Learning about Distress

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Abstract

I develop an analytically tractable dynamic asset pricing model to study expected returns of financially distressed firms in the presence of learning and investor activism. Learning critically affects distressed stocks’ valuations and risk exposures as information about solvency is essential for firm survival in distress. Informational externalities from active investors thus can also have first-order effects on distress risk premia. The presented model can shed light on a variety of empirical regularities related to financial distress, such as distressed firms’ apparent stock market underperformance, momentum return dynamics, and negative abnormal returns after private placements of public equity involving active investors.

Keywords: Expected Returns, Financial Distress, Learning, Momentum, Active Investors, Risk Premia

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1. Introduction

“After all, you only find out who is swimming naked when the tide goes out.”

Financial distress is a central phenomenon in financial economics that has gained renewed attention after the recent financial crisis and Great Recession. A key problem associated with financial distress is investors’ challenging task to determine firm solvency: lacking precise information, investors may be unable to distinguish solvent from insolvent firms and thus be unwilling to provide finance when it is needed the most. The availability of information and investors’ learning are thus essential for firm survival in distress, creating a feedback effect on firm outcomes — investors’ learning not only reduces subjective uncertainty but also affects a firm’s survival and thus, its actual riskiness.

In this paper I aim to shed light on the dynamic asset pricing implications of this learning-based feedback effect. I develop a continuous-time model of an indebted firm that, after receiving a negative shock to earnings, attempts to raise external funds to avoid default. Yet agents face uncertainty about the nature of the shock: earnings may be permanently depressed, rendering the firm fundamentally insolvent. Investors make dynamic decisions about capital injections and rationally update their beliefs about the firm’s future prospects in the presence of business-cycle risks. Apart from small investors that learn passively from public information I also consider large, active investors that can obtain superior information and assume a pivotal role in the firm’s access to finance. Despite the presence of both aggregate risks and learning about firm-specific prospects the setting is tractable and allows a parsimonious analysis of the optimal default boundaries. The model makes a large set of novel empirical predictions that shed light on several asset pricing puzzles related to financial distress, such as distressed firms’ apparent stock market underperformance, momentum return dynamics, and the connection between active investors and expected returns.

In a setting that has similarities with standard EBIT-based capital structure models

\[1\text{See, e.g., } \text{Dichev (1998), Campbell, Hilscher, and Szilagyi (2008), Avramov, Chordia, Jostova, and Philipov (2007, 2013), and Park (2013). I discuss related empirical evidence in more detail later in the introduction.}\]

\[2\text{See, e.g., } \text{Goldstein, Ju, and Leland (2001), Hackbarth, Hennessy, and Leland (2007), and Strebulaev (2007).}\]
this paper considers a firm with a stochastic earnings process that may reach states where the firm has to raise new equity to avoid defaulting on its existing perpetual debt. Investors are, however, uncertain about the firm’s underlying quality, which is determined by a hidden firm-specific Markov state that affects the firm’s future cash flow dynamics. Whereas good firms have rising expected earnings and are fundamentally solvent, bad firms have declining expected earnings and are truly insolvent. As investors cannot observe the hidden state directly, capital provision depends critically on investors’ current beliefs and, in a dynamic world, on expectations about the future speed of learning.

Even if investors currently have pessimistic beliefs about a firm’s prospects they may be willing to provide additional funds if the speed of learning is high and precise information about true firm quality is expected to arrive quickly. While waiting for more information for a short period of time may require injecting additional funds to keep the firm afloat, it gives equity investors an important benefit: default decisions can be delayed to a point in time when more information is available, which reduces the propensity of triggering default in error.

In contrast, supporting a distressed firm is less attractive when learning is slow. In this case, investors need to inject capital for an extended period of time before they can make a more informed default decision. However, the substantial capital they provide in the meantime may be wasted on a truly insolvent firm and thus constitute an involuntary subsidy to debt holders. Thus, holding current beliefs fixed, a slower speed of learning can render equity investors unwilling to provide more funds in the first place and lead to immediate default. As a result, equity holders’ default option value is highly sensitive to the speed of learning, not just to current beliefs.\footnote{As a firm’s default probability increases, learning speed in turn becomes a more and more important determinant of the stock price.} This basic insight has important implications for risk premia of firms with high default risk: risk exposures are affected not only by earnings risk and leverage but also by the exposure of learning speed to aggregate fluctuations. Bayesian filtering based on observed

\footnote{In this context it is important to distinguish investors’ conditional uncertainty about earnings prospects from the unconditional physical uncertainty of earnings. Whereas increased physical uncertainty raises the ex ante value of equity holders’ default option, a fast reduction in conditional uncertainty, that is, a better ability to predict future earnings, also increases the ex ante option value, as it improves investors’ dynamic financing and default decisions.}
earnings performance in turn dictates that learning speed is higher when good and bad firms have a higher propensity to exhibit differential earnings performance. In particular, if truly insolvent firms exhibit large downside risks during the test of an aggregate downturn, learning speed increases in these times and becomes counter-cyclical.\textsuperscript{4} Related to Schumpeter’s (1934) hypothesis that recessions can have a “cleansing effect,” truly insolvent firms then default more quickly in downturns, as their adverse performance separates them from good firms, which show more resilience. Expectations about a faster separation support distressed firms’ stock prices in these times, as the highlighted learning-based feedback effect operates.

As a result, learning can have substantive implications for distressed firms’ asset prices: exposures to business-cycle risk and risk premia can become negative, even when firms’ earnings processes are strongly pro-cyclical. If learning speed is not counter-cyclical but instead constant or moderately pro-cyclical, equity risk exposures are still reduced in comparison to the full information case. With increasing default risk, stock prices generically load more and more heavily on the cyclical movements of learning speed rather than just earnings risk. Learning may thus be a factor contributing toward the empirical finding that high default risk is associated with low future stock returns (Dichev, 1998; Campbell, Hilscher, and Szilagyi, 2008), a phenomenon referred to as the “distress anomaly.”\textsuperscript{5}

Going beyond an explanation for low risk premia, the theory can further provide a coherent story for the observed negative unconditional alphas on portfolios consisting of the most financially distressed stocks. When the quintile of stocks with the highest default risk has higher average default risk in downturns, learning can reduce the quintile portfolio’s exposure to business-cycle risks more in these times (the learning-based feedback effect increases with default risk). When market risk premia rise in downturns, these cyclical movements

\textsuperscript{4}See, e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014) for empirical evidence that cross-sectional dispersion in productivity is strongly countercyclical.

\textsuperscript{5}Gao, Parsons, and Shen (2013) provide evidence on the robustness of the distress anomaly based on a broad international data set of 39 countries. For US data Vassalou and Xing (2004) document that the size effect in expected returns exists only in segments of the market with high default risk, and that this is also largely the case for the book-to-market effect. Vassalou and Xing (2004) further find some evidence that distressed stocks with a low distance to default have higher returns, but this evidence comes entirely from small value stocks. Da and Gao (2010) further provide evidence that distressed firms’ stock returns in Vassalou and Xing (2004) are biased upwards by 1-month reversal and bid-ask bounce. Griffin and Lemmon (2002) document that among firms with the highest distress risk the difference in returns between high and low book-to-market securities is more than twice as large as that in other firms. Further, the authors find that firms with high distress risk exhibit the largest return reversals around earnings announcements.
imply a negative correlation between market risk premia and conditional betas of the quintile portfolio. As a result, unconditional CAPM regressions yield upward biased beta estimates for distressed firms, and negatively biased alpha estimates. The notion that conditional betas can contribute toward resolving the distress anomaly is further supported by empirical evidence in O’Doherty (2012) who finds that the correlation between market risk premia and conditional betas of distressed stocks can account for about two thirds of the negative unconditional alpha.

The same learning channel that dampens risk premia also implies that a momentum strategy that goes long recent winners and shorts recent losers among distressed firms can generate large expected returns. Positive correlations between past price changes and future expected returns obtain because firms that experienced negative contemporaneous returns have higher default risk, and thus, lower risk premia going forward, provided that learning is sufficiently fast in downturns. Consistent with this mechanism, Avramov, Chordia, Jostova, and Philipov (2007) find that both the extreme loser and winner portfolios consist of stocks with the lowest and the next-lowest credit rating, respectively. The authors further find that momentum profits are restricted to high credit risk firms and are nonexistent for firms of high credit quality. Avramov, Chordia, Jostova, and Philipov (2013) further document the importance of financial distress for the profitability of a variety of anomaly-based trading strategies, such as, earnings momentum, dispersion, idiosyncratic volatility, and capital investments. In addition, consistent with the theory’s link between risk premia and exposures to business cycle risks, Bansal, Dittmar, and Lundblad (2005) find that exposures to persistent changes in growth can explain a large fraction of the variation in expected returns across momentum portfolios.

While in my baseline model small investors learn passively from realized earnings performance, I also analyze an extension of the model that features a large active investor.

6See, e.g., Grant (1977) and Jagannathan and Wang (1996). Bandarchuk and Hilscher (2013) find that existing strategies that yield elevated momentum profits benefit from trading in stocks with more extreme past returns. High default risk and corresponding leverage constitute a natural economic reason for extreme stock returns — in the cross-section, extreme past returns are thus likely generated by firms with high default risk. Regressions reported in Bandarchuk and Hilscher (2013) further indicate that credit ratings have a significant effect on momentum profits even after controlling for idiosyncratic volatility and a measure of extreme past returns. Moreover, Avramov, Chordia, Jostova, and Philipov (2013) find that the effect of idiosyncratic volatility on expected returns is explained by measures of credit risk.
In practice, active investors, such as activist hedge funds or private equity funds, typically acquire substantial stakes in distressed firms’ private placements of public equity. In the model, these specialized investors can obtain additional information on a firm’s underlying state. The firm issues equity to such an investor at a discounted price when it requires new funds and small investors are uncertain about the firm’s solvency. Despite the discounted offer, involving the active investor is in the interest of existing shareholders who can free-ride by following the active investor’s lead going forward. As a result, the active investor assumes a pivotal role in the firm’s access to finance, which is shown to create an externality affecting other investors’ risk premia. In particular, if learning based on a firm’s public information is slow in aggregate downturns, the involvement of an active investor can help off-load systematic risk from equity to debt holders.

