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JACOBS LEVY EQUITY MANAGEMENT CENTER
FOR QUANTITATIVE FINANCIAL RESEARCH

Size Matters, If You Control Your Junk

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Motivation: The Size Premium

1. Banz (1981) found that small stocks in the U.S. have higher average returns than large stocks, a relation which is not accounted for by market beta

2. The size anomaly has become one of the focal points for discussions of market efficiency

3. The size factor has become one of the staples of current asset pricing models used in the literature
   - e.g., Fama and French (1993, 2014)

4. The size premium implies that small firms face larger costs of capital than large firms
   - Important implications for corporate finance, incentives to merge and form conglomerates, and broader industry dynamics

5. The size effect has had a large impact on investment practice:
   - Spawning an entire category of investment funds
   - Giving rise to indices
   - Serving as a cornerstone for mutual fund classification

Seven Criticisms of the Size Anomaly

1. It has a weak historical record
   - Many papers find that the size effect is simply not very significant
   - E.g., Israel and Moskowitz (2013)

2. It varies significantly over time, in particular weakening after its discovery in the early 1980s
   - The size effect has disappeared since the early 1980s
   - E.g., Dichev (1998), Chan, Karceski, and Lakonishok (2000), Horowitz, Loughran, and Savin (2000), Amihud (2002), Schwert (2003) and Van Dijk (2011)

3. It appears to be driven by "extreme" stocks
   - Removing stocks with less than $5 million in market cap or smallest 5% of firms causes the small firm effect to vanish
   - E.g., Horowitz, Loughran, and Savin (2000), Crain (2011) and Bryan (2014)

4. Predominantly resides in January
   - Premium seems to be in January, particularly in the first few trading days of the year, and is largely absent the rest of the time
   - E.g., Reinganum (1981), Roll (1981), Keim (1983), Gu (2003), Easterday, Sen, and Stephan (2009)

Seven Criticisms of the Size Anomaly – Cont'd

5. Size premium is not present for measures of size that do not rely on market prices
   - Non-price based measures of size do not yield a relation between size and average returns
   - E.g., Berk (1995, 1997)

6. Size premium is subsumed by proxies for illiquidity
   - Size may just be a proxy for a liquidity effect
   - E.g. Brennan and Subrahmanyam (1996), Amihud (2002), Hou and Moskowitz (2005), Sadka (2006), Ibbotson, Chen, Kim, and Hu (2013) , Pastor and Stambaugh (2003), Acharya and Pedersen (2005)
   - Crain (2011) summarizes the evidence on size and liquidity

7. Size premium is weak internationally
   - The size anomaly is weaker and not very robust in international equity markets, and hence the size effect may possibly be the result of data mining
   - E.g., Crain (2011) and Bryan (2014)

What We Do

We define a security's "quality" as characteristics that, all-else-equal, an investor should be willing to pay a higher price for:

- Stocks that are safe, profitable, growing, and well managed

Size and quality are negatively related

- Stocks with very poor quality (i.e., "junk") are typically very small, have low average returns, and are typically distressed and illiquid securities

We control for quality using the *Quality-Minus-Junk* (QMJ) factor proposed by Asness, Frazzini, and Pedersen (2014)

- Also look at sub-components based on profitability, profit growth, safety, and payout
- And do robustness checks using other measures of quality besides QMJ (e.g., Fama-French)

We examine the evidence on the size premium controlling for a security's quality

- We test whether the strong negative relation between size and quality explains the sporadic performance of the size premium and its challenges

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Summary of Results

1. Size matters: controlling for quality, a significant size premium emerges
   - Alphas of 5.9% per year, t-stat = 4.89 with QMJ in regression vs. 1.68% per year, t-stat 1.23 without it (using market, lagged market, HML and UMD and adding QMJ or not; all over the 7:1957-12:2012 period)
2. Stable through time and robust out of sample
3. Not concentrated in "extreme" stocks
4. More consistent across seasons and markets
5. Robust to non-price based measures of size
6. Not captured by an illiquidity premium
7. More consistent internationally

Road Map

- Defining quality and test portfolios
- Evidence: The size premium
- Evidence: The size premium controlling for quality/junk
- Conclusion

Defining Quality: Asness, Frazzini, and Pedersen (2014)

Gordon's growth model:

$$P = \frac{\text{dividend}}{\text{required return} - \text{growth}}$$

With very high tech math:

$$\frac{P}{B} = \frac{\text{profit/B} \times \text{dividend/profit}}{\text{required return} - \text{growth}} = \frac{\text{profitability} \cdot \text{payout ratio}}{\text{required return} - \text{growth}}$$

Defining Quality: Asness, Frazzini, and Pedersen (2014)

Gordon's growth model:

$$\frac{P}{B} = \frac{\text{profitability} \cdot \text{payout ratio}}{\text{required return} - \text{growth}}$$

Four quality measures:

Profitability: Gross profits, margins, earnings, accruals and cash flows; and focus on each stock's average rank across these metrics

Defining Quality: Asness, Frazzini, and Pedersen (2014)

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Profitability: Gross profits, margins, earnings, accruals and cash flows; and focus on each stock's average rank across these metrics

Growth: Prior five-year growth in each of our profitability measures

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Defining Quality: Asness, Frazzini, and Pedersen (2014)

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$$\frac{P}{B} = \frac{\text{profitability} \cdot \text{payout ratio}}{\text{required return} - \text{growth}}$$

Four quality measures:

Profitability: Gross profits, margins, earnings, accruals and cash flows; and focus on each stock's average rank across these metrics

Growth: Prior five-year growth in each of our profitability measures

Safety: We consider both return-based measures of safety (e.g., market beta and volatility) and fundamental-based measures of safety (e.g., stocks with low leverage, low volatility of profitability, and low credit risk)

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Defining Quality: Asness, Frazzini, and Pedersen (2014)

Gordon's growth model:

$$\frac{P}{B} = \frac{\text{profitability} \cdot \text{payout ratio}}{\text{required return} - \text{growth}}$$

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Safety: We consider both return-based measures of safety (e.g., market beta and volatility) and fundamental-based measures of safety (e.g., stocks with low leverage, low volatility of profitability, and low credit risk)

Payout: Fraction of profits paid out to shareholders. This characteristic is determined by management and can be seen as a measure of shareholder friendliness (e.g., if free cash flow increases agency problems)

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Data Sources and Portfolios

Data Sources

- Merged CRSP/ Xpressfeed Global, Common stocks
- Long sample: U.S., 1956 – 2012
- Broad sample: Global, 1986 – 2012, 24 Countries (MSCI Developed Markets)

Size: SMB (Small minus Big) factors

- Fama and French's SMB factors and a set of value-weighted decile portfolios based on market capitalization sorts
- Source: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
- We also compute non-price based SMBs (Total Assets, Employees , …)

Quality: QMJ (Quality minus Junk)

- Asness, Frazzini, and Pedersen (2014), formed by ranking stocks on measures of quality/junk based on their profitability, growth, safety, and payout
- Source: https://www.aqr.com/library/data-sets

Other Fama and French (1992, 2014) and Asness, Frazzini, and Pedersen (2014) factors, Frazzini and Pedersen (2013) BAB factors, credit portfolios and various liquidity measures

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Road Map

- Defining quality and test portfolios
- Evidence: The size premium
- Evidence: The size premium controlling for quality/junk
- Conclusion

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The Size Effect, 1926 – 2012

This table reports summary statistics on the size premium over time. Returns are monthly.

