A Multi-factor Model

\[ \tilde{R}_i = [b_{i1} \tilde{F}_1 + b_{i2} \tilde{F}_2 + \ldots + b_{in} \tilde{F}_n] + \tilde{\varepsilon}_i \]

Factors:

Returns on mutually exclusive and exhaustive market capitalization-weighted portfolios
Barra U.S. Stock Index Performance

FIGURE 2
ANNUAL EQUITY RETURNS
1986 - 1990

Year

Value
Growth
Medium
Small
Trustees' Commingled Fund - U.S. Portfolio
January 1985 through December 1989

<table>
<thead>
<tr>
<th>Securities</th>
<th>Unconstrained Regression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bills</td>
<td>14.69</td>
</tr>
<tr>
<td>Intermediate Bonds</td>
<td>-69.51</td>
</tr>
<tr>
<td>Long-term Bonds</td>
<td>-2.54</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>16.57</td>
</tr>
<tr>
<td>Mortgages</td>
<td>5.19</td>
</tr>
<tr>
<td>Value Stocks</td>
<td>109.52</td>
</tr>
<tr>
<td>Growth Stocks</td>
<td>-7.86</td>
</tr>
<tr>
<td>Medium Stocks</td>
<td>-41.83</td>
</tr>
<tr>
<td>Small Stocks</td>
<td>45.65</td>
</tr>
<tr>
<td>Foreign Bonds</td>
<td>-1.85</td>
</tr>
<tr>
<td>European Stocks</td>
<td>6.15</td>
</tr>
<tr>
<td>Japanese Stocks</td>
<td>-1.46</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>72.71</strong></td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td><strong>95.20</strong></td>
</tr>
</tbody>
</table>
Regression Analysis with a Constraint

Trustees' Commingled Fund - U.S. Portfolio
January 1985 through December 1989

<table>
<thead>
<tr>
<th></th>
<th>Unconstrained Regression</th>
<th>Constrained Regression</th>
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</thead>
<tbody>
<tr>
<td>Bills</td>
<td>14.69</td>
<td>42.65</td>
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<td>-68.64</td>
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<td>Long-term Bonds</td>
<td>-2.54</td>
<td>-2.38</td>
</tr>
<tr>
<td>Corporate Bonds</td>
<td>16.57</td>
<td>15.29</td>
</tr>
<tr>
<td>Mortgages</td>
<td>5.19</td>
<td>4.58</td>
</tr>
<tr>
<td>Value Stocks</td>
<td>109.52</td>
<td>110.35</td>
</tr>
<tr>
<td>Growth Stocks</td>
<td>-7.86</td>
<td>-8.02</td>
</tr>
<tr>
<td>Medium Stocks</td>
<td>-41.83</td>
<td>-43.62</td>
</tr>
<tr>
<td>Small Stocks</td>
<td>45.65</td>
<td>47.17</td>
</tr>
<tr>
<td>Foreign Bonds</td>
<td>-1.85</td>
<td>-1.38</td>
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<tr>
<td>European Stocks</td>
<td>6.15</td>
<td>5.77</td>
</tr>
<tr>
<td>Japanese Stocks</td>
<td>-1.46</td>
<td>-1.79</td>
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<tr>
<td>Total</td>
<td>72.71</td>
<td>100.00</td>
</tr>
<tr>
<td>R-squared</td>
<td>95.20</td>
<td>95.16</td>
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## Quadratic Programming

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<th>Quadratic Programming</th>
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<td>42.65</td>
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<td>-68.64</td>
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<td>Long-term Bonds</td>
<td>-2.54</td>
<td>-2.38</td>
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<tr>
<td>Corporate Bonds</td>
<td>16.57</td>
<td>15.29</td>
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<td>Mortgages</td>
<td>5.19</td>
<td>4.58</td>
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<td>109.52</td>
<td>110.35</td>
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<td>-1.38</td>
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<tr>
<td>European Stocks</td>
<td>6.15</td>
<td>5.77</td>
<td>0.15</td>
</tr>
<tr>
<td>Japanese Stocks</td>
<td>-1.46</td>
<td>-1.79</td>
<td>0</td>
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<tr>
<td><strong>Total</strong></td>
<td><strong>72.71</strong></td>
<td><strong>100.00</strong></td>
<td><strong>100.00</strong></td>
</tr>
<tr>
<td><strong>R-squared</strong></td>
<td><strong>95.20</strong></td>
<td><strong>95.16</strong></td>
<td><strong>92.22</strong></td>
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"An Algorithm for Portfolio Improvement"
Advances in Mathematical Programming and Financial Planning, 1987
## Asset Classes

“Determining a Fund's Effective Asset Mix”
(Dec. 1988)

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Index Description</th>
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</thead>
<tbody>
<tr>
<td><strong>Bills</strong></td>
<td>Salomon Brothers’ 90-day Bill Index</td>
</tr>
<tr>
<td><strong>Intermediate Government Bonds</strong></td>
<td>Shearson-Lehman Intermediate Government Bond Index</td>
</tr>
<tr>
<td><strong>Long-term Government Bonds</strong></td>
<td>Shearson-Lehman Long-term Government Bond Index</td>
</tr>
<tr>
<td><strong>Corporate Bonds</strong></td>
<td>Shearson-Lehman Corporate Bond Index</td>
</tr>
<tr>
<td><strong>Mortgages</strong></td>
<td>Shearson-Lehman Mortgage-backed Securities Index</td>
</tr>
<tr>
<td><strong>Large Value Stocks</strong></td>
<td>The Russell Price-driven Stock Index™</td>
</tr>
<tr>
<td><strong>Large Growth Stocks</strong></td>
<td>The Russell Earnings-growth Stock Index™</td>
</tr>
<tr>
<td><strong>Small Stocks</strong></td>
<td>The Russell 2000 Small Stock Index®</td>
</tr>
<tr>
<td><strong>Foreign Bonds</strong></td>
<td>Salomon Brothers’ Non-U.S. Government Bond Index</td>
</tr>
<tr>
<td><strong>Foreign Stocks</strong></td>
<td>Financial Times Actuaries Euro-Pacific Index</td>
</tr>
</tbody>
</table>
In-sample R2 Values
388 Pension Fund Manager Accounts