This informational externality in turn can generate the empirical phenomena that (1) financially distressed firms that issue discounted equity to new investors via private offerings of public equity have particularly low expected returns, and (2) the buy-and-hold returns after private offerings are negatively correlated with the discounts that were initially provided to the large investors that participated in these offerings. In the model, the firm is willing to offer larger discounts to active investors with greater skill. A larger discount thus forecasts more effective involvement of the active investor going forward, and thus a stronger effect on equity risk exposures. In other words, the involvement of an active investor is a state variable that affects distressed firms’ conditional expected returns. The highlighted informational

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8 See, e.g., Park (2013) for related empirical evidence.
9 Providing one investor with a large equity stake can help resolve free-rider problems present among small investors (see, e.g., Grossman and Hart (1980) and Shleifer and Vishny (1986)).
10 Consistent with the model, Brophy, Ouimet, and Sialm (2009) document that hedge funds tend to finance companies that have poor fundamentals and pronounced informational frictions, and require substantial discounts.
11 Park (2013) documents these empirical facts. Several other empirical papers analyze the relation between private placements and equity returns, and find related results. Krishnamurthy, Spindt, Subramaniam, and Woidtke (2005) find that shareholders not participating in private placements experience post-issue negative long-term abnormal returns. Hertzel, Lemmon, Linck, and Rees (2002) also document that public firms that place equity privately experience negative post-announcement stock-price performance. Hertzel and Smith (1993) provide empirical evidence that discounts provided in private placements reflect information costs borne by private investors, which is consistent with the mechanism in my model. Brophy, Ouimet, and Sialm (2009) also find that public companies that raise equity privately from large specialized investors such as hedge funds significantly underperform companies that obtain financing from other investors in the future.
externality can also shed some light on the intriguing empirical finding by Gao, Parsons, and Shen (2013) that the distress anomaly is especially strong among small stocks in North America and Europe, markets where distressed equity hedge funds are particularly active.\footnote{Since these are cross-country correlations several alternative explanations exist. For example, Gao, Parsons, and Shen (2013) interpret this correlation as consistent with investor overconfidence, a behavioral bias that could be more prevalent in the cultures of North America and Europe, which stress individualism.}

Related literature. This paper is generally related to a growing literature on learning in financial markets\footnote{See Pastor and Veronesi (2009) for a survey of this literature.} and the relationship between financing decisions and asset pricing\footnote{See, e.g., Hackethal, Miao, and Morellec (2006), Livdan, Sapriza, and Zhang (2009), and Gomes and Schmid (2010).}. Business-cycle risks of the type considered in this paper play a prominent role in dynamic capital structure models, such as Chen (2010) and Bhamra, Kuehn, and Streubel (2010). A paper related to mine that integrates both credit risk and learning in a dynamic economy is David (2008). Whereas in David (2008) agents learn about the hidden drifts of aggregate processes (such as real earnings and inflation), agents in my model learn about firm-specific states that determine future earnings dynamics. Critical for the results I obtain is the fact that I can solve for equity holders’ true endogenous default boundaries. In contrast, David (2008) makes exogenous assumptions on default decisions in order to solve for prices. In particular, by assuming the same default rules as in Merton (1974), David’s (2008) approach does not capture equity holders’ true option value.

My paper’s main objective is to provide a tractable model that illustrates the effects of learning and investor activism on financially distressed firms’ risk premia dynamics. I do not argue that the highlighted learning channel is the only plausible mechanism that can help explain the discussed empirical regularities, such as the distress anomaly. For example, investor irrationality or additional institutional frictions can certainly be contributing factors for these phenomena. Yet agents’ learning about firm solvency is an economic force that is generically relevant in financial distress and appears to warrant a rigorous formal analysis.

Regarding low returns for distressed firms, complementary theoretical explanations have been put forward that have other virtues and limitations. In contrast to my paper, these other theories do not analyze the effects of learning and investor activism on distress risk
premia and generally cannot produce negative expected returns. My paper further also provides theoretical reasons why distress might not only be associated with low returns but also negative alphas. George and Hwang (2010) argue that firms with high financial distress costs choose low leverage to avoid distress but retain exposure to the systematic risk of bearing such costs in low states, implying that they have higher expected returns than highly levered firms. Garlappi and Yan (2011) provide a model that shows how potential shareholder recovery upon resolution of financial distress (violation of the absolute priority rule) may effectively imply de-levering upon default, which can lead to lower expected returns for firms with high default probabilities. Similarly, Garlappi, Shu, and Yan (2008) argue that bargaining between equity holders and debt holders in default may account for low expected equity returns for firms with high default risk, given that shareholders can extract high benefits from renegotiation. McQuade (2013) provides a model where aggregate volatility risk commands a separate, negative market price and financially distressed stocks, due to their optionality, have lower volatility risk premia than financially healthy firms. Ozdagli (2013) emphasizes that risks under the physical and risk-neutral measure do not need to coincide, implying that the equity of firms with high physical default risk does not need to command high risk premia. Consistent with this idea, I provide a learning-based economic rationale for why financially distressed firms should indeed have low exposures to priced risk despite their high physical default risk. Finally, unrelated to financial distress, Johnson (2002) provides an alternative rational explanation of momentum effects based on stochastic expected growth rates.

In the following, I first present the baseline model that features passive learning from firms’ earnings performance. In section 3, I extend this setup to incorporate an active investor. Section 4 concludes.

Gao, Parsons, and Shen (2013) shed some light on the role of this channel by analyzing international data. Based on a broad data set of 39 countries the authors find that the distress anomaly is not related to a country’s creditor protection environment. Hackbarth, Haselmann, and Schoenherr (2013) provide evidence that weaker creditor rights can be associated with lower equity risk premia for distressed firms. However, the authors acknowledge that this finding does not resolve the distress anomaly, since it cannot explain why the CAPM and the three factor model yield negative alphas for financially distressed firms.
2. The Baseline Model

I consider a continuous time economy with exogenously specified aggregate dynamics and analyze asset pricing implications for marginal firms in the presence of learning about firm solvency.

2.1. Aggregate States and Stochastic Discount Factor

The aggregate state of the economy, denoted by $Z$, follows a two-state continuous time Markov chain with $Z \in \Omega^Z = \{G, B\}$, where $G$ refers to a high-growth state (boom) and $B$ refers to a low-growth state (recession). All agents can observe this aggregate state. I denote the transition rate between state $Z$ and $Z'$ by $\lambda (Z, Z')$, where $Z' \neq Z$. The stochastic discount factor (SDF) follows a Markov-modulated jump process,

$$
\frac{d\xi_t}{\xi_{t-}} = -r_f (Z_{t-}) dt + (e^{\phi(Z_{t-}, Z')} - 1) (dN_t (Z_{t-}, Z') - \lambda(Z_{t-}, Z') dt),
$$

where $r_f (Z)$ is the risk-free rate, $N_t (Z, Z')$ is a counting process that keeps track of the number of Markov chain jumps from state $Z$ to state $Z'$, and $\phi (Z, Z')$ represents the jump risk premia.\[17\]

The postulated SDF dynamics obtain for example in a Lucas-Breeden economy where the drift of aggregate consumption is governed by a continuous time Markov chain and agents have Duffie and Epstein (1992a,b) preferences. While sharing Markov chain dynamics with the SDFs in several recent credit risk models, this SDF deliberately abstracts from Brownian shocks, a key technical choice that ensures analytical tractability in my setting, which features learning. By considering a larger number of Markov states the model can in principle still capture rich dynamics and provide good approximations to more complex processes. For the purposes of this paper, however, I will focus on the case of two aggregate

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16In the following, all processes will be right continuous with left limits. Given a process $y_t$, the notation $y_{t-}$ will denote $\lim_{s \uparrow t} y_s$, whereas $y_t$ denotes $\lim_{s \downarrow t} y_s$.

17Thus, the jump intensity between states $Z$ and $Z'$ under the risk neutral measure is $e^{\phi(Z, Z')} \lambda(Z, Z')$.

states, which suffices to highlight the central insights and increases the transparency of the results.

2.2. The Firm

I follow standard EBIT-based capital structure models\textsuperscript{19} in that the earnings of a firm are split between a coupon promised to debt holders and a dividend paid to equity holders. Yet, since my paper’s focus is not on capital structure but the asset pricing implications of learning conditional on reaching distress, I take it as given that the firm issued debt in the past. For this reason, I also abstract from taxes and other frictions that might generate incentives to issue additional debt. Further, to set the channel apart from existing papers on the effects of violations of absolute priority, I assume that equity holders cannot renegotiate contracts with existing debt holders\textsuperscript{20}. An alternative interpretation of this setup is that required coupon payments represent operational cost that the firm has to pay to avoid defaulting on suppliers, customers, or employees. As the leading narrative, I will, however, refer to the receivers of coupon payments as debt holders. Throughout, management acts in the interest of shareholders. Further, in the baseline model, all agents have symmetric but potentially incomplete information.

At time $t = 0$, the firm has obligations that promise debt holders a constant perpetual coupon rate $c$. The firm’s before-interest earnings rates $X(z) \geq 0$ are governed by firm-specific Markov states $z \in \Omega^z$ the dynamics of which depend on the aggregate state $Z \in \Omega^Z$. Specifically, the transition rate from earnings state $z$ to state $z'$ is given by $\lambda_{z,z'}(Z)$. By conditioning the transition rates on the aggregate state $Z$ the setup can capture rich dependencies between the firm’s earnings dynamics and aggregate conditions. Let $\Omega = \Omega^Z \times \Omega^z$ denote the complete set of states for the vector $(z, Z)$.

To simplify the exposition, I will consider a particular stylized Markov state setup where I divide the set of all states $\Omega$ into four disjoint subsets, to which I refer as initial solvent states $\Omega_s$, opaque distressed states $\Omega_d$, and good and bad revealing sets of states $\Omega_g$ and $\Omega_b$, respectively. I discuss these subsets of states in more detail below. Figure \[ further

\textsuperscript{20}See, e.g., Garlappi, Shu, and Yan (2008), Garlappi and Yan (2011).
illustrates the setup.

**Initial solvent states.** At date $t = 0$, the firm is assumed to be in solvent states $\Omega_s$ in which it generates sufficiently high earnings to cover its interest expenses, that is, $X(z) > c$. In these states the firm pays positive dividends to shareholders and is thus unambiguously solvent.

**Opaque distressed states.** The firm can, however, receive negative shocks that push its earnings below the coupon rate $c$. In particular, the firm can transition into opaque distressed states $\Omega_d = \{dg, db\} \times \{G, B\}$, where earnings are given by $X_d < c$. In these states, the firm has to raise external funds in order to avoid default, but agents have incomplete information about the firm’s prospects, as they cannot directly observe whether a benign (transitory) or disastrous (permanent) shock hit.\(^{21}\)

Earnings do not directly reveal the underlying firm-state $z \in \{dg, db\}$ since $X(z) = X_d \forall (z, Z) \in \Omega_d$. After a benign shock, the firm is in a good firm-state, $z = dg$, where earnings are likely to recover quickly. After a disastrous shock, on the other hand, the firm is in a bad firm-state, $z = db$, where earnings are permanently depressed and recovery is impossible. To simplify the exposition I will refer to firms in states $dg$ and $db$ as “$dg$-firms” and “$db$-firms” respectively. These distressed states are defined such that, the firm is solvent in state $dg$ but insolvent in state $db$, that is, if equity investors could observe the true firm-state $z$, they would optimally choose to default immediately in firm-state $db$ and provide new funds in firm-state $dg$.\(^{22}\) Depending on investors’ beliefs about the underlying state, default thus can occur in the set of states $\Omega_d$. All investors are Bayesian learners and form rational beliefs about the underlying state based on available information, such as the passage time in the distressed state and other potential signals.