Panel A: Size premium over time

		SMB raw returns		SMB 4-factor alpha*	
		Mean	*t*-stat	Mean	*t*-stat
Full sample	1926:07-2012:12	0.23%	2.27	0.05%	0.48
January		2.30%	6.50	0.76%	2.02
Feb. - Dec.		0.04%	0.41	-0.13%	-1.34
Banz (1981)	1936:01-1975:12	0.16%	1.22	-0.03%	-0.29
Pre-&Post-Banz (1981)	1926:07-1935:12; 1976:01-2012:12	0.29%	1.92	0.11%	0.77
QMJ sample	1957:07-2012:12	0.22%	1.93	0.14%	1.23
January		2.08%	4.68	0.64%	1.35
Feb. - Dec.		0.06%	0.47	-0.05%	-0.45
Golden age	1957:07-1979:12	0.35%	2.00	0.25%	1.52
Embarrassment	1980:01-1999:12	-0.04%	-0.23	-0.11%	-0.64
Resurrection	2000:01-2012:12	0.42%	1.41	0.54%	2.06
BAB sample	1931:01-2012:12	0.29%	2.78	0.07%	0.72
FF 5-factor sample	1963:07-2012:12	0.25%	1.95	0.16%	1.31
Credit sample	1987:07-2012:12	0.14%	0.74	0.07%	0.40

*SMB 4-factor alpha is against the market, market lagged one month, HML and UMD.

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Results: Size Matters

Size matters: *controlling for quality, a significant size premium emerges*

This table shows monthly returns and alphas of size-sorted portfolios

Panel A: Adding QMJ

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + \mathrm{h} HML_t + \mathrm{m} UMD_t + \mathrm{q} QMJ_t + \varepsilon_t$$

	α	*t* (α)	β	*t* (β)	β$_{-1}$	*t* (β$_{-1}$)	h	*t* (h)	m	*t* (m)	q	*t* (q)	R^2
QMJ period	**0.0014**	**1.23**	0.17	6.36	0.13	5.42	-0.16	-3.96	0.00	0.13			0.15
(1957:07-2012:12)	**0.0049**	**4.89**	-0.04	-1.42	0.10	4.82	-0.24	-6.75	0.06	2.70	-0.74	-15.09	0.37
Golden age	**0.0025**	**1.52**	0.27	7.19	0.15	4.10	0.07	0.95	-0.09	-1.83			0.24
(1957:07-1979:12)	**0.0057**	**4.00**	0.07	1.96	0.14	4.70	-0.24	-3.73	-0.06	-1.39	-0.97	-10.73	0.48
Embarrassment	**-0.0011**	**-0.64**	0.04	0.97	0.18	5.05	-0.24	-3.56	-0.08	-1.63			0.18
(1980:01-1999:12)	**0.0050**	**3.06**	-0.14	-3.43	0.15	4.85	-0.42	-6.84	-0.06	-1.34	-0.83	-9.08	0.40
Resurrection	**0.0054**	**2.06**	0.25	4.25	0.10	1.75	-0.34	-4.46	0.14	3.00			0.25
(2000:01-2012:12)	**0.0089**	**4.04**	-0.17	-2.43	-0.03	-0.59	-0.18	-2.68	0.17	4.43	-0.84	-8.40	0.49

Panel B: Subcomponents of QMJ

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + \mathrm{h} HML_t + \mathrm{m} UMD_t + \mathrm{q} Q^*_t + \varepsilon_t$$

QMJ Period (1957:07 – 2012:12)	α	*t* (α)	β	*t* (β)	β$_{-1}$	*t* (β$_{-1}$)	h	*t* (h)	m	*t* (m)	q	*t* (q)	R^2
Q* = Profit	**0.0042**	**3.95**	0.06	2.36	0.11	5.07	-0.33	-8.04	0.03	1.24	-0.67	-10.98	0.28
Q* = Growth	**0.0020**	**1.80**	0.17	6.57	0.13	5.50	-0.27	-5.39	0.01	0.27	-0.26	-3.68	0.17
Q* = Safety	**0.0035**	**3.53**	-0.03	-1.12	0.10	4.82	0.20	4.61	0.05	1.98	-0.87	-14.94	0.36
Q* = Payout	**0.0044**	**4.60**	-0.12	-4.28	0.09	4.35	-0.28	-7.93	0.08	3.63	-0.70	-16.86	0.40

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Results: Size Matters

Size matters: *controlling for alternative measures of quality, a significant size premium emerges*

This table shows monthly returns and alphas of size-sorted portfolios

Panel C: Out of Sample and Other Measures of Quality

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + \mathrm{h} HML_t + \mathrm{m} UMD_t + \mathrm{b} BAB_t + \mathrm{d} Cred_t + \varepsilon_t$$

	α	*t* (α)	β	*t* (β)	β_{-1}	*t* (β_{-1})	h	*t* (h)	m	*t* (m)	b	*t* (b)	d	*t* (d)	R^2
1957:07-2012:12	**0.0014**	**1.23**	0.17	6.36	0.13	5.42	-0.16	-3.96	0.00	0.13					0.16
	0.0025	**2.42**	-0.12	-3.62	0.12	5.36	0.01	0.37	0.09	3.48	-0.43	-12.30			0.31
1931:01-1957:06	**0.0006**	**0.33**	0.07	2.11	0.14	5.39	0.29	5.47	0.01	0.13					0.30
	0.0016	**0.90**	-0.14	-2.55	0.16	6.32	0.08	1.22	0.04	1.04	-0.35	-4.99			0.36
1931:01-2012:12	**0.0007**	**0.72**	0.19	10.09	0.13	7.54	0.03	1.09	-0.01	-0.28					0.17
	0.0023	**2.50**	-0.13	-4.77	0.14	8.85	0.01	0.24	0.07	3.39	-0.42	-14.85			0.33
1987:07-2012:12	**0.0005**	**0.27**	0.11	2.77	0.13	3.39	-0.31	-5.23	0.04	1.15					0.17
	0.0035	**2.12**	0.04	1.13	0.08	2.10	-0.28	-5.02	0.07	2.15			-0.12	-7.82	0.31
	0.0032	**2.12**	-0.27	-5.35	0.06	1.97	-0.06	-1.13	0.19	5.65	-0.45	-8.58	-0.08	-5.74	0.45

