).

Overall, the analysis suggests that whereas quantities and prices of assets traded in competitive financial markets remain similar, agents’ utility from performance-sensitive VC compensation and aggregate VC activity is significantly affected by imperfect risk sharing.

6. Conclusion

This paper presents a model that can explain central empirical facts about the magnitude and cyclicality of VC activity. The framework reveals that boom-bust cycle dynamics of VC activity and uninsurable idiosyncratic risks associated with VC contracts lead to substantial discounts in the value agents assign to VC investments’ contributions to consumption. Despite these risk discounts, I find that VC investments have a significantly larger impact than suggested by empirical magnitudes of aggregate VC fund commitments and exit valuations. When compared to estimates of the costs of business cycles (see, e.g., [Lucas, 2003]), the magnitudes of the results suggest that regulatory changes, such as those that appear to have led to the emergence of the VC industry\(^{70}\) can have relevant implications for the macroeconomy. The proposed dynamic general equilibrium model is highly tractable, even in the presence of imperfect risk sharing, and could be used as a stepping stone to evaluations of policies affecting VC activity and entrepreneurship, including patent regulation and investment mandates for institutional investors. Such endeavors are left for future research.

\(^{70}\)See footnote\(^5\)
A. Appendix

A.1. Tables

Table 4
Aggregate Consumption Growth
Panels (a) and (b) report the state-contingent values of the consumption growth drifts and the local risk exposures estimated in Chen (2010).

<table>
<thead>
<tr>
<th></th>
<th>(a) Consumption growth drifts: $\mu(Z)$</th>
<th>(b) Local risk exposures: $\sigma(Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>low</td>
</tr>
<tr>
<td>high</td>
<td>0.041</td>
<td>0.048</td>
</tr>
<tr>
<td>$\mu$ med</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>low</td>
<td>$-0.004$</td>
<td>$-0.011$</td>
</tr>
</tbody>
</table>

Table 5
Stationary Distribution of Markov States
The table reports the stationary distribution of the Markov generator matrix estimated in Chen (2010).

<table>
<thead>
<tr>
<th></th>
<th>Unconditional probability: $\Pr[Z]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
</tr>
<tr>
<td>high</td>
<td>0.016</td>
</tr>
<tr>
<td>$\mu$ med</td>
<td>0.059</td>
</tr>
<tr>
<td>low</td>
<td>0.016</td>
</tr>
</tbody>
</table>

Table 6
State-dependent Aggregate Growth
Panels (a) and (b) report the exogenous and endogenous components of aggregate growth (across all industries $\Psi = \Psi' \cup \Psi''$). Values are reported in percentage points.

<table>
<thead>
<tr>
<th></th>
<th>(a) Exogenous drift: $\delta(Z)$</th>
<th>(b) Endogenous drift: $\log([\kappa \int_{\Psi} h(v, Z)dv] \sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>low</td>
</tr>
<tr>
<td>high</td>
<td>3.52</td>
<td>4.18</td>
</tr>
<tr>
<td>$\mu$ med</td>
<td>1.35</td>
<td>1.36</td>
</tr>
<tr>
<td>low</td>
<td>$-0.88$</td>
<td>$-1.53$</td>
</tr>
</tbody>
</table>
Panels (a) and (b) report the risk-free rate and the white-collar wage-to-output ratio in each of the nine Markov states.

<table>
<thead>
<tr>
<th>(a) Risk-free rate: $r_f(Z)$</th>
<th>(b) White-collar wages: $\hat{w}_W(Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>high 0.037</td>
<td>high 0.106</td>
</tr>
<tr>
<td>med</td>
<td>med</td>
</tr>
<tr>
<td>0.023</td>
<td>0.098</td>
</tr>
<tr>
<td>low</td>
<td>low</td>
</tr>
<tr>
<td>0.008</td>
<td>0.092</td>
</tr>
</tbody>
</table>

Table 8
State-contingent Outcomes in Industries $\Psi''$

The table reports the state-contingent values of productivity in industries $\Psi''$, which follow from the calibrated relation:

$$\theta(v, Z) = a + b \cdot (\mu(Z) - \min_{\forall Z}\{\mu(Z)\}),$$

where, $a = 0.062$, $b = 0.11$, and where $\mu(Z)$ are the consumption growth drifts from Chen (2010).

<table>
<thead>
<tr>
<th>Productivity: $\theta(v, Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>low</td>
</tr>
<tr>
<td>high 0.068</td>
</tr>
<tr>
<td>med</td>
</tr>
<tr>
<td>0.066</td>
</tr>
<tr>
<td>low</td>
</tr>
<tr>
<td>0.063</td>
</tr>
</tbody>
</table>

A.2. Definition of Allocation and Decentralized Equilibrium

**DEFINITION 1** (Allocation). An allocation in this economy is given by stochastic processes of agents’ consumption $[C_j(t)]_{j \in \Psi, t=0}^\infty$, where $j = (S, i'_S, i''_S)$, quantities of superior and regular venture ideas undertaken in each industry, $[n_1(v, t), n_2(v, t)]_{v \in \Psi, t=0}^\infty$, matching with VC funds in each industry $[\iota_1(v, t)]_{v \in \Psi, t=0}^\infty$, qualities of leading-edge intermediate goods in each industry $[q(v, t)]_{v \in \Psi, t=0}^\infty$, and quantities of intermediate goods produced in each industry $[x(v, t)]_{v \in \Psi, t=0}^\infty$.