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\(^{21}\)Here, a disastrous shock affects a single firm, although the arrival intensities of these shocks are allowed to depend on the aggregate state $Z$. See, e.g., [Rietz (1988), Barro (2009), Wachter (2013)] for papers on the asset pricing implications of disasters affecting aggregate consumption dynamics.

\(^{22}\)Investors’ decision problem would be degenerate if the equity value in firm-state $dg$ was also non-positive, since in that case the distressed firm would be always insolvent.
Opaque distressed states

Hidden firm-state $z \in \{dg, db\}$

Revealing states

$\Omega_g$ and $\Omega_b$ are absorbing in the sense that once the firm reaches one of these sets, it will stay in this set with probability one. This assumption by itself does not put any restrictions on earnings dynamics as it does not restrict how the sets $\Omega_g$ and $\Omega_b$ are constructed. Yet, to ensure consistency with the above-mentioned defining properties of the distressed states $dg$ and $db$, I restrict earnings in the sets $\Omega_g$ and $\Omega_b$ as follows: while earnings in the good set, $\Omega_g$, are above the required coupon payments, implying solvency, earnings in the bad set, $\Omega_b$, are below $X_d$, implying insolvency. Thus, when the firm’s earnings recover or deteriorate further relative to $X_d$, agents can infer whether the firm was previously hit by a benign or a disastrous shock.\(^{23}\)

\[^{23}\]These clear-cut differences in earnings levels across the sets $\Omega_g$ and $\Omega_b$ are not essential for the results but yield tractability. For example, one could add noise by assuming that the set $\Omega_b$ also contains states where earnings are above $X_d$. Yet, to be consistent with the definition of $db$ as a firm-state where the firm is
**Discussion:** Markov state dynamics. To illustrate results in the most parsimonious way I consider a Markov state setup where current states are observable until the firm is hit by a negative shock that causes earnings to drop below the required coupon payments. While, more generally, the underlying firm-state could already be unobservable before earnings drop below the coupon, the firm would still be unambiguously solvent at that time, as dividends are locally strictly positive. In other words, uncertainty about the hidden firm-state could not raise doubts about solvency unless earnings also fall below the required coupon $c$. Once this happens, equity holders would again form beliefs about the hidden state and evaluate solvency, just as captured in the considered Markov state setup.

2.3. Analysis

Solving for equity values backwards, I first analyze the revealing sets of states $\Omega_b$ and $\Omega_g$, then opaque distressed states $\Omega_d$, and finally the initial solvent states $\Omega_s$. The decision problem at the heart of the analysis arises in opaque distressed states, where agents are generally uncertain about the firm’s fundamental solvency.

2.3.1. Revealing States

In the long-run, the firm either has defaulted while being in a opaque distressed state $\Omega_d$, or has reached a state in the revealing sets $\Omega_b$ or $\Omega_g$. When the firm reaches a state in the bad revealing set $\Omega_b$, equity holders optimally choose to default immediately, since the firm is known to be insolvent. Equity values in the good revealing states $\Omega_g$ are characterized in the following proposition.

**PROPOSITION 1** (Equity value in good revealing states). In good revealing states $(z,Z) \in \Omega_g$ the firm’s equity value is given by

\[
V(z_t, Z_t) = E_t \left[ \int_0^\infty \frac{\xi(Z_\tau)}{\xi(Z_t)} (X(z_\tau) - c) \, d\tau \right].
\]

(2)

Insolvent, the probability that earnings recover and rise above the coupon rate $c$ would have to be sufficiently small.
The corresponding Hamilton-Jacobi-Bellman equation implies that the function \( V(z, Z) \) solves the following system of linear equations for all \((z, Z) \in \Omega_g:\)

\[
0 = X(z) - c - r_f(Z) V(z, Z) + \sum_{z' \in \Omega^x} \lambda_{zz'}(Z) (V(z', Z) - V(z, Z)) \\
+ \lambda(Z, Z') e^{\phi(Z, Z')} (V(z, Z') - V(z, Z)).
\]  

(3)

The firm’s state-contingent equity risk premium \( r_p(z, Z) \) is given by:

\[
r_p(z, Z) = -\lambda(Z, Z') \left( e^{\phi(Z, Z')} - 1 \right) \left( \frac{V(z, Z')}{V(z, Z)} - 1 \right).
\]  

(4)

Proof. The result follows immediately from the Hamilton-Jacobi-Bellman equation. Since, by assumption, \( X(z) > c \) for all \((z, Z) \in \Omega_g\), the firm does not default in the set \( \Omega_g \).  

The formula for a firm’s equity risk premium provided in equation (4) reflects the intuitive notion that if stock prices \( V \) are positively exposed to business cycle risks, that is, changes in the macro state \( Z \), stocks command a positive risk premium.

2.3.2. Opaque Distressed States

In opaque distressed states \( \Omega_d \) investors are generally uncertain about the underlying firm-state \( z \in \{db, dg\} \). Let \( \pi_t \) denote the probability under agents’ filtration \( F_t \) that the firm is fundamentally solvent, that is, \( \pi_t \equiv \Pr[z = dg|F_t] \). The following Lemma characterizes the evolution of posterior beliefs.

**Lemma 1** (Posterior beliefs). *If at time \( \tilde{t} \) the firm jumps from a solvent firm-state \( z = s \) to a state with distressed earnings \( X_d \), investors assign the following probability to the event that the firm is fundamentally solvent \((z = dg)\):*

\[
\pi_{\tilde{t}} = \frac{\lambda_{s,dg}(Z_{\tilde{t}-})}{\lambda_{s,dg}(Z_{\tilde{t}-}) + \lambda_{s,db}(Z_{\tilde{t}-})}.
\]  

(5)
For $t > \tilde{t}$, posterior beliefs $\pi_t$ evolve as follows:

$$d\pi_t = \theta(Z) (1 - \pi_t) \pi_t dt - \pi_t dN_t^{db} + (1 - \pi_t) dN_t^{dg},$$

(6)

where $\theta(Z)$ is defined as:

$$\theta(Z) = \sum_{(z',Z) \in \Omega_b} \lambda_{db,z'}(Z) - \sum_{(z',Z) \in \Omega_g} \lambda_{dg,z'}(Z),$$

(7)

and where $N_t^z$ for $z \in \{dg, db\}$ is a counting process that switches from 0 to 1 when the firm jumps from an opaque distressed state $z \in \{dg, db\}$ to the corresponding revealing set of states ($\Omega_g$ or $\Omega_b$).\(^{24}\)

**Proof.** The results follow from Bayes’ law and the martingale property of $\pi$. \(\blacksquare\)

$\theta(Z)$ in equation (4) captures the relative propensity with which good and bad firms show improving and deteriorating earnings performance, respectively. Correspondingly, $\theta(Z)$ determines whether beliefs are revised upwards or downwards as a function of passage time without changes in earnings. $\theta(Z)$ generally depends on the current state of the aggregate economy. For example, $\theta(G)$ takes a negative value if during economic booms ($Z = G$) good firms’ earnings are more likely to recover than bad firms’ earnings are to deteriorate further. Passage time without earnings changes is then regarded as negative news during booms and thus leads to downward revisions in posterior beliefs $\pi_t$. On the other hand, $\theta(B)$ takes a positive value if, during economic downturns ($Z = B$), bad firms’ earnings are more likely to fall further than good firms’ earnings are to recover. Correspondingly, constant earnings are viewed as a sign of resilience in recessions, leading to improvements in posterior beliefs $\pi_t$.\(^{25}\)

Based on the belief dynamics described in Lemma \[\|\] I can now characterize the firm’s equity value in opaque distressed states. I introduce the superscript $R$ to indicate that this equity value will also constitute a reservation value in the extended model that features an

\(^{24}\) After a jump to the good revealing set $\Omega_g$ the probability that the firm was previously hit by a benign shock jumps to one (that is, $\pi = 1$). In contrast, after a jump to the bad revealing set $\Omega_b$ this probability jumps to zero (that is, $\pi = 0$).

\(^{25}\) In the special case where $\theta(Z) = 0$ passage time alone does not alter beliefs $\pi_t$. 

14
active investor (introduced in Section 3).

**PROPOSITION 2** (Equity values in opaque distressed states). The firm’s equity value in opaque distressed states is given by

\[
V^R(\pi_t, Z_t) = \max_{\{r^*\}} E_t \left[ \int_t^{r^*} \frac{\xi(Z_r)}{\xi(Z_t)} (X(z_r) - c) \, dr \right] = v^R(\pi_t, Z_t)^+.
\] (8)

The corresponding Hamilton-Jacobi-Bellman equation yields the following set of ODEs that the function \( v^R(\pi, Z) \) solves:

\[
0 = X_d - c - r_f(Z) v^R(\pi, Z) + v^R_{\pi}(\pi, Z) \theta(Z) (1 - \pi) \pi \\
+ \sum_{z' \neq db} \pi \lambda_{dg,z'}(Z) (V(z', Z) - v^R(\pi, Z)) \\
- \sum_{z' \neq db} (1 - \pi) \lambda_{db,z'}(Z) v^R(\pi, Z) \\
+ \lambda(Z, Z') c_{\theta}(Z, Z') \left( v^R(\pi, Z') 1_{\{\pi \geq \pi^R(Z')\}} - v^R(\pi, Z) \right).
\] (9)

Equity holders optimally default in aggregate state \( Z \) when posterior beliefs \( \pi \) fall below an endogenous cutoff value \( \pi^R(Z) \). In states \( Z \) where \( \theta(Z) < 0 \) the following conditions apply: \( 0 = v^R_{\pi}(\pi^R(Z), Z) \) and \( 0 = v^R(\pi^R(Z), Z) \). In states \( Z \) where \( \theta(Z) > 0 \), the following conditions apply: \( v^R(\pi^R(Z), Z) = 0 \) and \( \lim_{\pi \uparrow 1} v^R(\pi, Z) = V^R(dg, Z) \), where \( V^R(dg, Z) \) is characterized in Appendix A.2. If \( \theta(Z) = 0 \) then the ODE for state \( Z \) simplifies to a nonlinear equation.

**Proof.** See Appendix A.2. ■

Equity holders’ optimal default strategy can be summarized by the state-contingent belief thresholds \( \pi^R(Z) \). Investors provide additional equity as long as beliefs are above these thresholds and let the firm default otherwise.