Results: Size Matters

Size matters: *controlling for alternative measures of quality, a significant size premium emerges*

This table shows monthly returns and alphas of size-sorted portfolios

Panel D: Fama and French (2014) 5-Factor Model and Quality

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + \mathrm{h} HML_t + \mathrm{m} UMD_t + \mathrm{r} RMW_t + \mathrm{c} CMA_t + qQMJ_t + \mathrm{b} BAB_t + \mathrm{d} Cred_t + \varepsilon_t$$

	α	*t* (α)	β	*t* (β)	β_{-1}	*t* (β_{-1})	h	*t* (h)	m	*t* (m)	r	*t* (r)	c	*t* (c)	q	*t* (q)	b	*t* (b)	R^2
1963:07-2012:12	**0.0016**	**1.31**	0.17	6.13	0.14	5.33	-0.17	-3.87	0.01	0.52									0.16
	0.0033	**2.82**	0.11	4.04	0.14	5.63	-0.09	-1.52	0.04	1.57	-0.54	-9.74	-0.15	-1.81					0.28
	0.0054	**4.92**	-0.07	-2.25	0.10	4.46	-0.30	-5.30	0.08	3.18	0.15	1.82	0.06	0.70	-0.89	-10.12			0.38
	0.0031	**2.86**	-0.11	-3.11	0.12	5.42	0.00	0.09	0.10	3.88	-0.35	-6.41	-0.01	-0.13			-0.37	-9.41	0.37
	0.0047	**4.36**	-0.16	-4.69	0.10	4.62	-0.18	-3.06	0.11	4.29	0.08	0.96	0.09	1.15	-0.64	-6.64	-0.24	-5.61	0.41

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + \mathrm{h} HML_t + \mathrm{m} UMD_t + i\overline{QIndex}_t + qQMJ_t + \varepsilon_t$$

	α	*t* (α)	β	*t* (β)	β_{-1}	*t* (β_{-1})	h	*t* (h)	m	*t* (m)	i	*t* (i)	q	*t* (q)	R^2
1957:07-2012:12	**0.0014**	**1.23**	0.17	6.36	0.13	5.42	-0.16	-3.96	0.00	0.13					0.16
	0.0041	**3.60**	0.00	0.08	0.11	4.41	-0.03	-0.69	0.07	2.80	-0.51	-12.04			0.32
	0.0055	**5.15**	-0.08	-2.62	0.10	4.40	-0.17	-4.00	0.09	3.62	-0.24	-4.65	-0.57	-8.78	0.40

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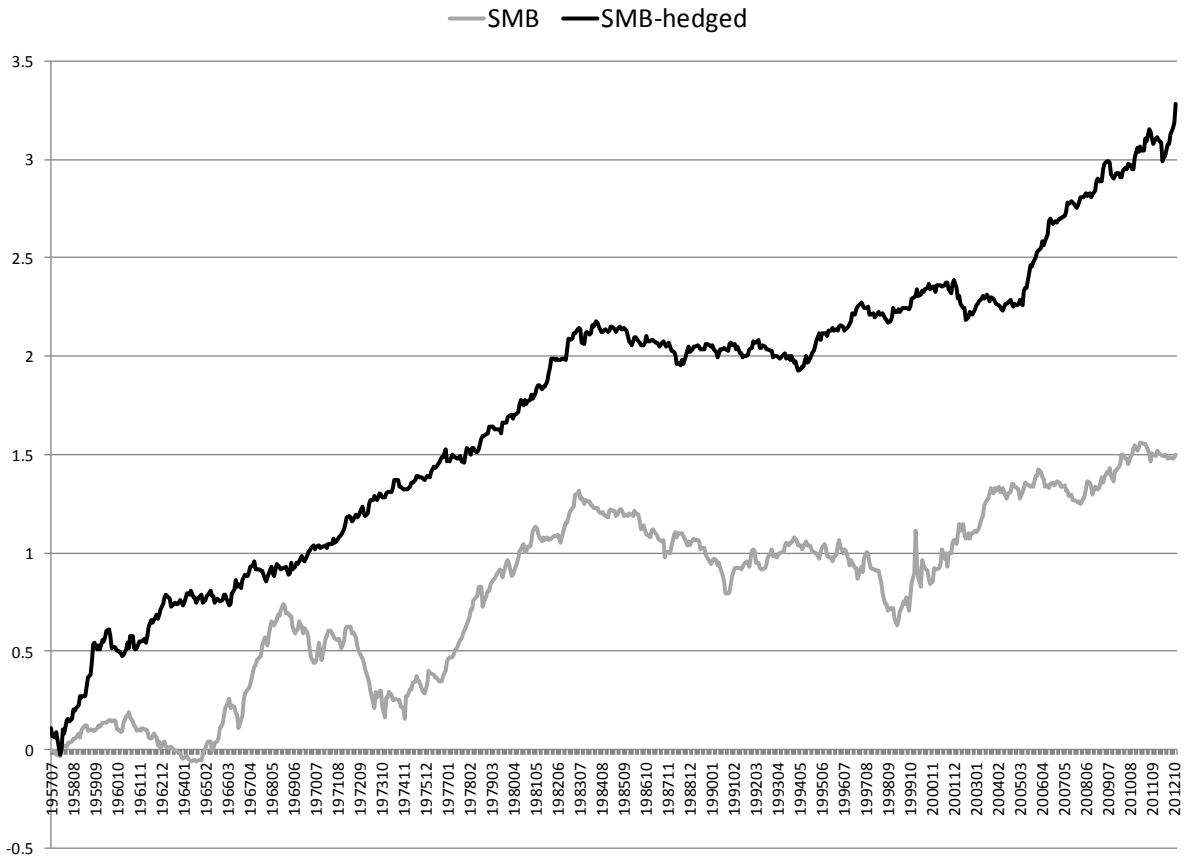
Results: Size Matters