**DEFINITION 2** (Decentralized Equilibrium). An equilibrium in this economy is given by an allocation, and stochastic processes for wage rates $[w_B(t), w_W(t)]_{t=0}^\infty$, the venture ownership stakes

45
obtained by network VCs and entrepreneurs in each industry, the stochastic discount factor \([\xi(t)]_{t=0}^{\infty}\), and intermediate goods prices denoted by \([p^x(v,t)]_{v \in \Psi, t=0}^{\infty}\), such that firms’ pricing, production, and venture funding maximizes their value; VC funds’ operating and funding decisions maximize their value; agents choose their paths of consumption optimally; labor markets, intermediate good markets, and the final good market clear.

A.3. Incumbent Optimization and Proof of Proposition 2

First, I show why a firm does not fund additional venture ideas in a product line where it already owns the best patent. The setup ensures that an incumbent’s maximization problem is separable across industries, since each incumbent is restricted to operate in a finite number of industries \(v\), which have zero measure. Let \(i \geq 1\) denote the number of innovations that the current incumbent’s best patent is ahead relative to the next-best patent owned by a different firm, that is, the firm’s patent yields quality advantage of a factor \(\kappa(v,t) = \kappa^i\). The value the incumbent firm obtains from this patent is given by:

\[
P_i(v,t) = \mathbb{E}_t \left[ \int_t^{\tau^*} \frac{\xi_t Y_t}{\xi_t} \left( 1 - \frac{1}{\kappa^i} \right) d\tau + \frac{\xi_{s^*}}{\xi_t} P_{i+1}(v,s^*) \right],
\]

where the distributions of \(\tau^* \equiv \inf \{ \tau : M(v,\tau) > M(v,t) \}\) and \(s^* \equiv \inf \{ s : \kappa(v,s) > \kappa(v,t) \}\) are affected by the incumbents optimal venture funding policy. Suppose the associated value function takes the form:

\[
P_i(v,Y_t,Z_t) = Y_t \cdot \hat{P}(v,Z_t) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\hat{\kappa}}}.\]

Since incumbents and non-incumbent firms have symmetric access to regular venture ideas, the per-venture success rate and the per-venture funding cost are equal for both types of firms. To compare incumbent and non-incumbent firms’ incentives to fund regular venture ideas it thus suffices to compare each firm type’s gain conditional on a venture success. If an incumbent firm
succeeds in innovating again in the same product line, its value jumps by:

\[
\tilde{P}_{i+1}(v,Y,Z) - \tilde{P}_i(v,Y,Z) = Y \cdot \tilde{P}(v,Z) \frac{1 - \frac{1}{\kappa+1}}{1 - \frac{1}{\kappa}} - Y \cdot \tilde{P}(v,Z) \frac{1 - \frac{1}{\kappa}}{1 - \frac{1}{\kappa}}
\]

\[
= Y \cdot \tilde{P}(v,Z) \frac{1}{\kappa^i}
\]

\[
< Y \cdot \tilde{P}(v,Z),
\]

where the last inequality holds since \(\kappa > 1\). Thus, under the conjectured value function, incumbent firms gain less than non-incumbent firms, which gain \(Y \cdot \tilde{P}(v,Z)\) conditional on a venture success. As the free-entry condition (19) implies that non-incumbent firms just break even at the equilibrium per-venture success rate and per-venture funding cost, incumbent firms cannot break even when funding ventures in their own product line, as they gain less from innovating (see inequality (35)). Thus, incumbent firms optimally refrain from funding additional ventures in the product line where they already own the best patent.

Finally, I verify the conjectured value function (34), which also proves Proposition 2. As shown above, under this conjectured value function, incumbents optimally do not fund additional venture ideas in their own product line, and the arrival rate of new innovations \(\bar{h}\) in the product line is determined as described in Proposition 1. Substituting the conjectured value function (34) into the HJB equation associated with (33) yields the following linear system of equations determining the values \(\tilde{P}(v,Z)\) for all \(Z \in \Omega\):

\[
0 = 1 - \frac{1}{\kappa^i} - (r_f(Z) + \gamma \sigma^2(Z) + \tilde{h}(v,Z) - \mu(Z)) \tilde{P}(v,Z) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}}
\]

\[
- \sum_{Z' \neq Z} \lambda_{ZZ'} e^{\xi(Z,Z')} \left( \tilde{P}(v,Z') - \tilde{P}(v,Z) \right) \frac{1 - \frac{1}{\kappa^i}}{1 - \frac{1}{\kappa}},
\]

confirming that the conjecture (34) is indeed a solution to the firm’s HJB equation. Given that at date \(t = 0\) every incumbent’s patent has a quality advantage of a factor \(\kappa\), the only relevant case is \(\kappa(v,t) = \kappa\). Finally, setting \(i = 1\) in equation (36), rearranging, and defining the diffusion- and
jump-risk premia

\[ rp^D (v, Z) = \gamma \sigma^2 (Z), \quad rp^J (v, Z) = \sum_{Z' \neq Z} \lambda_{ZZ'} \left( e^{\xi(Z,Z')} - 1 \right) \left( 1 - \frac{\tilde{P}(v, Z')}{\tilde{P}(v, Z)} \right) \]  

(37)
yields equation (23) in Proposition (2).