### 2.3.3. Initial Solvent States

Finally, equity values in initial solvent states \( \Omega_s \) can be characterized analogously to those in states \( \Omega_g \) and \( \Omega_d \) (see Propositions 1 and 2). To economize on space I relegate this part
of the analysis to Appendix A.1.

2.4. A Calibration of the Baseline Model

In this section I parameterize the model to illustrate the asset pricing implications of learning when firms are in distressed states. I discuss a baseline calibration as well as comparative statics that illustrate the impact of alternative parameter choices. Parameter values of the baseline calibration are listed in Table 1.

2.4.1. Choosing Parameters

**Stochastic discount factor.** I use the values from Chen, Xu, and Yang (2012) and Chen, Cui, He, and Milbradt (2014) to calibrate the parameters of the aggregate economy. According to this calibration, the risk-free rate is set to be constant at 5%. Transition rates between aggregate states $G$ and $B$ are set such that expansions and recessions last, on average, 10 years and 2 years, respectively. Finally, the jump risk premium is chosen so that risk premia are consistent with a Markov-modulated long-run risk model that is calibrated to match the equity premium.

**Opaque distressed states $\Omega_d$.** I normalize coupon payments $c$ to one and calibrate the shortfall in earnings in opaque distressed states $\Omega_d$ using statistics provided in Campbell, Hilscher, and Szilagyi (2008). To capture the notion that firms are distressed in states $\Omega_d$, I calibrate the parameter $X_d$ to match a ratio of net income to market value of assets of approximately $-2.5\%$, which is representative for firms that have high default risk but do not file for bankruptcy with certainty. This value is half a standard deviation above the mean value for firms in Campbell, Hilscher, and Szilagyi’s (2008) “Bankruptcy Group” and

---

26 As discussed in Section 2.1, the SDF in my model deviates from the one in Chen, Xu, and Yang (2012) and Chen, Cui, He, and Milbradt (2014) in that it does not feature Brownian shocks. The parameters corresponding to Brownian shocks are thus also not part of my calibration.

27 For a consumption process with a growth drift of $+0.05$ in booms and $-0.01$ in recessions and a leverage parameter for dividends of 3.5 (as in Bansal and Yaron, 2004) this SDF generates an average equity risk premium of 6%.
about one standard deviation below the average value across all firms. Using this ratio, I back out an earnings shortfall of 21% in opaque distressed states, which corresponds to choosing $X_d = 0.79$.

Further, consistent with the description of opaque distressed states in Section 2.2, earnings evolve according to a Markov-modulated jump process that depends on the underlying hidden firm state:

$$\frac{dX_t}{X_t} = (e^{\delta_z} - 1) dN^z_t, \text{ for } z \in \{db, dg\}. \quad (10)$$

A log-change in earnings of size $\delta_z$ occurs when the counting process $N^z_t$ jumps and the firm moves from the hidden state $z \in \{db, dg\}$ to a revealing set of states, that is, $\Omega_b$ or $\Omega_g$. These earnings changes occur with Poisson intensities $\lambda_{db,b}(Z)$ and $\lambda_{dg,g}(Z)$, respectively. In the baseline calibration I choose symmetric values for $\delta_z$, that is, $\delta_{dg} = -\delta_{db} = \delta$. Below, I further discuss the extent to which asymmetric values for $\delta_z$ would affect the results.

**Revealing sets of states $\Omega_g$ and $\Omega_b$.** I calibrate $\delta$ and the price-dividend ratios after recovery from distress (after a jump to $\Omega_g$) so that the average price-dividend ratio and equity risk premium match those of the market portfolio, as estimated by Bansal and Yaron (2004, Table IV). The mean price-dividend ratio is 26.56 and the average equity risk premium is 6.33%. After recovery from distress, a firm’s key asset pricing moments are thus representative of the average firm in the market. Given the paper’s focus on distress it is convenient to calibrate price-dividend ratios in post-recovery states $\Omega_g$ directly, as they encode all information about dynamics in states $\Omega_g$ relevant for equity valuations in opaque distressed

\footnote{Campbell, Hilscher, and Szilagyi (2008) find that the average ratio of net income over market value of total assets is $-4\%$ for firms in the “Bankruptcy Group,” which is defined as the sample of firms in months immediately preceding a bankruptcy filing. The standard deviation of this ratio in the Bankruptcy Group is $3\%$, implying that I calibrate to a ratio of $-2.5\%$. This number ($-2.5\%$) is also approximately one standard deviation below the average ratio of net income over market value of assets in the total sample.}

\footnote{Using a recovery rate of 41.3\% and approximating the market value of assets as the recovery value of debt based on a promised risk-free perpetuity value of $\frac{c_{r\tau}}{r_{p}} = \frac{1}{m_{0.05}} = 20$, one obtains the relation $X_{d-c} = X_{d} - c_{r\tau} = -0.025$, or equivalently, $X_d = 1 - 0.025 \cdot 20 = 0.413 = 0.7935$. The recovery rate of 41.3\% is the long-term mean recovery rate based on data provided in Moody’s “Corporate Default and Recovery Rates, 1920-2008,” which is also documented in Chen (2010).}
Table 1

Baseline Parameters. The table lists parameter values of the baseline calibration. The transition density between aggregate states $\lambda(Z, Z')$, the jump risk premium $\exp(\phi)$, and the risk-free rate $r_f$, are taken from Chen, Xu, and Yang (2012) and Chen, Cui, He, and Milbradt (2014). Coupon payments $c$ are normalized to one. Earnings in opaque distressed states, $X_d$, are chosen to match a ratio of net income to market value of assets of approximately $-2.5\%$, which is representative of firms with high default risk (see summary statistics in Campbell, Hilscher, and Szilagyi 2008). The parameter $\delta$ and price-dividend ratios after a jump into the recovery set $\Omega_g$ are calibrated to match the equity risk premium and the average price-dividend ratio of the market portfolio, as estimated by Bansal and Yaron (2004). Poisson intensities for $dg$-firms are chosen to match average earnings volatility of $30\%$ and an average equity risk premium of $6.33\%$, consistent with the one after a jump to the recovery set $\Omega_g$. Poisson intensities for $db$-firms are chosen to match an average earnings volatility of $30\%$ and average systematic volatility of $10\%$ at a one-year horizon. These earnings volatilities are chosen to be consistent with calibrations in the existing literature (e.g., Bhamra, Kuehn, and Strebulaev 2010, Chen, Cui, He, and Milbradt 2014).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable</th>
<th>State $G$</th>
<th>State $B$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Economy</td>
<td>Transition rates for aggregate states</td>
<td>$\lambda$</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Jump-risk premium</td>
<td>$\exp(\phi)$</td>
<td>2.0</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Risk-free rate</td>
<td>$r_f$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Firm</td>
<td>Earnings in opaque distressed states</td>
<td>$X_d$</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Size of log-earnings changes</td>
<td>$\delta$</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P-D ratio after recovery (jump to $\Omega_g$)</td>
<td>$\frac{V}{X-c}$</td>
<td>28.51</td>
<td>16.80</td>
</tr>
<tr>
<td></td>
<td>Poisson intensity of recovery by $dg$-firm</td>
<td>$\lambda_{dg,g}$</td>
<td>1.10</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>Poisson intensity of decline by $db$-firm</td>
<td>$\lambda_{db,b}$</td>
<td>0.70</td>
<td>2.50</td>
</tr>
</tbody>
</table>

Moreover, since, by definition, equity prices are zero after a jump into the bad revealing set $\Omega_b$, no additional structure needs to be imposed on earnings in this set.\(^{34}\)

Poisson intensities for $dg$-firms. The level and variation of Poisson intensities $\lambda_{dg,g}$ across the two aggregate states $G$ and $B$ are chosen to match an unconditional earnings

\(^{30}\)The parameter choices $X_d = 0.79$, $\delta = 0.3$, and $c = 1$, determine the level of dividends immediately after recovery (after a jump to $\Omega_g$). Dividends and P-D ratios pin down equity values. As shown in Proposition 2, equity values in distressed states $\Omega_d$ can be characterized as a function of the average equity value after recovery — all relevant properties of earnings dynamics in the set of states $\Omega_g$ are encoded in these prices.

\(^{31}\)Details about earnings dynamics in the set $\Omega_b$ do not affect equity valuations in distressed states $\Omega_d$, since, by definition, earnings prospects in the set $\Omega_b$ are so poor that immediate default is optimal.
volatility of 30% and an average equity risk premium of 6.33% for firms that are believed to be a $dg$-firm with probability one. Recent papers that calibrate structural credit risk models feature similar earnings volatility. Note that to match a risk premium of 6.33% earnings have to be calibrated to be procyclical — as a result, $dg$-firms are more likely to recover during aggregate booms than during recessions.

**Poisson intensities for $db$-firms.** The Poisson intensities $\lambda_{db,b}$ are chosen to match an average earnings volatility of 30% and an average systematic volatility of 10% at a one-year horizon. As in the case of $dg$-firms, these numbers are broadly consistent with choices made in other papers in the literature. Since the firm is insolvent for $\pi \to 0$ and thus does not have a well-defined equity risk premium, I target systematic earnings volatility in the case of $db$-firms. The corresponding parameter choices imply strongly pro-cyclical earnings dynamics for $db$-firms: on average, a $db$-firm exhibits earnings declines after 7 months if the economy is currently in a recession; this number is 16 months for a boom.

**Asymmetry of business cycles.** Since recessions are short-lived relative to aggregate booms (unconditional probability of 1/6 vs. 5/6, respectively) the calibration is inherently asymmetric. Thus, when $db$-firms and $dg$-firms have the same average earnings volatility and pro-cyclical earnings, their Poisson intensities are naturally not completely symmetric.

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32 That is, average local earnings volatility of a $dg$-firm is given by $E[\text{Std}[d\log X|Z]|dg] = 0.30 dt$.

33 That is, $\lim_{\pi \uparrow 1} E[r_{\pi}(\pi, Z)] = 6.33\%$.

34 For example, in Chen, Cui, He, and Milbradt (2014) local idiosyncratic volatility is 22.5% and local systematic volatility is on average 10.8%. In Bhamra, Kuehn, and Streubel (2010) these numbers are 22.6% and 10.1%, respectively. Both models additionally feature pro-cyclical variation in earnings growth drifts, which yields additional variation at lower frequencies.

35 That is, average local earnings volatility of a $db$-firm is given by $E[\text{Std}[d\log X|Z]|db] = 0.30 dt$.

36 Whereas instantaneous earnings risk is purely idiosyncratic, cumulative earnings changes are correlated with the aggregate conditions since the Poisson intensities $\lambda_{db,b}$ vary with the aggregate state $Z$. Let $\tau_G$ denote the cumulative time spent in aggregate state $Z = G$ over the next year, that is, $\tau_G = \int_t^{t+1} 1_{\{Z_s = G\}} ds$ and let $\Delta x = \log (X_{t+1}/X_t)$. I compute the average of systematic volatility $\text{Std}[\mathcal{E}_t[\Delta x|\tau_G]]$ across aggregate states $Z_t \in \{G, B\}$.