Controlling for quality, the premium is *stable through time and robust out of sample*

The figure plots the cumulative sum of returns over time of (i) SMB hedged with QMJ and (ii) SMB unhedged



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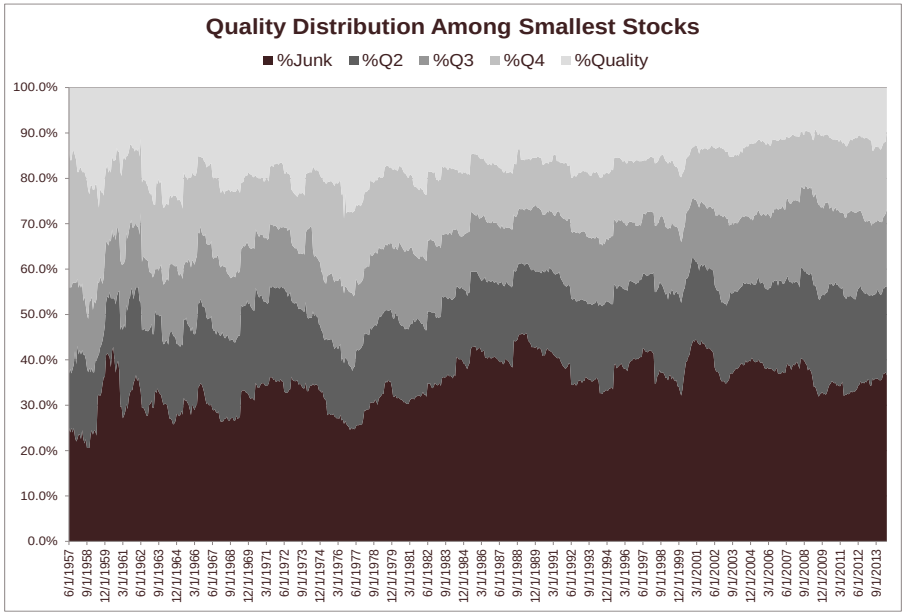
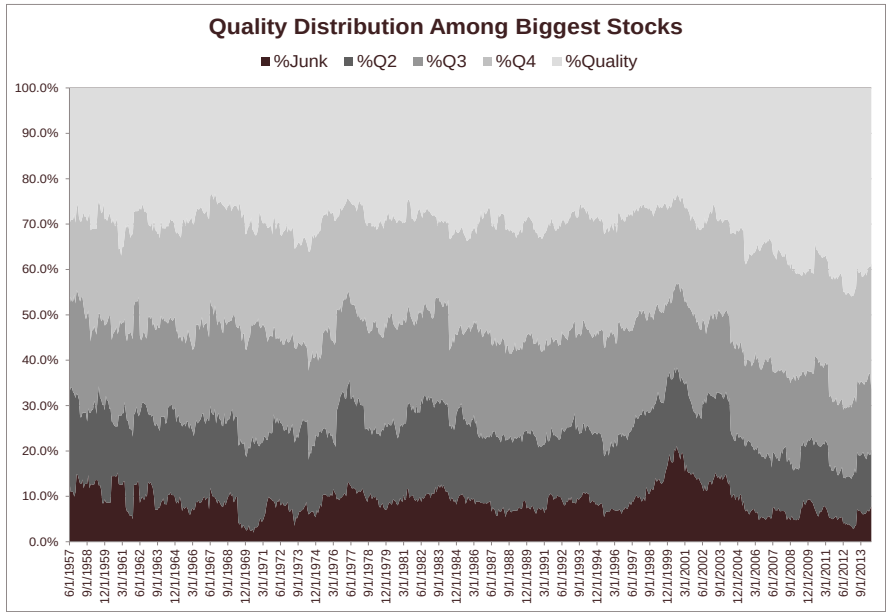
Results: Why Size Matters After Controlling for Quality

Distribution of quality/junk among large and small stocks

- Junk stocks are typically very small, have low average returns, and are typically distressed and illiquid securities
- These characteristics drive the strong negative relation between size and quality and the returns of these junk stocks chiefly explain the sporadic performance of the size premium





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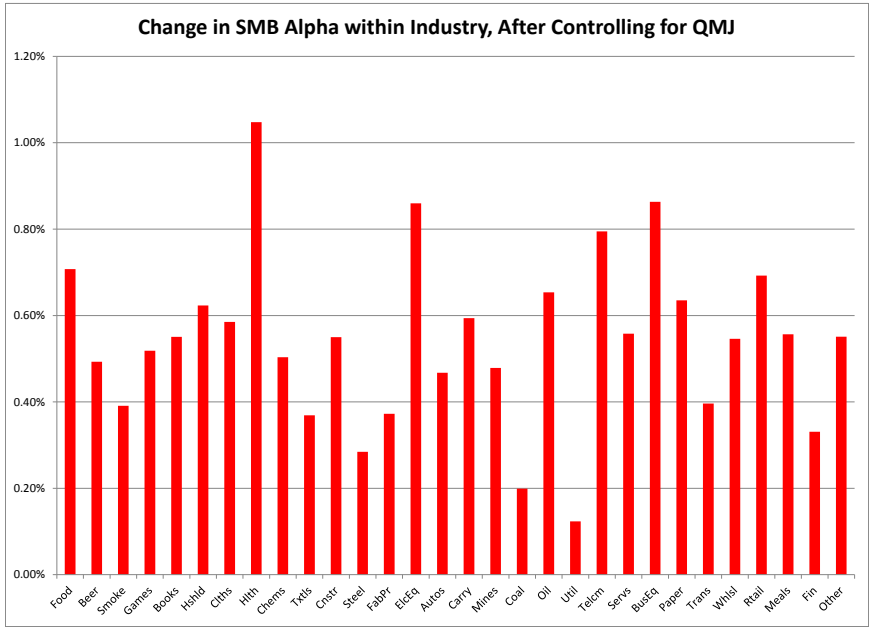
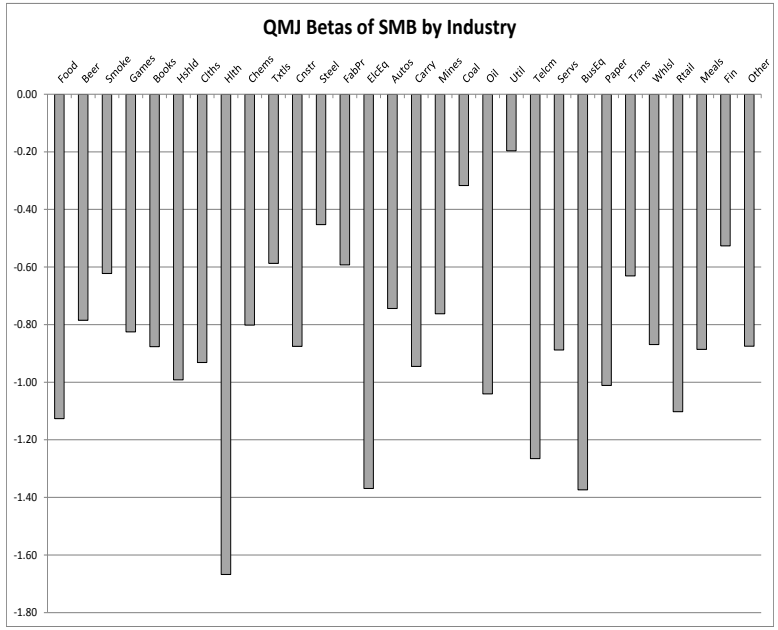
Results: Size Matters in Each Industry