A.4. Proof of Proposition 3

In an economy where incumbent firms in each industry \( v \in \Psi \) have a quality advantage of a factor \( \kappa \) relative to the next-best producer, incumbents’ profit maximization in combination with the market clearing condition for blue-collar labor (27) yield a symmetric allocation of blue-collar labor across industries, that is, \( x(v, t) = 1 \) for all \( v \in \Psi \). Plugging this result into the final good production technology yields:

\[ Y_t = A_t Q_t, \]  

(38)

where I define:

\[ Q_t \equiv \exp \left( \int_{\Psi} \log [q(v,t)] dv \right). \]  

(39)

Innovation arrivals in each industry \( v \) follow idiosyncratic Poisson processes with hazard rates \( \bar{h}(v, Z) \) defined in equation (21). The aggregate quality level \( Q_t \) then grows at the state-dependent rate

\[ \frac{dQ_t}{Q_t} = \log [\kappa] \int_{\Psi} \bar{h}(v, Z_t) dv. \]  

(40)

Thus, final good output follows the stochastic differential equation

\[ \frac{dY_t}{Y_t} = \left( \delta (Z_t) + \log [\kappa] \int_{\Psi} \bar{h}(v, Z_t) dv \right) dt + \sigma (Z_t) dB_t. \]  

(41)
A.5. Proof of Proposition 4

In the baseline analysis of the model, agents can optimally share risks by trading state-contingent claims to future compensation from labor contracts, in particular, white-collar and blue-collar wages, and venture ownership shares provided to network VCs and entrepreneurs. As a result of perfect risk sharing, agents consume constant fractions of aggregate output. If at date 0 agent \( j = (S, i'_S, i''_S) \) has a wealth share \( \varpi_j \), then perfect risk sharing implies that this agent consumes at any time \( t \geq 0 \):

\[
C_j(t) = \varpi_j Y_t. \tag{42}
\]

The agent’s value function is given by

\[
J(\varpi_j, Y_t, Z_t) = \mathbb{E}_t \left[ \int_t^\infty m(\varpi_j Y_\tau, J_\tau) \, d\tau \right], \tag{43}
\]

which yields the associated Hamilton-Jacobi-Bellman (HJB) equation:

\[
0 = m(\varpi_j Y, J(\varpi_j, Y, Z)) + J_Y(\varpi_j, Y, Z) Y \mu(Z) + \frac{1}{2} J_{YY}(\varpi_j, Y, Z) Y^2 \sigma^2(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} \left( J(\varpi_j, Y, Z') - J(\varpi_j, Y, Z) \right). \tag{44}
\]

Due to the isoelastic properties of the setup, I conjecture the solution for \( J \) takes the form

\[
J(\varpi_j, Y, Z) = F(Z) \frac{Y^{1-\gamma}}{1-\gamma} \varpi_j^{1-\gamma}. \tag{45}
\]

Substituting the conjecture \( J(\varpi_j, Y, Z) = F(Z) \frac{Y^{1-\gamma}}{1-\gamma} \varpi_j^{1-\gamma} \) into the HJB equation yields the following system of equations that \( F(Z) \) solves for all \( Z \in \Omega \):

\[
0 = \left( \frac{\beta(1-\gamma)}{\rho} F(Z)^{-\frac{\varpi_j}{1-\gamma}} - 1 \right) + (1-\gamma) \mu(Z) - \frac{1}{2} \gamma(1-\gamma) \sigma^2(Z) \right) F(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} \left( F(Z') - F(Z) \right). \tag{46}
\]
Duffie and Epstein (1992b) show that household maximization implies that a state-pricing process $\xi_t$ may be written as follows:

$$\xi_t \equiv \exp \left[ \int_0^t m_j(C_j(\tau), J_\tau) \, d\tau \right] \cdot m_C(C_j(t), J_t). \quad (47)$$

Using the value function (45) and the consumption policy (42) we obtain

$$\xi_t = (\varpi_j Y_t)^{-\gamma} \beta F(Z_t)^{1-\frac{\rho}{1-\gamma}} e \left\{ \int_0^t \left( \frac{\beta(1-\gamma-\rho)}{\rho} F(Z_\tau) - \frac{\beta(1-\gamma)}{\rho} \right) \, d\tau \right\}. \quad (48)$$

Applying Itô’s lemma yields

$$\frac{d\xi_t}{\xi_{t-}} = -r_f(Z_t) \, dt - \gamma \sigma(Z_t) \, dB_t + \sum_{Z' \neq Z_{t-}} \left( \left( \frac{F(Z')}{F(Z_{t-})} \right)^{1-\frac{\rho}{1-\gamma}} - 1 \right) \left( dN_t(Z_{t-}, Z') - \lambda_{Z_{t-}Z'} \, dt \right), \quad (49)$$

where

$$r_f(Z) = -\frac{\mathbb{E}_t \frac{d\xi_t}{\xi_{t-}}}{\xi_{t-}} = \frac{\beta(1-\gamma)}{\rho} - \frac{\beta(1-\gamma-\rho)}{\rho} F(Z)^{-\frac{\rho}{1-\gamma}} + \gamma \mu(Z) - \frac{1}{2} \gamma (1+\gamma) \sigma^2(Z) - \sum_{Z' \neq Z} \lambda_{Z'Z} \left( \left( \frac{F(Z')}{F(Z)} \right)^{1-\frac{\rho}{(1-\gamma)}} - 1 \right). \quad (50)$$