37 As noted in Section 2, the specifications for earnings dynamics in Bhamra, Kuehn, and Streubel (2010) and Chen, Cui, He, and Milbradt (2014) are not identical to the ones postulated in this paper, as they feature diffusion risk and abstract from learning about hidden states. See also footnote 34.

38 These expected arrival times account for mean-reversion in the aggregate state $Z$. Formally, I compute the conditional expectation $E[\int_t^{\tau^*} \tau dN^{db}_t | Z_t]$ for $Z_t \in \{G, B\}$, where $\tau^*$ denotes the first time after date $t$ that the counting process $N^{db}_t$ jumps.
either. While the intensity of an earnings change is highest for \( db \)-firms in recessions, the average arrival time of earnings news (across all \( Z \)) is actually longer for \( db \)-firms (14 months) than for \( dg \)-firms (13 months). This difference arises because booms tend to last longer and \( dg \)-firms have a high intensity of recovery during these times.

In unreported calibrations I confirm that in an economy with symmetric business cycles qualitatively similar results can obtain when \( dg \)-firms’ and \( db \)-firms’ Poisson intensities are symmetric mirror images of each other. However, since the asymmetry of business cycles is an empirically robust phenomenon, it is also a feature of the baseline calibration.

\[ \text{2.4.2. Results of the Calibration} \]

The main results of the calibration can be best understood by studying the dependence of equilibrium equity prices on the two state variables, aggregate conditions \( Z \) and agents’ beliefs about the firm’s fundamental solvency \( \pi \). The left-hand side panel of Figure II plots stock valuations as a function of these two states in a region close to the default boundaries. The right-hand side panel plots equity risk premia. The two lines in each panel refer to the two aggregate states, \( G \) and \( B \).

Belief dynamics. The calibration implies the intuitive notion that distressed firms that show resilience in downturns by maintaining stable earnings exhibit upward revisions in posterior beliefs over time. In other words, in a downturn, passage time without earnings changes improves agents’ posterior beliefs and correspondingly leads to an increase in equity values. In contrast, in booms, stable earnings are considered a bad signal, since truly solvent firms are likely to recover quickly when general economic conditions are good. In booms, passage time without earnings changes is thus a sign of weakness and associated with deteriorating beliefs and declining equity values. Referring to the left-hand side panel of Figure II in recessions (\( B \)), valuations thus drift upwards on the equity value function.

\[ \text{39} \] That is, a parameterization where booms and recessions have equal probability (\( \lambda(G, B) = \lambda(B, G) \)), and where \( dg \)-firms and \( db \)-firms have pro-cyclical intensities that are mirror images of each other, i.e., \( \lambda_{dg,g}(B) = \lambda_{db,b}(G) \) and \( \lambda_{dg,g}(G) = \lambda_{db,b}(B) \).

\[ \text{40} \] See, e.g., Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014), Kehrig (2015).

\[ \text{41} \] Statements about the evolution of beliefs as a function of passage time refer to the evolution of beliefs absent earnings changes. Unconditionally, beliefs are a martingale.

\[ 20 \]
absent earnings changes. In contrast, in booms (\(G\)), valuations drift downwards absent earnings changes, and smoothly approach zero, where equity holders trigger default.\footnote{Given the considered parameter choices, smooth pasting only applies in state \(Z = G\), not in state \(Z = B\). In state \(Z = B\), passage time causes beliefs to drift upwards, since \(db\)-firms are more likely to exhibit a decline in recessions than \(dg\)-firms are to exhibit a recovery.}

**Equity valuations.** Equity valuations in both aggregate states are naturally increasing in beliefs \(\pi\). For optimistic beliefs (high \(\pi\)) stock valuations are pro-cyclical, that is, for given beliefs \(\pi\), they are higher in booms (\(G\)) than in recessions (\(B\)). Yet the opposite is true in a region close to the default boundaries. Here, the equity value is lower in booms. This is true despite the fact that earnings are highly pro-cyclical in the calibration. As I will explain below, endogenous default decisions informed by learning generate this surprising result. As a benchmark it is useful to keep in mind that pro-cyclicality of earnings ensures that the *unlevered* firm would always have higher valuations in booms than in recessions.\footnote{The unlevered firm’s equity value is given by the probability weighted average of the equity values in state \(dg\) and \(db\), that is, \(V_U(\pi, Z) = (1 - \pi)V_U(db, Z) + \pi V_U(dg, Z)\), where the subscript \(U\) indicates the unlevered equity value. Pro-cyclicality of earnings implies that \(V_U(db, G) > V_U(db, B)\) and \(V_U(dg, G) > V_U(dg, B)\), such that \(V_U(\pi, G) > V_U(\pi, B)\) for all \(\pi\).}

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**FIGURE II**
The graphs plot the firm’s equity values (left-hand side panel) and equity risk premia (right-hand side panel) as a function of the posterior probability that the firm is fundamentally solvent, \(\pi_t = \Pr[z = dg | F_t]\), and the aggregate state \(Z_t \in \{G, B\}\). The plot of equity values focuses on a region close to the default boundaries to highlight the behavior of stock prices when default risk is high. All parameter values are provided in Table 1.

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21
Risk premia and momentum. The described inversion of valuations close to the default boundaries has striking implications for firms with high default risk: equity risk premia turn negative, as shown in the right-hand side panel of Figure [II]. Whereas risk premia are high for optimistic beliefs — for $\pi = 1$ the average risk premium is 6.33% — they turn negative when beliefs become sufficiently pessimistic.

As a result, the model can generate momentum dynamics for distressed firms. Negative updating about firm quality leads not only to negative contemporaneous returns, but also to lower expected equity returns going forward, as the firm gets closer to the default boundary. “Recent losers” thus have lower expected returns going forward. Conversely, good news that is associated with positive price changes pushes the firm away from the default boundary and leads to higher future expected returns for “recent winners.” A momentum strategy that goes long recent winners and shorts recent losers can therefore generate a large positive spread in expected returns. While the results reconcile positive correlations between recent price changes and future expected returns for financially distressed firms, they suggest that the phenomenon becomes less pronounced the further the firm moves away from the default boundary, where default risk is lower. These implications of the calibration shed light on the empirical findings of Avramov, Chordia, Jostova, and Philipov (2007) who document that both the long and the short portfolio of momentum strategies consist of stocks with high default risk and that momentum profits are restricted to high credit risk firms and are nonexistent for firms of high credit quality. According to the calibration, distressed firms are good candidates for portfolio strategies that sort on past returns, since their past returns are strongly correlated with future expected returns. Moreover, evidence in Bansal, Dittmar, and Lundblad (2005) supports the theory’s link between risk premia and exposures to business cycle risks. The authors find that exposures to persistent changes in growth can explain a large fraction of the variation in expected returns across momentum portfolios.

Economic mechanism. As highlighted above, the firm would always have pro-cyclical valuations absent leverage, since $X(z)$ is calibrated to be strongly pro-cyclical. Yet, with leverage and the potential need for external finance, risk is also affected by investors’ financing and default decisions. These decisions, in turn, depend not only on current beliefs about the firm’s prospects but also on expectations about the future arrival of information. Even
if current beliefs are pessimistic (low \( \pi \)), the firm may be able to attract new funds if information that helps separate solvent from insolvent firms is expected to arrive in the near-term. Supporting the firm a little longer is worthwhile in this case as newly arriving information will help equity holders make better financing decisions than under the current information set. In particular, information that helps separate good from bad firms allows equity holders to avoid losses from mistakenly subsidizing insolvent firms and abandoning solvent firms. Holding current beliefs \( \pi \) fixed, current equity prices are therefore increasing in the expected future speed of learning.

In the context of the business cycle, the cyclical properties of learning therefore have first-order effects on the cyclicality of stock valuations. When, in the face of an aggregate downturn, truly insolvent firms are more likely to exhibit deteriorating earnings performance, this increases equity values conditional on current beliefs \( \pi \) — ceteris paribus, equity holders rather quickly observe bad performance by truly insolvent firms, as this signal helps separate solvent from insolvent firms. This basic benefit of learning implies that, although worse performance in downturns increases distressed firms’ equity risk exposures. For low \( \pi \), risk premia even turn negative as, close to default, the highlighted learning-based feedback effect becomes a more and more important determinant of stock prices.

Figure III illustrates these effects based on comparative statics with respect to the parameter \( \lambda_{db,b}(B) \): higher downside risk for insolvent firms in recessions, \( \lambda_{db,b}(B) \), reduces equity risk premia. Conversely, lower downside risk and a corresponding slower arrival of information, leads back to the standard result, obtained for example in Merton (1974), that equity risk premia increase dramatically as the firm approaches default. Insolvent firms’ downside risk in recessions, which is associated with dispersion in realized earnings, is thus an important determinant of risk premia when agents learn about firm solvency. Moreover, there is strong empirical evidence of rising dispersion and downside risk in recessions.

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44 The reader might wonder why equity risk premia reach a ceiling of 10% in the good aggregate state of the low parameterization. For beliefs below the threshold \( \pi^R(B) \) the firm defaults immediately after a jump to the recession state \( B \). The corresponding stock return is thus given by \(-100\%\) for all \( \pi < \pi^R(B) \), implying a constant risk premium in this region of beliefs.

45 Using micro-level Census data, Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2014) estimate a two-state Markov switching model at a quarterly frequency and find that idiosyncratic volatility increases by 310% in recessions.
The graphs plot equity risk premia as a function of the posterior probability that the firm is fundamentally solvent, \( \pi_t = \Pr [z = dg | F_t] \), and the aggregate state \( Z_t \in \{ G, B \} \). The graphs illustrate the dependence of equity risk premia on truly insolvent firms’ downside risk in downturns by solving the model for different values of \( \lambda_{db,b}(B) \). The labels low, med, and high refer to the parameter values 1.8, 2.5, and 3.2 for \( \lambda_{db,b}(B) \), respectively. All other parameters remain at their baseline values listed in Table 1. The graph on the left-hand side plots equity risk premia as a function of beliefs in the boom state \( G \). The right-hand side panel plots equity risk premia in the recession state \( B \).

**Downside risk and information.** Given that equity holders optimally default after observing a log-decline in earnings of size \( \delta_{db} \), the actual parameter value for \( \delta_{db} \) does not further affect stock valuations, since equity holders are protected by limited liability. In fact, equity holders would value the stock in the same way if information from some other source became available with arrival intensity \( \lambda_{db,b} \), such as, for example, a negative update to some other accounting line item. Fundamentally, \( \lambda_{db,b} \) thus represents the arrival intensity of a negative signal that reveals poor earnings prospects and corresponding insolvency. Comparative statics with respect to \( \lambda_{db,b} \) thus more generally trace out the effects of downside risks emerging from any source of news about future earnings.