Controlling for quality, the size premium is *robust to the specification*

This figure plots the improvement in SMB alphas (relative to the Fama and French factors market, market lagged a month, HML, and UMD) after controlling for QMJ within 30 industries





Results: Many Sizes Matter

Controlling for quality, the size premium is *robust to non-price based measures of size*

The table reports regression results for the P1-P10 value-weighted spread portfolios sorted using non-priced based measures of size

Panel A: Non-priced based size premia

$$P1-P10_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + h HML_t + m UMD_t + \varepsilon_t$$

	Size measure:	Book assets		Sales		Book equity		PP&E		Employees		Market cap	
		α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$
1957:07-2012:12	QMJ sample	0.0017	0.96	0.0002	0.10	0.0004	0.22	0.0008	0.00	0.0000	0.01	0.0004	0.20
1957:07-1979:12	Golden age	0.0037	1.52	0.0023	1.04	0.0028	1.06	0.0041	1.84	0.0019	1.00	0.0019	1.00
1980:01-1999:12	Embarrassment	-0.0016	-0.63	-0.0033	-1.34	-0.0048	-1.95	-0.0020	-0.83	-0.0035	-1.40	-0.0035	-1.40
2000:01-2012:12	Resurrection	0.0053	1.38	0.0027	0.75	0.0057	1.71	0.0013	0.41	0.0038	1.07	0.0038	1.07

Panel B: Non-priced based size premia, controlling for QMJ

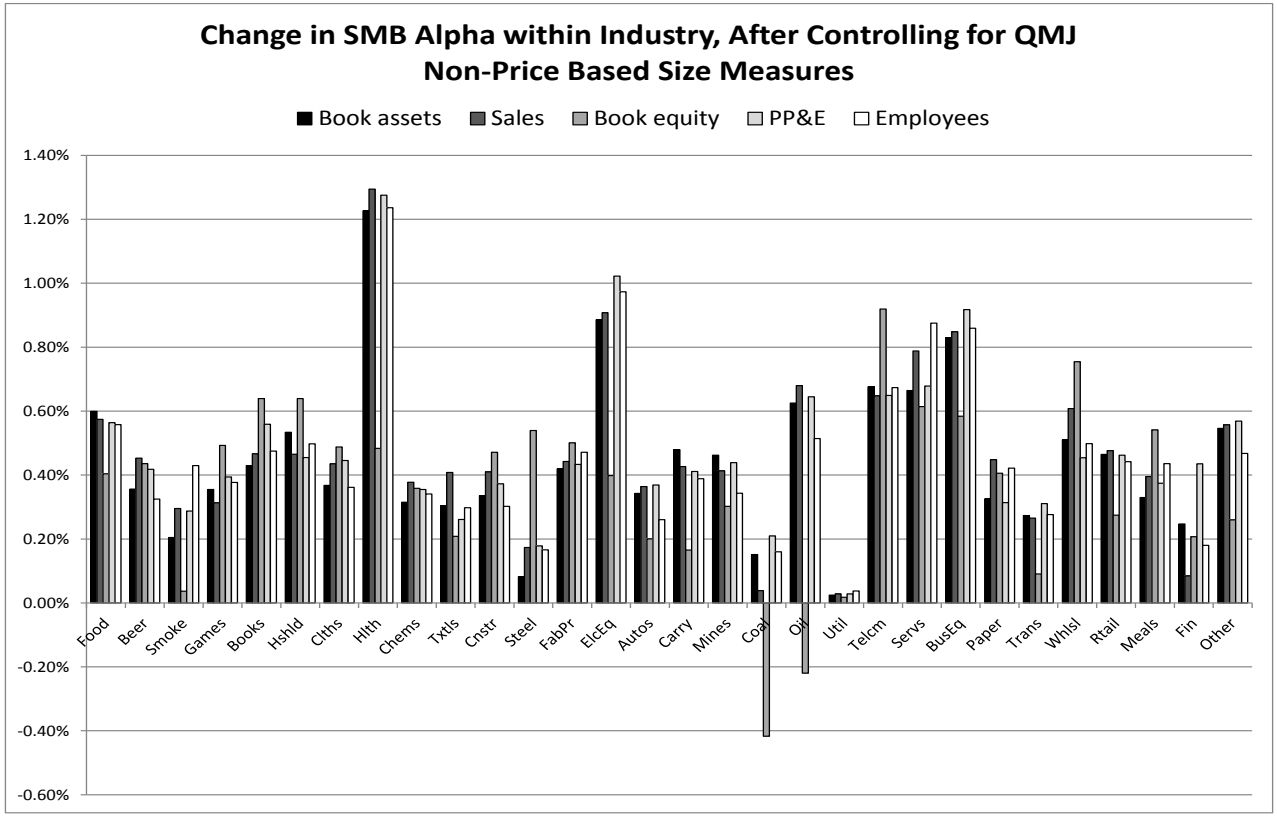
$$P1-P10_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + h HML_t + m UMD_t + q QMJ_t + \varepsilon_t$$