Note that the growth of $\xi_t$ is independent of an agent’s wealth share $\varpi_j$. Thus, all agents in the economy assign identical values to payoffs in all future states of the world, and have no incentives to deviate from the consumption policy (42). Defining

$$\zeta(Z, Z') \equiv \left( 1 - \frac{\rho}{1-\gamma} \right) \log \left( \frac{F(Z')}{F(Z)} \right). \quad (51)$$

and rearranging (49) yields the result stated in Proposition 4.

A.6. Imperfect Risk Sharing

For the analysis of the model discussed in Section 5 the contractual space and the logical order of events within a period $[t, t + dt)$ are specified as follows:
1. Agents complete all financial transactions and choose their consumption for the period. Financial transactions can specify payments conditional on the paths of the variables \( \{Y_r\}_{r \geq t} \), \( \{Z_r\}_{r \geq t} \), \( \{\iota_1(v, \tau)\}_{v \in \Psi, \tau \geq t} \), \( \{M(v, \tau)\}_{v \in \Psi, \tau \geq t} \), and \( \{M_1(v, \tau)\}_{v \in \Psi, \tau \geq t} \), where \( M_1(v, t) \) is a counting process keeping track of the number of new patents created from funding the most productive entrepreneurs’ venture ideas in industry \( v \). VC funds and firms can obtain funding commitments from investors as a function of these variables. In return, investors obtain venture ownership shares.

2. The random variables \( E_t \) and \( V_t \) are realized, that is, nature chooses the network VCs with connections and the most productive entrepreneurs in the period.

3. Whenever VC intermediation obtains \( (\iota_1(v, t) = 1) \), the connected network VC and the most productive entrepreneur have to be compensated with venture equity, as specified in the baseline model. Network VCs that obtained connections choose whether to facilitate the matching process, which determines the realizations of the variables \( \{\iota_1(v, t)\}_{v \in \Psi} \).

4. State-contingent funding commitments for VC funds are fulfilled (contingent on \( \iota_1(v, t) = 1 \)) and are used to pay for the implementation of venture ideas of the most productive entrepreneurs and for VC management fees.

5. State-contingent funding commitments for regular venture ideas are fulfilled and can be used to pay for investment cost.

6. Innovation arrivals as reflected by jumps in the variables \( \{M(v, t)\}_{v \in \Psi} \) are realized and patents are allocated to ventures as described in the baseline setup.

For the analysis of the economy with incomplete risk sharing it is useful to first consider a setting where the first element of the double index identifying agents, \( i_S' \), takes values in a discrete set (instead of the continuum \( [0, 1] \)). In particular, define \( \Delta i_S' \equiv \frac{1}{K+1} \) and let \( i_S' \in \{0, \frac{1}{K}, \frac{2}{K}, ..., 1\} \). To establish the solution for the model with double continuums of agents, I take the limit \( K \to \infty \), or equivalently, \( \Delta i_S' \downarrow 0 \).

**Probabilities of receiving venture opportunities.** Consistent with the baseline setup, the first indices \( i_E' \) (for entrepreneurs) and \( i_V' \) (for network VCs) identifying agents with special venture
opportunities are again randomly selected according to a uniform distribution: each possible value of \(i'_E \) and \(i'_V \) has an equal probability \(\Delta i'_E = \Delta i'_V \) of being selected in each period. In addition, the specification determining the second element, \(i''_S\), remains exactly as specified in Sections 2.4 and 2.5. Recall that if a particular first index \(i'_E \) is selected by nature in a given period, all entrepreneurs with this first index obtain a positive shock in some industry; which entrepreneur obtains this shock in which industry is pinned down by the second index element \(i''_E = v\). An entrepreneur with the second index element \(i''_E = v\) then has a probability \(\Delta i''_E \) of becoming the most productive entrepreneur in industry \(v\) in a given period. Analogously, a network VC with the second index element \(i''_V = v\) has a probability \(\Delta i''_V \) of receiving a useful connection in industry \(v\).

**Consumption and portfolio policies.** I start with the conjecture that, in equilibrium, all agents rebalance their portfolios at the beginning of each period to always invest all their tradable wealth in the “market portfolio” of all tradable assets, the price of which is denoted by \(P_T\). For example, immediately after experiencing a positive idiosyncratic venture success as an entrepreneur, an agent sells her equity stake to other agents at the market price and rebalances to again hold the market portfolio. Further, conjecture that all agents choose the same consumption-to-tradable-wealth ratios, which implies that the wealth shares also pin down the fraction of output \(Y\) that each agent consumes. Below, I will verify the optimality of these conjectured consumption and portfolio policies.