**Relation to the business cycle literature.** A central prediction of the baseline calibration is that default thresholds are pro-cyclical, that is, the threshold at which default occurs is lower in downturns, as capital providers are more lenient with respect to current performance in these times. This prediction is consistent with recent findings of the literature on the cyclical nature of the productivity distribution, in particular Kehrig (2015), who...
estimates establishment-level productivity levels in the U.S. manufacturing sector from 1972 to 2009 using confidential micro-level Census data. Kehrig's (2015) empirical results suggest a \textit{pro-cyclical} survival cutoff at the bottom tail of the productivity distribution. If failure is triggered when a firm’s equity value turns zero, this result implies that, as a function of current productivity, equity valuations close to default are \textit{higher} in recessions, consistent with the result of the baseline calibration.

Whereas Kehrig (2015) interprets this empirical finding as evidence of economic factors that lead to “sullying” recessions, my model reveals that “cleansing” recessions (Schumpeter, 1934) can actually generate this phenomenon as well. Since investors expect to learn about insolvent firms during the test of an aggregate downturn, they are more willing to support firms of uncertain quality that currently show mediocre performance\footnote{Whereas Schumpeter (1934) was concerned with the general economic benefits of abandoning inefficient firms in downturns, my paper focuses primarily on equity valuations. If, as discussed at the beginning of section 2.2 we interpret $c$ as operational cost then equity value is equal to total firm value, and, absent other frictions, maximizing equity value coincides with maximizing total surplus.} — a high expected speed of learning increases the equity value in downturns and warrants more lenience with respect to current performance. Yet note that this learning-based feedback effect does not predict fewer defaults in recessions. On the contrary, it is the very fact that equity holders expect to trigger numerous defaults after more extreme declines that justifies more lenience with respect to current performance.

\textbf{Implications for empirical asset pricing tests.} When relating the predictions of the model to the empirical asset pricing literature it is important to note that empirical papers study expected returns of financially distressed stocks at the \textit{portfolio level}. Studies such as Campbell, Hilscher, and Szilagyi (2008) sort the cross-section of stocks based on default likelihood measures and then form portfolios based on breakpoints (e.g., quintile breakpoints). The degree to which firms in these portfolios are financially distressed varies naturally over the business cycle: the average firm in the bottom quintile tends to have higher default risk during a recession than during a boom. Given the calibration’s prediction that firms that are closer to default have lower exposures to business-cycle risks, this implies that the quintile portfolio of the most distressed stocks will have \textit{lower} exposures to these risks in downturns. Since market risk premia tend to be high in recessions, the model can thus rationalize a nega-
tive correlation between market risk premia and portfolio betas at a business cycle frequency. This negative correlation implies that unconditional CAPM regressions yield positively biased betas and negatively biased alphas, helping reconcile the empirical finding that the portfolio of stocks with the highest default risk generates negative unconditional alphas in the data.

While estimating conditional exposures to business-cycle risks should eliminate this mis-specification problem, doing so is a challenging task. First, estimating time-varying betas with high-frequency return data generally leads to biased estimates, as business cycle risks materialize at low frequencies and since the coexistence of both high-frequency and low-frequency shocks obfuscates estimation. Second, estimating exposures to business cycle risks requires long time-series of data, as a business cycle generates essentially only one data point. The result is low power, even with several decades of return data.

Despite these empirical challenges, existing empirical evidence supports the notion that cyclical variation in betas can help reconcile negative unconditional distress alphas. Specifically, O’Doherty (2012) finds that the conditional betas of the quintile of stocks that are most distressed are lower in downturns and are negatively correlated with measures of the market risk premium. O’Doherty (2012) further estimates that this negative correlation can account for about two thirds of the unconditional distress anomaly alpha. Yet, facing limited data availability, O’Doherty’s (2012) estimation approach uses daily return data, which is not ideal.

The role of external finance. In search for a theoretical explanation for cyclical variation in betas O’Doherty (2012) notes that noisier information about a firm’s underlying asset values increases equity values and decreases betas when equity is a European call option on a firm’s final asset payoff, as in Johnson’s (2004) extension of Merton (1974). According to this argument, noisier information in recessions would help decrease betas in these times, a prediction that appears to be at odds with the results discussed here. The difference arises because in a European call setting the firm has no need to raise external finance, implying

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47 See, e.g., Grant (1977) and Jagannathan and Wang (1996).
that there is also no notion of financial distress. Equity holders make only one default decision at the terminal date, when the final payoff is perfectly observable.\footnote{Default occurs at the terminal date if, at that time, the perfectly observable asset value is below the face value of debt.} Prior noisy information thus cannot harm access to finance or value. The learning-based feedback effect, which is at the core of my model, is absent.

The two settings also generate markedly different predictions along other dimensions. In my model, equity risk premia can fall below asset risk premia and even become negative as the firm’s default probability increases. As a result, momentum dynamics can arise. In contrast, in the discussed European call setting, equity risk premia are higher than asset risk premia and increase monotonically with leverage, creating the opposite relation. In addition, my model is consistent with the empirical finding that the distress anomaly is particularly strong among net-issuers,\footnote{See \cite{Park2013}.} that is, distressed firms that seek external finance. In contrast, external finance is absent in the European call setting. Finally, the learning-based feedback effect of my model can shed light on the role of active investors in facilitating external finance and affecting risk premia, a feature I will discuss in more detail in the next section.

3. Externalities from Active Investors

In the following model extension I illustrate how, through the highlighted learning-based feedback effect, information-gathering “active investors” can create a significant externality on other investors’ risk exposures and expected returns. The results of this extended model can shed light on the puzzling empirical finding that regular shareholders not participating in private placements of public equity (PIPEs) experience post-issue negative abnormal returns (see, e.g. \cite{Hertzel2002, Krishnamurthy2005, Park2013}).

In the model extension, a private placement of public equity to a large, active investor with access to superior information generates an externality: using her superior information, the active investor can better discern solvent from insolvent firms, which improves financing and liquidation decisions and thereby affects other investors.
3.1. The Extended Model

An active investor is endowed with an information production technology that generates a perfectly precise signal of the hidden firm state $z \in \{db, dg\}$ with Poisson arrival rate $a$, where $a$ may depend on the aggregate state $Z$. The active investor is a financial firm that maximizes the market value of its portfolio.

Contracting. The analysis considers contracts between management and the active investor that provide the active investor with new equity shares that yield an ownership share $\omega$ at a purchase price $\bar{\kappa}$. Management acts in the interest of existing shareholders. Contracts are limited to one-time provisions of new equity, potentially at a price that differs from the current market price. Management does not renegotiate the contract in the future, which implies that future equity injections by the active investor occur at regular market prices. In principle, the transaction with the active investor may occur at any point in time $\bar{t} > 0$. Yet there is no reason to involve the active investor before the firm enters an opaque distressed state ($\Omega_d$), since the active investor’s information technology is only useful in states in which the underlying firm-state $z$ is unobservable.

I treat the size of the active investor’s ownership share $\omega$ as an exogenous parameter. In practice, ownership shares may be affected by regulations, as well as by the trade-off between incentives provision on the one hand and potential costs of un-diversification on the other. Yet, since these forces could coexist without affecting the qualitative results I aim to illustrate in this section, I abstract from them to keep the model simple and tractable. Further, as shown below, the chosen parameter value for $\omega$ will not affect equity risk premia, which are the object of interest in this section.

In practice, private investments in public equity ("PIPEs") are a typical form of distressed equity investments by active investors. Management is directly involved in PIPE transactions and has the fiduciary duty to act in the interest of existing shareholders. In these transactions, management typically deliberately offers profits to active investors by providing them with securities at discounted prices.$^52$

$^52$See, e.g., Park [2013].
Information environment. Changes to the active investor’s ownership in the firm are assumed to be publicly observable. The resulting lack of “noise” in the system implies that the active investor cannot extract rents by trading against less informed investors — all market participants understand that the active investor only trades for informational reasons. Yet, the active investor can still generate profits, since management is willing to provide equity to the active investor at a discounted price. Doing so is consistent with shareholder value maximization, since, once exposed to the firm, the active investor will have incentives to inform ongoing financing and default decisions. Thus, the active investor becomes truly “active” after obtaining a stake in a company, as she is involved in evaluating and influencing firm policies.

I abstract from noise in the system (e.g., noise traders or liquidity shocks to active investors) for two reasons. First, regulation in the United States requires that investors acquiring more than 5% of a firm’s equity with the intent to exert control have to file a schedule 13d with the SEC within 10 days. In addition, investors have to re-file these forms in case of material changes to their positions (1%). These required regulatory filings ensure that information about an active investor’s exposure becomes public at a high-enough frequency to help inform other equity holders’ future financing and default decisions, which is the central externality channel in the presented setup.

Second, whereas adding noise to the system would be an interesting extension in its own right, it would be distracting in the context of this section, which aims to shed light on empirical regularities of buy-and-hold returns after PIPE transactions. A buy-and-hold strategy does not correspond to random or selective buy-and-sell orders that are typical features of noisy rational expectations models. By definition, buy-and-hold strategies do not encompass trades against informed active investors. Finally, adding noise to the system would also come at the cost of reducing the tractability of the model.

Cash. The firm can hold new funds provided by the active investor as cash that is invested at the risk-free rate. In case the firm depletes its cash it can attempt to raise additional funds, just as in the baseline model of section 2. Management can pay out cash as a dividend at any point in time. In particular, in case of a strategic liquidation, management can pay out cash before triggering default.
The assumption that cash can be paid out as a dividend at any time simplifies the analysis but is not essential for the highlighted results. If management could not pay out cash, the firm could naturally only default after depleting the funds provided by the active investor. At that time, the same optimal default strategies and valuations would apply as in the setup considered here. The optimal default strategies in the two setups — with and without cash payouts — are thus closely related.