	Size measure:	Book assets		Sales		Book equity		PP&E		Employees		Market cap	
		α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$	α	$t(\alpha)$
1957:07-2012:12	QMJ sample	0.0083	5.98	0.0067	5.52	0.0066	4.98	0.0058	4.57	0.0068	5.78	0.0064	5.78
1957:07-1979:12	Golden age	0.0084	3.97	0.0062	3.43	0.0083	3.76	0.0083	4.27	0.0055	3.27	0.0055	3.27
1980:01-1999:12	Embarrassment	0.0094	4.15	0.0086	4.37	0.0065	3.19	0.0072	3.24	0.0087	4.41	0.0087	4.41
2000:01-2012:12	Resurrection	0.0115	3.98	0.0088	3.55	0.0112	4.66	0.0056	2.19	0.0102	4.13	0.0102	4.13

Results: Many Sizes Matter

Controlling for quality, the size premium is *robust to non-price based measures of size*

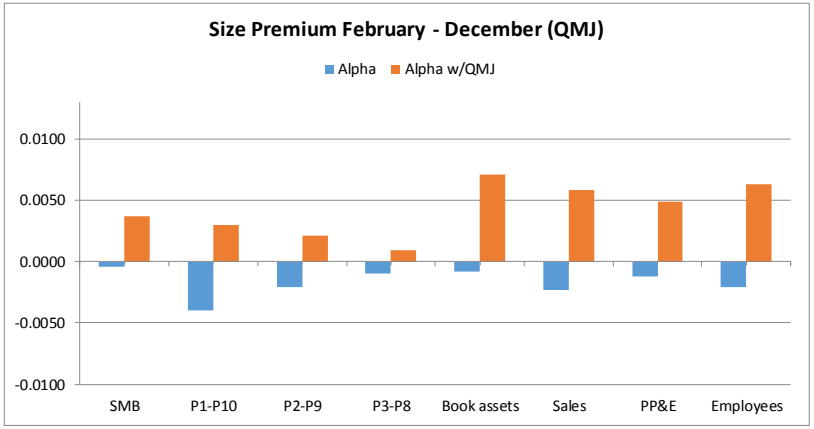
The figure plots the improvement in SMB alphas (relative to the Fama and French factors RMRF, RMRF lagged a month, HML, and UMD)

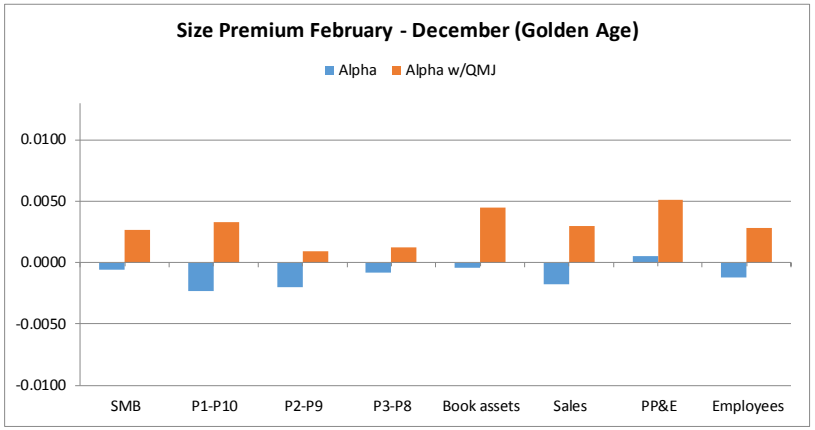


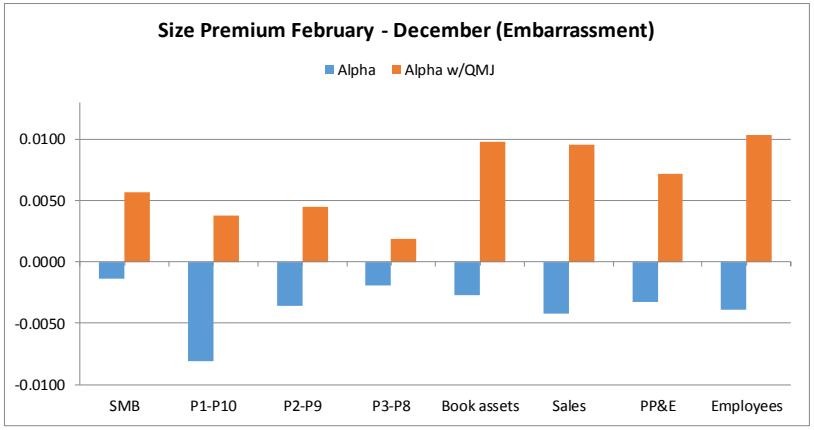
Results: Size Matters Through the Year

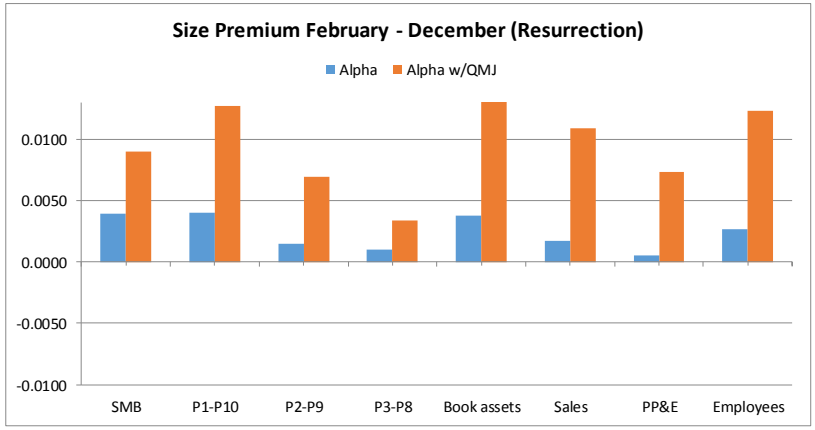
Controlling for quality, the size premium is *more consistent across seasons*

These figures plot the alphas outside of January from February to December of various size portfolios