Let \(\varpi(S, i'_S, i''_S, t)\) denote the share of total tradable wealth owned by agent \((S, i'_S, i''_S)\) at time \(t\) scaled by \(\frac{4}{\Delta i'_S di''_S}\), such that the actual wealth share is: \(\varpi(S, i'_S, i''_S, t) \frac{\Delta i'_S di''_S}{4} \). By definition, wealth shares add up to one:

\[
\sum_{S \in \{B, W, V, E\}} \sum_{i'_S \in \{0, \frac{1}{K}, \ldots, 1\}} \int_0^1 \varpi(S, i'_S, i''_S, t) \frac{\Delta i'_S di''_S}{4} = 1. \tag{52}
\]

To economize on notation, I will use a subscript \(j\) to refer to an agent identified by a tuple \((S, i'_S, i''_S)\) going forward. An agent’s tradable wealth at time \(t\) is denoted by \(W_j(t)\) and is given by:

\[
W_j(t) = P_T(t) \varpi_j(t) \frac{\Delta i'_S di''_S}{4}. \tag{53}
\]
The assets that are part of the tradable market portfolio pay an aggregate dividend that is equal to the aggregate output $Y$. Thus, an agent holding a fraction $\varpi_j(t) \frac{\Delta \delta^{u'}_S}{4}$ of the market portfolio obtains a dividend $Y_t \varpi_j(t) \frac{\Delta \delta^{u'}_S}{4}$. Identical consumption-to-wealth ratios and market clearing imply that agents consume this dividend, that is,

$$C_j(t) = Y_t \varpi_j(t) \frac{\Delta \delta^{u'}_S}{4}. \quad (54)$$

**Evolution of tradable wealth shares.** The set of assets in positive net supply that are tradable changes over time. The venture ownership stakes network VCs and entrepreneurs will obtain through future idiosyncratic shocks are not traded until after ventures succeed and ownership stakes can be sold in the competitive market. It is useful to specify the market values of different vintages of tradable market portfolios. I define $P_{T,\tau}(t)$ as the time-$t$ market value of the portfolio of all assets in positive net supply that have been tradable since time $\tau$, where $t \geq \tau$.

Let $\vartheta_j(v,t) \in [0,1]$ denote the equity stake agent $j$ obtains if she receives a VC-related opportunity in industry $v$ in period $[t, t + dt)$, which is relevant only for entrepreneurs and network VCs (that is, $\vartheta_j(v,t) = 0$ for white-collar and blue-collar agents). Further, let $1_j(v,t)$ be an indicator variable that takes the value one if agent $j$ actually obtains a VC-related opportunity in industry $v$ in period $[t, t + dt)$. An agent obtains equity worth $\vartheta_j(v,t) P(v,t) dv$ when succeeding with a VC-backed venture in industry $v$. Given the conjectured consumption and portfolio policies an agent’s tradable wealth thus evolves as follows:

$$dW_j(t) = \varpi_j(t) dP_{T,t}(t) \frac{\Delta \delta^{u'}_S}{4} + \int_\Psi dM_1(v,t) 1_j(v,t) \vartheta_j(v,t) P(v,t) dv. \quad (55)$$

The total fraction of venture equity that entrepreneurs and network VCs will obtain jointly in industry $v$ at time $t$ is given by

$$\vartheta(v,t) = \begin{cases} \frac{\pi_1(v,t)}{n_1(v,t) P(v,t)} & \text{if } h_1(v,t) > 0, \\ 0 & \text{otherwise.} \end{cases} \quad (56)$$

These equity shares are distributed according to the bargaining power parameter $\varrho$. The remainder
share of the equity is held by diversified investors and is part of the market portfolio of tradable assets.

The dynamics of the value of the market portfolio of tradable assets can be decomposed into the value change of existing tradable assets and the arrival of new venture equity sold by successful entrepreneurs and network VCs:

\[
\begin{align*}
    dP_T(t) &= dP_{T,t}(t) + \int_{\Psi} \vartheta(v,t)dM_1(v,t)P(v,t)dv \\
    &= dP_{T,t}(t) + \int_{\Psi} \vartheta(v,t)h_1(v,t)P(v,t)dvdt \\
    &= dP_{T,t}(t) + \pi_1(v,t)dvdt,
\end{align*}
\]

where the first step obtains since \(E[dM_1(v,t)] = h_1(v,t)dt\), and the law of large numbers applies \(^{71}\) and the second step obtains due to the definition of \(\vartheta(v,t)\) provided in (56) and since \(\pi_1(v,t) = 0\) when \(h_1(v,t) = 0\). Thus, we obtain the following law of motion for \(\varpi_j(t)\):

\[
\begin{align*}
    d\varpi_j(t) &= \left( \frac{W_j(t)}{P_T(t)} \right) dP_{T,t}(t) + \int_{\Psi} \vartheta_j(v,t)\varpi_j(v,t)P(v,t)dv \\
    &= \varpi_j(t)dP_{T,t}(t) + \int_{\Psi} \vartheta_j(v,t)\varpi_j(v,t)P(v,t)dv \\
    &= \int_{\Psi} dM_1(v,t)\varpi_j(v,t)P(v,t)dv \\
    &\quad - \varpi_j(t)\int_{\Psi} \pi_1(v,t)dvdt.
\end{align*}
\]

which yields:

\[
\begin{align*}
    d\varpi_j(t) &= \int_{\Psi} dM_1(v,t)\varpi_j(v,t)P(v,t)dv \\
    &\quad - \varpi_j(t)\pi_1(v,t)dvdt.
\end{align*}
\]