**Bargaining power.** Management acting in the interest of existing equity holders always has the outside option not to issue shares to the active investor. The value of the equity under this scenario is given by the reservation value $V^R$, characterized in Proposition 2, which puts an upper bound on the price discount management is willing to offer. How the surplus from the active investor’s involvement is split between existing shareholders and the active investor depends on assumptions about bargaining power. Yet, while the distribution of bargaining power affects the initial purchase price $\bar{\kappa}$, it does not influence the results for equity risk premia discussed in the analysis below.\[53\]

### 3.2. Analysis

The active investor’s information production technology becomes useful as soon as the firm enters an opaque distressed state $\Omega_d$, where investors face uncertainty about firm solvency.\[54\] As discussed in Lemma 1, investors’ beliefs after a jump to an opaque distressed state are given by $\pi_0(Z)$. The reservation value is then simply given by the solution for the equity value under the baseline model, that is, $V^R(\pi_0(Z_t), Z_t)$. Let $V(\pi, Z, \bar{\kappa})$ denote the total equity value at the time when the active investor acquires a stake in the firm at price $\bar{\kappa}$. The issuance of new equity to the active investor implies a cash injection, generating an initial excess cash position of $\bar{\kappa}$. Let $Div_t$ denote a discrete payout of excess cash at time $t$ as a dividend, where $0 \leq Div_t \leq \kappa_t$. In opaque distressed states the firm’s cash balance

\[53\]Empirically, the dilution of existing shareholders in PIPE deals suggests that active investors are in fact able to extract rents (see, e.g., Park, 2013).

\[54\]As there is no uncertainty about solvency in the initial solvent states $\Omega_s$, there is also no benefit from undertaking a PIPE transaction.
evolves as follows:

\[
d\kappa_t = \begin{cases} 
(r_f(Z)\kappa_t + X_d - c) \, dt - Div_t & \text{for } \kappa_t > 0, \\
0 & \text{for } \kappa_t = 0. 
\end{cases}
\]  
(11)

The following proposition characterizes the solution to the active investor’s dynamic decision problem after acquiring an \( \omega \)-share of the firm’s equity.

**PROPOSITION 3** (Valuations in the presence of an active investor). In opaque distressed states \( \Omega_d \) the active investor’s value is given by

\[
V^A(\pi_t, Z_t, \kappa_t) = \max_{\{\tau^*, Div\}} E_t \left[ \int_t^{\tau^*} \frac{\xi(Z_{\tau})}{\xi(Z_t)} \omega \left( X(z_{\tau}) - c - \frac{1}{d\tau} Div_{\tau} \right) d\tau \right] = v^A(\pi_t, Z_t)^+ + \omega \kappa_t.
\]  
(12)

The corresponding Hamilton-Jacobi-Bellman equation yields the following set of ODEs that the function \( v^A(\pi, Z) \) solves for \( Z \in \{G, B\} \):

\[
0 = \omega(X_d - c) - r_f(Z) v^A(\pi, Z) + v^A(\pi, Z) \theta(Z)(1 - \pi) \pi \\
+ \sum_{z' \neq \text{dg}} \pi \lambda_{d\pi, z'}(Z) \left( \omega V(z', Z) - v^A(\pi, Z) \right) \\
- \sum_{z' \neq \text{db}} (1 - \pi) \lambda_{d\pi, z'}(Z) v^A(\pi, Z) \\
+ a(Z) \left( \pi \omega V^R(dg, Z) - v^A(\pi, Z) \right) \\
+ \lambda(Z, Z') e^{\phi(Z, Z')} \left( v^A(\pi, Z') 1_{\{\pi \geq \pi^A(Z')\}} - v^A(\pi, Z) \right). 
\]  
(13)

The active investor optimally proposes a liquidating dividend (for \( \kappa_t > 0 \)) or refuses to inject additional funds (for \( \kappa_t = 0 \)) immediately after obtaining a bad signal, or when posterior beliefs \( \pi \) fall below an endogenous cutoff value \( \pi^A(Z) \). In states \( Z \) where \( \theta(Z) < 0 \) the following conditions apply: \( 0 = v^A_{\pi}(\pi^A(Z), Z) \) and \( 0 = v^A(\pi^A(Z), Z) \). In states \( Z \) where \( \theta(Z) > 0 \), the following conditions apply: \( v^A(\pi^A(Z), Z) = 0 \) and \( \lim_{\pi \uparrow 1} v^A(\pi, Z) = \omega V^R(dg, Z) \), where \( V^R(dg, Z) \) is characterized in Appendix A.2. If \( \theta(Z) = 0 \) then the ODE for state \( Z \) simplifies to a nonlinear equation. The firm’s total equity value is given by \( V(\pi_t, Z_t, \kappa_t) = \frac{1}{\omega} V^A(\pi_t, Z_t, \kappa_t) \). Equity holders optimally agree to default or liquidate the firm when the
active investor proposes to do so. Equity holders further choose to default when in aggregate state $Z$ posterior beliefs $\pi$ fall below the endogenous cutoff value $\pi_A(Z)$.

Proof. See Appendix A.3.

Informational Externality. Since there are no non-informational reasons to trade, the active investor will obtain a price of zero when attempting to sell her equity stake, as other market participants infer that the active investor must have received a negative signal. Thus, instead, the active investor optimally publicly proposes liquidation after receiving a bad signal or when beliefs drop below the endogenous default boundaries $\pi_A(Z_t)$. Otherwise, it is optimal for the active investor to keep her equity share and stay involved. Small equity holders follow the active investor, as they know that the active investor aims to maximize shareholder value, given her equity exposure and her inability to make trading gains. Through this externality channel, equity holders can effectively free ride on the active investor’s information production after the active investor has obtained a stake in the company. Due to this ability to free ride, small equity holders have the same optimal default boundaries as the active investor.

Purchase price. As discussed above, the initial purchase price depends on the distribution of bargaining power. If the active investor has all the bargaining power, existing shareholders obtain exactly their reservation value $V^R$, that is, the purchase price $\bar{\kappa}$ is set such that existing shareholders’ stake is worth $V^R$ after the transaction. For an $\omega$-share in the firm’s post-issuance equity the active investor’s purchase price is then given by:

$$\bar{\kappa} = \frac{V^R(\pi, Z)}{(1 - \omega)} - v(\pi, Z)^+.$$  \hspace{1cm} (14)

In this case, existing shareholders’ ability to free ride after the transaction does not increase their shareholder value ex ante, as the price discount given to the active investor exactly offsets the future benefits of free riding. Nonetheless, immediately after the transaction, shareholders’ risk exposures are affected significantly by the involvement of the active investor, as highlighted in the next subsection.
In this context, it is worth highlighting that negative contemporaneous returns for existing shareholders at PIPE announcements do not provide conclusive evidence that management is acting against shareholders’ interests. By conditioning on firms that issue discounted equity to an active investor, the econometrician systematically sorts on firms that just received a negative shock and for that reason approach an active investor. In other words, under the counter-factual of no active investor involvement, existing equity holders could have lost at least as much value.

3.3. A Parameterized Illustration

In this section, I discuss an adjusted parameterization of the model to illustrate how the involvement of an information-producing active investor affects the equity risk premia of distressed firms. Throughout, I will focus the analysis on risk premia associated with the firm’s non-cash assets. To determine the mechanical effect of cash on the overall risk premium, one simply has to take the value-weighted average of the risk premium on the firm’s non-cash assets and its cash investments.

I focus on non-cash assets for several reasons. First, I aim to isolate the effects of the informational channel highlighted in this paper, which does not affect the risk premium on cash investments. Second, in the model, the firm could pay out cash as a dividend at any point in time without positively or negatively affecting shareholder value — the size of the cash position is thus not uniquely determined. Third, in practice, the firm has in principle some freedom to choose from a range of fairly-priced marketable securities which can carry different risk premia. Finally, it is worth noting that, in practice, cash positions after PIPE transactions are not necessarily large, since injected cash is typically used to make lumpy debt payments, a feature that is not present in my model with continuous debt payments.

Parameter choices. I use the same parameter values as in the baseline calibration listed in Table 1 except for two adjustments: First, I assume that the active investor’s information acquisition technology is expected to generate a signal about firm solvency after 3 months, independent of the aggregate state $Z$ (that is, $a(Z) = 4$ for all $Z$). Second, I reduce truly

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55See, e.g., Park (2013) for related empirical evidence.
insolvent firms’ downside risk in recessions to $\lambda_{db,b}(B) = 0.8$. This parameter change strongly reduces the pro-cyclicality of truly insolvent firms’ earnings and lowers the speed of learning from earnings in recessions. I make this adjustment to illustrate how the active investor’s involvement can still help reduce risk premia when a firm’s earnings are not as informative in downturns as in the baseline calibration.

### FIGURE IV

The graphs plot equity risk premia as a function of the posterior probability that the firm is fundamentally solvent, $\pi_t = \Pr[z = dg|F_t]$, the aggregate state $Z_t \in \{G,B\}$, and the involvement of an active investor. The labels "With AI" and "Without AI" refer to the two cases with and without the involvement of the active investor. The active investor is expected to receive a signal after 3 months ($a(Z) = 4$, for all $Z$) and truly insolvent firms’ downside risk in recessions is given by $\lambda_{db,b}(B) = 0.8$. All other parameter values are as listed in Table I.

**Risk premia, alphas, and default boundaries.** Figure IV illustrates that, under this parameterization, equity risk premia increase dramatically close to the default boundaries. The reason for this change is a slower speed of learning from earnings in downturns, due to lower downside risk $\lambda_{db,b}(B)$ relative to the baseline calibration (recall the related comparative statics in Figure III). Yet Figure IV also reveals that equity risk premia are substantially lower with the involvement of the active investor.

After the completion of a PIPE transaction that exposes the active investor to the firm, risk premia thus can decline discontinuously, even if agents had perfectly anticipated the active investor’s involvement. For example, when the active investor obtains equity at a discounted price and can extract the additional surplus it generates, equity risk exposures
before the transaction are identical to the ones under permanent autarky (that is, without
the involvement of an active investor). Yet, as soon as the active investor has received
the discounted equity, risk exposures change discontinuously, reflecting the active investor’s
influence going forward. As a result, estimating betas based on return data that precedes
the transaction lead to biased results, in particular, negative alphas.

With the active investor, default thresholds also become more lenient (that is, \( \pi^A(Z) < \pi^R(Z) \)). Improved access to information increases equity holders’ option value, which justifies
this additional lenience. Importantly, since it is likely that the active investor generates an
informative signal in the near-term, the probability that the firm ever finds itself in the range
of beliefs with elevated risk premia is reduced markedly.

**Limiting cases and the cyclicality of activism.** In fact, in the limiting case where the
active investor is expected to obtain a signal very quickly (\( a(Z) \to \infty \) for all \( Z \)), equity risk premia become flat over the whole range of beliefs. Thus, even with very high default risk,
risk premia do not increase, counter to the predictions of standard theories. Strikingly, a
constant speed of learning in this limiting case effectively neutralizes the leverage effect that
usually causes risk premia to rise with default risk. Further, if investor activism is stronger
in recessions (\( a(B) > a(G) \)), then learning can induce negative equity risk premia close to
default, even in the considered parameterization where learning from earnings is slow in
downturns. Overall, the active investor can thus boost the value of her stake not only by
increasing expected dividends but also by reducing discount rates.

**Empirical predictions.** The highlighted results imply a set of unique predictions for the
expected buy-and-hold returns after distressed equity issuances to active investors. Risk
premia are functions of both the involvement and the skill level of the active investor and
generally change discontinuously after a PIPE transaction is completed, even if the transac-
tion is anticipated. These predictions can shed light on the empirical finding that distressed
firms that announce issuances of private placements of public equity have particularly low

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56 Equity holders anticipate that, immediately after the transaction, their equity stake will be worth exactly
the reservation value \( V^R \), since the active investor can extract the additional equity value it generates.
57 That is, elevated relative to the equity risk premium for \( \pi = 1 \), which is calibrated to be on average
6.33%. 