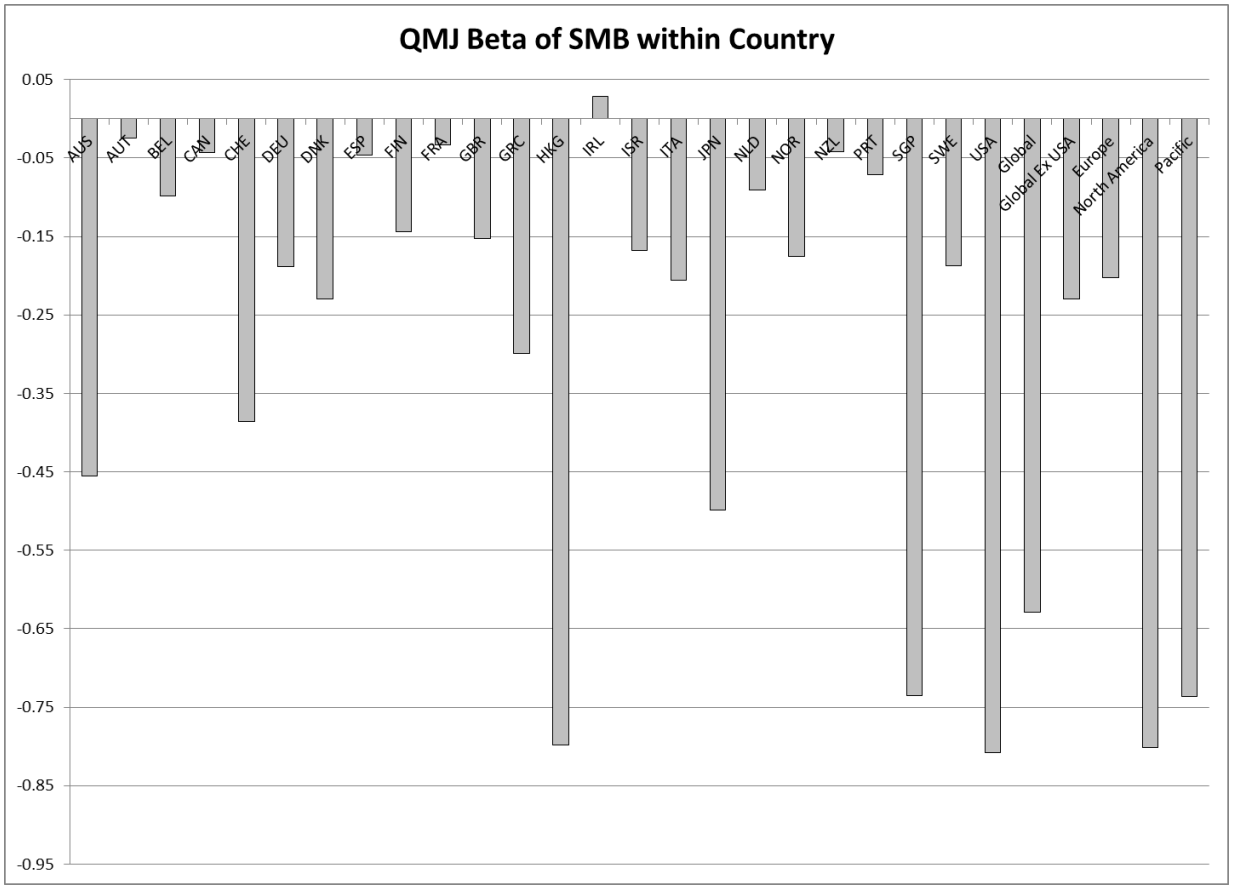




Results: Size Matters Globally

Controlling for quality, the size premium is *more consistent across markets*

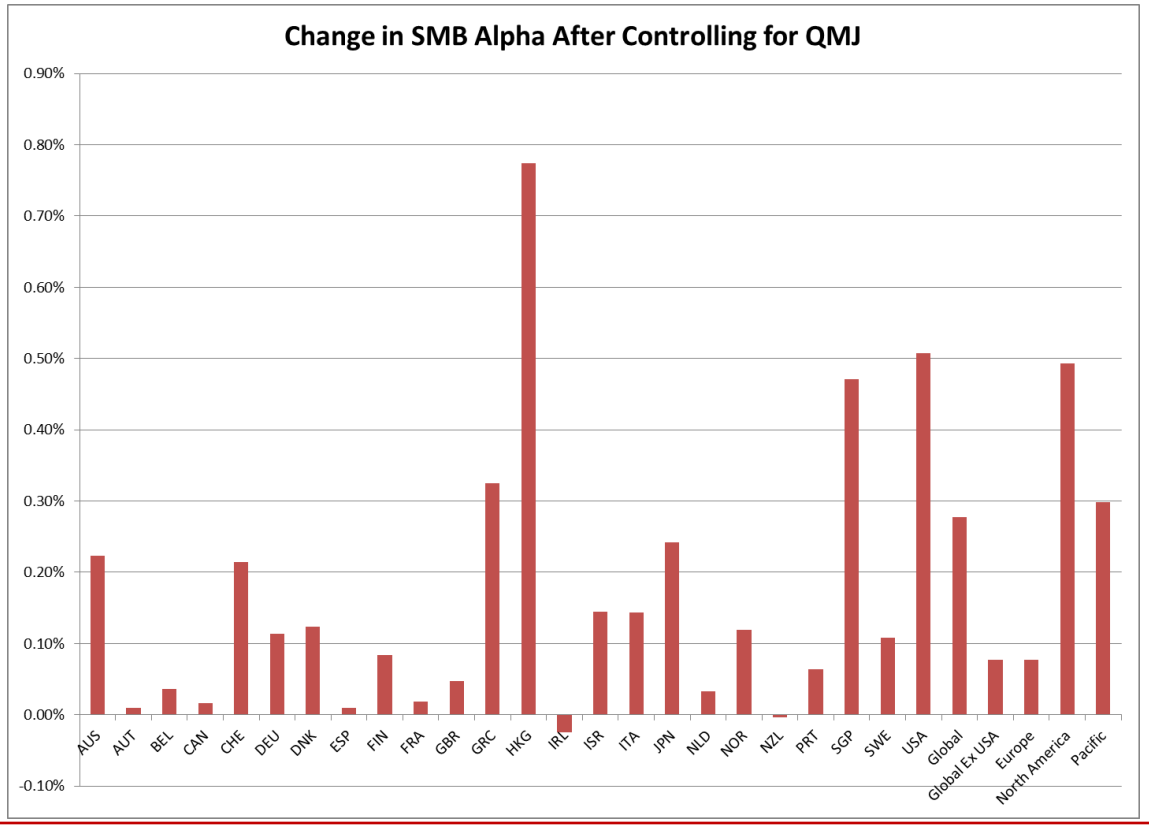
This figure plots loadings of SMB alphas on QMJ within 24 developed markets