The first term reflects idiosyncratic risk — each agent \(j\) obtains a venture equity share \(\vartheta_j(v,t) > 0\) in at most one industry \(v\) and thus, may be exposed to venture success risk in that particular industry. The second term reflects the decline in an agent’s wealth share due to the venture successes of other

\(^{71}\)In the calibration, \(dM_1(v,t)\), with \(v \in \Psi'\) is a collection of identically distributed and pairwise uncorrelated random variables with common finite mean and variance. In industries \(v \in \Psi''\) VC funds are not active, such that \(h_1(v,t) = 0\) and \(dM_1(v,t) = 0\) at all times.
agents (network VCs and entrepreneurs) in the economy.

**Consumption dynamics.** Given the consumption policy \([54]\), agent \(j\)’s consumption follows:

\[
\frac{dC_j(t)}{C_j(t)} = \left( \mu(t) - \frac{\int_{\varpi} \pi_1(v, t) dv}{P_T(t)} \right) dt + \sigma(t) dB_t + \frac{\int_{\varpi} dM_1(v, t) \mathbb{1}_J(v, t) \theta_j(v, t) P(v, t) dv}{\varpi_j(t) P_T(t) \frac{\Delta \varpi^S_T \varpi^S}{4}}.
\]  

(60)

**Value function.** Under the conjectured consumption policy an agent’s value function is given by:

\[
J(\varpi_j(t), Y_t, Z_t) = \mathbb{E}_t \left[ \int_t^\infty m(C_j(\tau), J, \varpi) d\tau \right],
\]

(61)

which yields the following HJB equation for entrepreneurs with a second index element \(i''_E = v\) and network VCs with a second index element \(i''_v = v\):

\[
0 = m(C_j, J(\varpi_j, Y, Z)) + J_Y(\varpi_j, Y, Z) Y \mu(Z) - J_{\varpi_j}(\varpi_j, Y, Z) \varpi_j \frac{\int_{\varpi} \pi_1(v, Z) dv}{P_T(Z)}
\]

\[
+ \frac{1}{2} J_{YY}(\varpi_j, Y, Z) Y^2 \sigma^2(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'}(J(\varpi_j, Y, Z') - J(\varpi_j, Y, Z))
\]

\[
+ \left( J(\varpi_j, Y, Z) - J(\varpi_j, Y, Z) \right) \bar{i}_1(v, Z) h_1(v, Z) \Delta i''_S,
\]  

(62)

where I use the fact \(E[dM_1(v, t)|Z] = \bar{i}_1(v, Z) h_1(v, Z)\), and that \(\Pr[1_j(v, t) = 1] = \Delta i''_S\), if agent \(j\) is an entrepreneur or a network VC with second index element \(i''_E = v\) and \(i''_v = v\), respectively. Note that for white-collar and blue-collar agents we have \(\theta_j(v, Z) = 0\), such that the last term in the previous equation drops out. Conjecture the solution for \(J\) takes the form:

\[
J(\varpi_j, Y, Z) = F(Z) \frac{(\varpi_j Y)^{1-\gamma}}{1-\gamma}.
\]  

(63)

Substituting this conjecture into the HJB equation yields:

\[
0 = \left( \frac{\beta(1 - \gamma)}{\rho} \left( F(Z)^{-\frac{1}{1-\gamma}} - 1 \right) + (1 - \gamma) \left( \mu(Z) - \frac{\int_{\varpi} \pi_1(v, Z) dv}{P_T(Z)} \right) - \frac{1}{2} \gamma(1 - \gamma) \sigma^2(Z) \right) F(Z)
\]

\[
+ \sum_{Z' \neq Z} \lambda_{ZZ'} \left( F(Z') - F(Z) \right) + \Delta i''_S \left( \left( 1 + \frac{\theta_j(v, Z) \bar{P}(v, Z) dv}{\varpi_j P_T(Z) \frac{\Delta \varpi^S_T \varpi^S}{4}} \right)^{1-\gamma} - 1 \right) F(Z) \bar{i}_1(v, Z) h_1(v, Z).
\]  

(64)
For $\gamma > 1$, the value of the term

$$
\left(1 + \frac{\varrho_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \Delta i_s^X d\mu^S} \right)^{1-\gamma} - 1
$$

(65)

is bounded from below by $-1$ and from above by $0$. Thus, we obtain in the relevant limiting case:

$$
\lim_{\Delta i'_S \downarrow 0} \left\{ \Delta i'_S \cdot \left(1 + \frac{\varrho_j(v, Z) \tilde{P}(v, Z_t) dv}{\varpi_j \tilde{P}_T(Z_t) \Delta i_s^X d\mu^S} \right)^{1-\gamma} - 1 \right\} = 0.
$$

(66)

It follows that this term drops from equation (64) in the relevant case when $\lim_{\Delta i'_S \downarrow 0}$, and $F(Z)$ solves:

$$
0 = \left( \beta(1 - \gamma) \left( F(Z) - \frac{\mu}{1-\gamma} - 1 \right) + (1 - \gamma) \left( \mu(Z) - \int \dot{\varrho}_1(v, Z) dv \right) - \frac{1}{2} \gamma (1 - \gamma) \sigma^2(Z) F'(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)) \right).
$$

(67)

Note that this equation is independent of an agent’s skill type $S$ and her wealth share $\varpi_j$, implying that all agents share the same $F(Z)$ function, consistent with the conjecture of the value function.