EXHIBIT VII
R² values for manager-fund combinations

Number of Managers

0 10 20 30 40 50 60 70 80 90 100
Percentage due to Asset Allocation (R²)
Asset Classes

“Asset Allocation: Management Style and Performance Measurement”
Winter 1992

- Bills
- Intermediate-Term Government Bonds
- Long-Term Government Bonds
- Corporate Bonds
- Mortgage-Related Securities
- Large-Capitalization Value Stocks
- Large-Capitalization Growth Stocks
- Medium-Capitalization Stocks
- Small-Capitalization Stocks
- Non-U.S. Bonds
- European Stocks
- Japanese Stocks
Figure 4
TRUSTEES' COMMINGLED- U.S. PORTFOLIO
STYLE COMPOSITION
FIGURE 8
STYLES, 1985-1989
161 GROWTH EQUITY FUNDS

Percent of Total

Bills
Intermediate Governments
Long Governments
Corporate Bonds
Mortgages
Value Stocks
Growth Stocks
Medium Stocks
Small Stocks
Foreign Bonds
Europe Stocks
Japan Stocks

Selection 10.1%
Style 89.9%
FIGURE 9
STYLES, 1985-1989
118 GROWTH AND INCOME EQUITY FUNDS

Percent of Total
FIGURE 11

STYLES, 1985-1989
NINETEEN BALANCED FUNDS

Percent of Total

Bills
Intermediate Governments
Long Governments
Corporate Bonds
Mortgages
Value Stocks
Growth Stocks
Medium Stocks
Small Stocks
Foreign Bonds
Europe Stocks
Japan Stocks

R2 = 89.0 %
FIGURE 12
STYLES, 1985-1989
FIFTY-FOUR BOND HIGH-QUALITY FUNDS

Percent of Total

- Bills
- Intermediate Governments
- Long Governments
- Corporate Bonds
- Mortgages
- Value Stocks
- Growth Stocks
- Medium Stocks
- Small Stocks
- Foreign Bonds
- Europe Stocks
- Japan Stocks

Selection 11.3%
Style 88.1%
FIGURE 13
STYLES, 1985-1989
FIVE CONVERTIBLE BOND FUNDS

Bills
Intermediate Governments
Long Governments
Corporate Bonds
Mortgages
Value Stocks
Growth Stocks
Medium Stocks
Small Stocks
Foreign Bonds
Europe Stocks
Japan Stocks

Percent of Total

Selection 11.2%
Style 88.8%
Figure 16
FIDELITY MAGELLAN FUND
CUMULATIVE RETURN DIFFERENCE: FUND VERSUS BENCHMARK

Mean: 0.18 %/mo  Standard Deviation = 1.48 %/mo  t(avg) = 0.84
Figure 17
FIDELITY MAGELLAN FUND
CUMULATIVE RETURN DIFFERENCE: FUND VERSUS STYLE

Mean: 0.57 %/mo  Standard Deviation = 1.05 %/mo  t(avg) = 3.76
FIGURE 3
TRUSTEE'S COMMINGLED - U.S. PORTFOLIO
JANUARY 1985 - DECEMBER 1989

Style based on 60 monthly returns

Selection 7.0%
Style 92.2%
Figure 18
AVERAGE TRACKING ERROR
636 MUTUAL FUNDS, 1985-1989

Mean = -0.074 %/mo (annualized: 0.888% /year)
Style and Performance Analysis

- Performance in month $t$ vs. Style at end of month $t-2$
- Out-of-sample $R^2$ values
- Style Analysis
  - $\lambda$ number of months utilized
  - $\lambda$ exponential weighting of monthly observations
- Aggregate results for individual funds
A Large Pension Fund

Total Fund
Compound Return, 200909

<table>
<thead>
<tr>
<th></th>
<th>Value of $100</th>
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</thead>
<tbody>
<tr>
<td>Total</td>
<td>123.34</td>
</tr>
<tr>
<td>Style</td>
<td>128.81</td>
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<tr>
<td>Difference</td>
<td>-5.47</td>
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</table>
SH4KA1: TOTAL DOMESTIC EQUITY Performance, 200909

# # #

<table>
<thead>
<tr>
<th>Statistics</th>
<th>FirstMth</th>
<th>LastMth</th>
<th>NumMths</th>
<th>PctStyle</th>
<th>SelMn</th>
<th>SelSD</th>
<th>t-stat</th>
<th>SelPctl</th>
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<tbody>
<tr>
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<td>200410</td>
<td>200909</td>
<td>60</td>
<td>99.1</td>
<td>0.00</td>
<td>1.47</td>
<td>0.00</td>
<td>50.2</td>
</tr>
</tbody>
</table>
Holdings-based vs. Returns-based Style Analysis Models
(from Morningstar website)

Holdings-based
- Uses characteristics of underlying securities

Returns-based
- More widely used among financial professionals, because the input data (monthly returns) is (sic) widely available

Morningstar has long been a proponent of holdings-based style analysis but recognizes that there may be situations where returns-based analysis can also be helpful.

The company's institutional research platform … combines advanced holdings-based and returns-based style analysis.

Ideally, practitioners should use both approaches.