Results: Size Matters Globally

Controlling for quality, the size premium is *more consistent across markets*

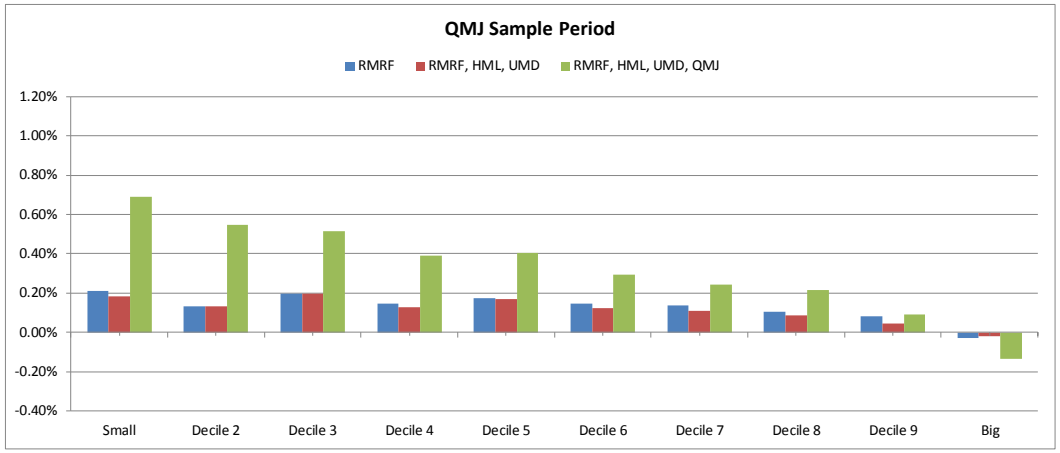
This figure plots the improvement in SMB alphas (relative to the Fama and French factors market, market lagged a month, HML, and UMD) after controlling for QMJ

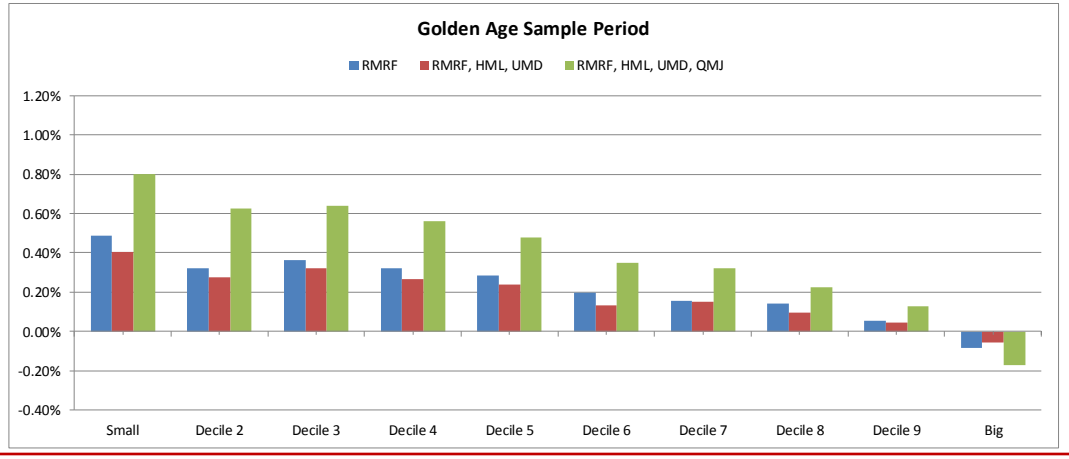


Results: Size Matters Not Just in the Extremes

Controlling for quality, the size premium is *not concentrated in "extreme" stocks*

This figure plots alphas of each size decile with respect to three factor models





Results: Size Matters Beyond Liquidity

Controlling for quality, the size premium is *not captured by an illiquidity premium*

The table reports regression results for the size premium, SMB, on the factors RMRF, its lagged value, HML, UMD, and various proxies for liquidity and liquidity risk

Panel A: Adding Liquidity

$$SMB_t = \alpha + \beta\, RMRF_t + \beta_{-1} RMRF_{t-1} + h HML_t + m UMD_t + l_1 LIQRISK_t + l_2 STREV_t + l_3 LIQ_t + q QMJ_t + \varepsilon_t$$

	α	$t(\alpha)$	l_1	$t(l_1)$	l_2	$t(l_2)$	l_3	$t(l_3)$	q	$t(q)$
QMJ sample	**0.0012**	**0.95**								
1968:01-2012:12	**0.0006**	**0.42**	-0.04	-1.22	0.11	2.72	0.14	3.89		
	0.0047	**3.90**	-0.02	-0.79	0.08	2.26	0.04	1.40	-0.71	-12.89
January	**0.0048**	**0.91**								
1968:01-2012:12	**0.0039**	**0.68**	0.06	0.68	0.11	0.55	0.16	1.38		
	0.0012	**0.22**	0.03	0.39	0.05	0.29	0.08	0.75	-0.65	-2.77
Feb.-Dec.	**-0.0003**	**-0.22**								
1968:01-2012:12	**-0.0005**	**-0.35**	-0.06	-1.69	0.05	1.23	0.12	3.42		
	0.0043	**3.48**	-0.04	-1.16	0.04	1.18	0.03	1.05	-0.69	-12.04
Golden age	**0.0035**	**1.45**								
1968:01-1979:12	**0.0008**	**0.31**	-0.06	-0.75	0.30	3.07	0.24	2.68		
	0.0044	**1.92**	0.09	1.26	0.20	2.27	0.12	1.49	-0.88	-6.47
Embarrassement	**-0.0011**	**-0.64**								
1980:01-1999:12	**-0.0002**	**-0.12**	-0.18	-3.85	0.02	0.21	-0.02	-0.42		
	0.0051	**3.05**	-0.14	-3.38	0.07	1.10	-0.01	-0.22	-0.80	-8.84
Resurrection	**0.0054**	**2.06**								
2000:01-2012:12	**0.0062**	**2.49**	0.04	0.62	0.02	0.39	0.34	4.94		
	0.0087	**3.98**	0.06	1.14	0.01	0.27	0.20	3.20	-0.74	-7.27

Conclusions

We find that controlling for a security's quality unlocks a large and significant size premium

- Quality minus Junk has a positive E[r]
- Small is junky very consistently (time, calendar, industry, geography)

When controlling for quality, the size premium is (2. – 7. from earlier):

2. Stable through time and robust out of sample
3. Not concentrated in "extreme" stocks
4. More consistent across seasons and markets
5. Robust to non-price based measures of size
6. Not captured by an illiquidity premium
7. More consistent internationally

Our results make risk-based explanations for the size effect more challenging

- High Sharpe ratio, e.g., Hansen and Jagannathan (1997)
- It is the low-volatility, high-quality stocks that drive the high expected returns (no ICAPM)
- The size effect has always presented a challenge to theory, the challenge just got bigger

To end on a sobering note, how implementable these results are after trading costs is still to be determined…