**Verifying the conjectured equilibrium for $\lim_{\Delta i'_S \downarrow 0}$.** I verify whether any individual agent has an incentive to deviate from the posited behavior, given that all other agents behave as conjectured.

The marginal utility process of agents following the conjectured portfolio and consumption policies may be written as follows:

$$
\xi_{j,t} \equiv \exp \left[ \int_0^t m_J(C_j(\tau), J_\tau) d\tau \right] m_C(C_j(t), J_t).
$$

(68)

Using the value function (63) associated with the conjectured policies, where $F$ solves equation (67), we obtain

$$
\xi_{j,t} = (\varpi_j Y_t)^{-\gamma} \beta F(Z_t)^{1-\frac{\mu}{1-\gamma}} \left\{ \int_0^t \left( \frac{\beta(1-\gamma - \rho)}{\rho} F(Z_\tau)^{1-\frac{\mu}{1-\gamma}} - \frac{\beta(1-\gamma)}{\rho} \right) d\tau \right\}.
$$

(69)

Applying Itô’s lemma yields for entrepreneurs with a second index element $i''_E = v$ and network...
VCs with a second index element \( i'_{v'} = v \):

\[
- \frac{d \xi_{jt}}{\xi_{jt}} = \beta \left( 1 - \gamma \right) - \beta \left( 1 - \gamma - \rho \right) F(Z)^{1 - \frac{\gamma}{\tau}} + \gamma \left( \mu(Z) - \frac{f_{\psi} \tilde{p}_{1}(v, Z) dv}{P_{T}(Z)} \right) - \frac{1}{2} \gamma (1 + \gamma) \sigma^{2}(Z) \\
- \sum_{Z' \neq Z} \lambda_{Z'Z} \left( \frac{F(Z')}{F(Z)} \right)^{1 - \frac{\gamma}{\tau}} - 1 \\
- \Delta i'_{S} \left( 1 + \frac{\partial_{j}(v, Z) \tilde{P}(v, Z_{t}) dv}{\omega_{j} \tilde{P}_{T}(Z_{t}) \Delta i'_{S}} \right)^{-\gamma} - 1 \tilde{v}_{1}(v, Z) h_{1}(v, Z).
\]

(70)

where, for the relevant limiting case \( \Delta i'_{S} \downarrow 0 \), the last term drops out, since:

\[
\lim_{\Delta i'_{S} \downarrow 0} \left\{ \Delta i'_{S} \left( 1 + \frac{\partial_{j}(v, Z) \tilde{P}(v, Z_{t}) dv}{\omega_{j} \tilde{P}_{T}(Z_{t}) \Delta i'_{S}} \right)^{-\gamma} - 1 \tilde{v}_{1}(v, Z) h_{1}(v, Z) \right\} = 0.
\]

(71)

Note that for white-collar and blue-collar agents we have \( \partial_{j}(v, Z) = 0 \), such that the last term also drops out. It follows that all agents agree on the risk-free rate \( r_{f}(Z) = \frac{\varepsilon_{t} d_{j_{t}}}{\xi_{jt}} \) independently of their skill \( S \) and \( \omega_{j} \). For entrepreneurs with a second index element \( i'_{v'} = v \) and network VCs with a second index element \( i'_{v'} = v \), the dynamics for \( \xi_{jt} = v \) are given by:

\[
\frac{d \xi_{jt}}{\xi_{jt}} = - r_{f}(Z_{t}) dt - \gamma \sigma(Z_{t}) dB_{t} + \sum_{Z' \neq Z_{t}} \left( \frac{F(Z')}{F(Z_{t})} \right)^{1 - \frac{\gamma}{\tau}} - 1 \left( dN_{t}(Z_{t} \rightarrow Z') - \lambda_{Z_{t}Z'} dt \right) \\
+ 1_{j}(v, t) \left( 1 + \frac{\partial_{j}(v, Z) \tilde{P}(v, Z_{t}) dv}{\omega_{j}(t) \tilde{P}_{T}(Z_{t}) \Delta i'_{S}} \right)^{-\gamma} - 1 \right) dM_{1}(v, t).
\]

(72)

For white-collar and blue-collar agents the last term again drops out. A central feature of the analysis with imperfect risk sharing is that agents cannot trade claims that pay conditional on the joint future realizations of their idiosyncratic venture opportunities \( 1_{j}(v, t) \) and \( \partial_{j}(v, t) \) and VC-backed venture successes \( dM_{1}(v, t) \). Yet consider the set of tradable claims, which can pay a cash flow \( CF \) at some future date \( \tau > t \) contingent on realizations of the aggregate state variables \( Y \) and \( Z \), the quality ladder counting processes \( \{ M(v, \tau) \}_{v \in \Psi} \), the VC-backed patent counting processes \( \{ M_{1}(v, \tau) \}_{v \in \Psi} \), and the VC intermediation variables \( \{ \nu_{1}(v, \tau) \}_{v \in \Psi} \). Formally, any tradable claim to a cash flow is specified as a general function of these state variables:

\[
CF_{\tau} = CF(Y_{\tau}, Z_{\tau}, \{ \nu_{1}(v, \tau) \}_{v \in \Psi}, \{ M(v, \tau) \}_{v \in [0, 1]}, \{ M_{1}(v, \tau) \}_{v \in [0, 1]})
\]

(73)
Define “ns” (no success) as the event where
\[
\int_t^\tau \int_{\Psi} \mathbb{1}_j(v,s) \vartheta_j(v,Z_s) dM_1(v,s) = 0,
\] (74)
that is, agent \(j\) does not obtain idiosyncratic venture equity gains between time \(t\) and time \(\tau\).

Define \(PNS_j(t,\tau,Z)\), as the probability of this event, which will generally depend on the current state \(Z\) and the type of the agent \(j\). \(PNS_j(t,\tau,Z)\) solves for all \(Z\):
\[
0 = \frac{\partial PNS_j(t,\tau,Z)}{\partial t} - \frac{1}{dt} \mathbb{E} \left[ \int_t^\tau \mathbb{1}_j(v,s) \vartheta_j(v,Z) dM_1(v,t) \right] PNS_j(t,\tau,Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (PNS_j(t,\tau,Z') - PNS_j(t,\tau,Z))
\] (75)
with \(PNS_j(\tau,\tau,Z) = 1\). Note that in the relevant case where \(\Delta i'_S \downarrow 0\), we obtain for entrepreneurs with a second index element \(i''_E = v\) and network VCs with a second index element \(i''_V = v\):
\[
\lim_{\Delta i'_S \downarrow 0} \left\{ \frac{1}{dt} \mathbb{E} \left[ \int_t^\tau \mathbb{1}_j(v,s) \vartheta_j(v,Z) dM_1(v,t) \right] \right\}
= \lim_{\Delta i'_S \downarrow 0} \left\{ \Delta i'_S \vartheta_j(v,Z) \bar{\iota}_1(v,Z) h_1(v,Z) \right\}
= 0.
\] (76)

For white-collar and blue-collar agents this term also takes the value zero. It follows that \(PNS_j(t,\tau,Z) = 1\). Agents then value a claim to \(CF_\tau\) as follows:
\[
\mathbb{E}_t \left[ \frac{\xi_{j,\tau}}{\xi_{j,t}} CF_\tau \right] = \mathbb{E}_t \left[ \frac{\xi_{j,\tau}}{\xi_{j,t}} CF_\tau \middle| nS \right] PNS_j(t,\tau,Z_t) + \mathbb{E}_t \left[ \frac{\xi_{j,\tau}}{\xi_{j,t}} CF_\tau \middle| nS^c \right] (1 - PNS_j(t,\tau,Z_t))
= \mathbb{E}_t \left[ \frac{\xi_{j,\tau}}{\xi_{j,t}} CF_\tau \middle| nS \right],
\] (77)
that is, the tradable claim is valued by all agents as if the evolution of their \(\xi_{j,t}\) was given by:
\[
\frac{d\xi_{j,t}}{\xi_{j,t}} = - r_f (Z_t) dt - \gamma \sigma (Z_t) dB_t
+ \sum_{Z' \neq Z_t} \left( \left( \frac{F(Z')}{F(Z_t)} \right)^{1-r_f} - 1 \right) (dN_t(Z_t-Z',Z') - \lambda_{Z_t-Z'} dt).
\] (78)
The prices agents assign to tradable claims are thus independent of their own wealth share $\varpi_j$ and their skill $S$. Since all agents agree on the prices of tradable claims they have no incentive to deviate from the conjectured portfolio- and consumption policies. Thus, the overall wealth distribution does not constitute a state variable, and the only relevant aggregate state variables are $Z$ and $Y$.

**Valuing growth contributions of aggregate VC investment.** We have proven that for any agent with a current wealth share $\varpi_j(t)$ the solution for the value function $J$ is given by (63), where $F(Z)$ solves (67). Under the alternative scenario where starting from date $t$, agents do not expect any future innovations from VC investments, but future state-contingent resource allocations remain unchanged, all agents still have the same tradable wealth shares, as all agents hold the market portfolio, and thus, their tradable wealth is equally affected, in percentage terms, by this change in expectations. The current level of output $Y_t$ is also unaffected by this change in expectations. However, the function $F(Z)$ now solves:

$$
0 = \left( \frac{\beta(1-\gamma)}{\rho} \left( F(Z)^{1-\gamma} - 1 \right) + (1-\gamma) \left( \mu(Z) - \log [\kappa] \int_{\Psi} \tilde{i}_1(v, Z) h_1(v, Z) dv \right) 
- \frac{1}{2} \gamma(1-\gamma) \sigma^2(Z) \right) F(Z) + \sum_{Z' \neq Z} \lambda_{ZZ'} (F(Z') - F(Z)),
$$

(79)

which accounts for the lower growth expectations. Overall, each agent’s value function is equally affected, in percentage terms, by the change in expectations, since each agent experiences an identical change in the $F(Z)$ function. Since agents have identical preference parameters, they are also willing to give up the same fraction of lifetime consumption to avoid losing the growth contributions from VC investments.

**References**


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