35
estimated abnormal returns going forward.\textsuperscript{58} The model’s mechanism is further also consistent with the finding that the degree of dilution of existing shareholders is negatively related to future expected returns.\textsuperscript{59} If the discounts provided to active investors are reflective of their skill (that is, their access to information) then discounts and future risk premia should be related according to the presented theory.

Note that the presence of the active investor only affects risk premia for sufficiently pessimistic beliefs. For $\pi \to 1$, risk premia with and without active investor become identical. Information production by the active investor does not affect equity valuations absent doubts about firm solvency, as it is not expected to generate a feedback effect on future financing and default decisions. Yet, when there is sufficient uncertainty about firm solvency, the active investor’s externality has a large impact on valuations and risk exposures.

Alternative channels through which active investors may affect firm value include proxy fights, shareholder proposals to replace management, and the alike. Whereas these alternative types of investor activism could be beneficial whenever the firm faces operating decisions, they do not necessarily relate to firms in financial distress. Further, active investors’ externality is likely particularly significant in the case of financial distress, where the equity value is almost completely determined by the default option value, which in turn is highly sensitive to the active investor’s information.

**Redistributional effects for cash flows and risk exposures.** The active investor’s information not only creates surplus but also has redistributional effects. In particular, debt holders of insolvent firms are ceteris paribus worse off, since better informed equity holders quickly abandon insolvent firms, and thus stop subsidizing debt holders via equity injections. The analysis shows that although information production can have similarly adverse effects on debt holders as risk shifting [Jensen and Meckling, 1976], it does not require any change in the underlying assets. Further, the active investor’s effect on financing and default policies leads to a reallocation of risk exposures from equity holders to debt holders.


\textsuperscript{59}See Park (2013).
Robustness: Markov state dynamics & precautionary information production. As previously discussed in Section 2.2, it is not essential that uncertainty about the underlying firm state arises only in opaque distressed states. The Markov chain setup could be extended to capture the notion that investors already face uncertainty about the firm’s true state before the firm lacks sufficient funds to make debt payments. In this case, it would be useful to involve the active investor as soon as this uncertainty arises, even if equity holders do not face an immediate default decision at that point in time.\footnote{As long as the firm generates positive dividends it is always optimal not to default, independent of the underlying hidden firm state.} This is the case, because precautionary information production by the active investor would become valuable as soon as there is also uncertainty about solvency (rather than just uncertainty about firm prospects). Yet, as discussed above, the benefit of this information would naturally increase the more distressed the firm becomes.

4. Conclusion

The quality of investors’ information about firm solvency is essential for firm survival in distress — lacking precise information, investors may refuse to provide new funds to distressed firms and thereby trigger default. In this paper I develop a tractable dynamic model to shed light on the asset pricing implications of this feedback effect. The model reveals that learning can rationalize low and even negative expected equity returns for distressed firms, and that informational externalities from active investors can have first-order effects on risk premia dynamics. The theory can shed light on a variety of asset pricing puzzles related to financial distress, such as distressed firms’ apparent stock market underperformance, momentum return dynamics, and the connection between active investors and expected returns after private placements of public equity.
A. Proofs

A.1. Equity Value in Initial Solvent States

PROPOSITION 4 (Equity value in initial solvent states). In the solvent states \((z, Z) \in \Omega_s\) the firm’s equity value is given by

\[
V (z_t, Z_t) = \max_{\{\tau^*\}} E_t \left[ \int_t^{\tau^*} \frac{\xi (Z_\tau)}{\xi (Z_t)} \left( \left( X (z_\tau) - c \right) d\tau \right) \right],
\]

(15)

The corresponding Hamilton-Jacobi-Bellman equation implies that the function \(V (z, Z)\) solves the following system of equations for all \((z, Z) \in \Omega_s\):

\[
0 = X (z) - c - r_f (Z) V (z, Z) + \sum_{z' \in \{dg, db\}} \lambda_{zz'} (Z) \left( V^R (\pi_t (Z), Z) - V (z, Z) \right) + \sum_{z' \in \Omega^Z \setminus \{dg, db\}} \lambda (z, z') (V (z', Z') - V (z, Z)) + \sum_{Z' \in \Omega^Z} \lambda (Z, Z') e^{\delta (Z, Z')} (V (z, Z') - V (z, Z)).
\]

(16)

Proof. The result follows immediately from the Hamilton-Jacobi-Bellman equation. The equity value in opaque distressed states, \(V^R\), is characterized in Proposition 2.

A.2. Proof of Proposition 2

Equity value in good distressed states \((dg, Z)\). It is useful to first characterize the firm’s equity value in states \((dg, Z)\):

\[
V^R (dg, Z_t) = E_t \left[ \int_t^\infty \frac{\xi (Z_\tau)}{\xi (Z_t)} (X (z_\tau) - c) d\tau \right].
\]

(17)
The corresponding HJB equation implies that the function $V^R (dg, Z)$ solves the following system of linear equations for all $Z$:

\[
0 = x - c - r_f (Z) V^R (dg, Z) + \sum_{(z', Z) \in \Omega_g} \lambda_{dg, z'} (Z) \left( V^R (z', Z) - V^R (dg, Z) \right) \\
+ \lambda (Z, Z') e^{\phi(z, Z')} \left( V^R (dg, Z) - V^R (Z, Z') \right).
\]  
(18)

**Boundary Conditions.** If $\theta (Z) < 0$ in state $Z$, then $v^R (\pi_t, Z)$ satisfies the smooth pasting and value matching conditions,

\[
0 = \theta (Z) \cdot (1 - \pi^{R*} (Z)) \cdot \pi^{R*} (Z) \cdot v^R (\pi^{R*} (Z), Z),
\]  
(19)

\[
0 = v^R (\pi^{R*} (Z), Z).
\]  
(20)

If $\theta (Z) > 0$, then $\pi^{R*} (Z)$ has to be chosen such that $v^R (\pi^{R*} (Z), Z) = 0$ and $v^R (1, Z)$ matches $V (dg, Z)$. If $\theta (Z) = 0$, then the ODE for state $Z$ simplifies to a nonlinear equation.

To verify these boundary conditions, let $\bar{V}^R (\pi_t, Z_t, \tau^R)$ denote the equity value given beliefs $\pi_t$, the aggregate state $Z_t$, and given that the agent follows a strategy of abandoning the firm at time $t + \tau^R$ if no jump to any other state occurs in the time between $t$ and $t + \tau^R$.

The first-order necessary condition for $\tau^R$ yields

\[
\left. \frac{\partial V^R (\pi_t, Z_t, \tau^R)}{\partial \tau^R} \right|_{\tau^R = \tau^{R*}} = 0.
\]  
(21)

A change in variables yields alternatively

\[
\left. \left( \frac{\partial \bar{V}^R (\pi_t, Z_t, \tau^R)}{\partial \pi^R} \cdot \frac{d\pi^R (\tau^R, \pi_t, Z_t)}{d\tau^R} \right) \right|_{\tau^R = \tau^{R*}} = 0,
\]  
(22)

where I define the function $\pi^R (\tau^R, \pi_t, Z_t)$ as follows:

\[
\pi^R (\tau^R, \pi_t, Z_t) = \left( 1 + e^{(-\tau^R \theta(Z_t))} \cdot \frac{1 - \pi_t}{\pi_t} \right)^{-1},
\]  
(23)
implying that for all $\pi_t \in (0, 1)$ we have $\frac{d\pi^R}{d\tau^R} > 0$. Thus, for $\pi_t \in (0, 1)$ the first-order necessary condition may also be written as

$$\frac{\partial V^R (\pi_t, Z_t, \pi^R)}{\partial \pi^R} \bigg|_{\pi^R = \pi^R_*(Z_t)} = 0,$$  \hspace{1cm} (24)

where I define

$$\pi^R_*(Z_t) \equiv \pi^R (\tau^R_*, \pi_t, Z_t).$$  \hspace{1cm} (25)

Notice that for $\theta (Z) > 0$, any $\tau^R \geq 0$ will correspond to $\pi^R \geq \pi_t$, since waiting time $\tau^R$ increases the conditional probability $\pi_t$. Let $V^R (\pi_t, Z_t)$ denote the value function from the optimal solution of the equity holders’ problem. Given the assumption that the equity value is strictly positive at $\pi_t$, i.e. $V^R (\pi_t, Z_t) > 0$, we obtain

$$V^R (\pi^R (\tau^R, \pi_t, Z_t), Z_t) \geq V^R (\pi_t, Z_t) > 0, \text{ for all } \tau^R \geq 0,$$  \hspace{1cm} (26)

given that $\frac{\partial V^R (\pi, Z_t)}{\partial \pi} \geq 0$ for all $\pi \in [\pi_t, 1]$, since $\pi^R (\tau^R, \pi_t, Z_t) \geq \pi_t$ for all $\tau^R \geq 0$. Thus, given that $V^R (\pi, Z_t) > 0$ and $\theta (Z) > 0$, the equity value after any positive waiting time is also positive, and thus it is optimal not to abandon the firm as long as it stays in the current state, that is, it is optimal to set $\tau^R_* = \infty$. Since

$$\lim_{\tau^R \to \infty} \pi^R (\tau^R, \pi_t, Z_t) = 1,$$  \hspace{1cm} (27)

the ODE simplifies in the limit $\tau^R \to \infty$ to the non-linear equation that the function $V (dg, Z_t)$ solves, implying that

$$\lim_{\pi \to 1} V^R (\pi_t, Z_t) = V (dg, Z_t).$$  \hspace{1cm} (28)

On the other hand, for $\theta (Z) < 0$, it follows that $\frac{d\pi^R}{d\tau^R} < 0$, implying that waiting time corresponds to lower conditional probabilities $\pi_t$. By assumption, at $\pi = 0$ the equity value is zero ($V (db, Z) = 0$), and the firm is abandoned. Further, by assumption, we have
\( V^R(1, Z) = V(dg, Z) > 0 \). It is optimal to abandon at \( \pi^{R*} \) where \( \pi^{R*} \) satisfies

\[
V^R(\pi^{R*}, Z) = 0,
\]

and where the smooth pasting condition

\[
\left. \frac{\partial V^R(\pi, Z_t)}{\partial \pi} \right|_{\pi_t=\pi^{R*}} = 0,
\]

is satisfied. If smooth pasting was not satisfied then there could be an optimal cutoff \( \pi^{R^*} \) where the resulting value function \( V^{R^*} \) satisfies \( V^{R^*}(\pi^{R^*}, Z) = 0 \) and

\[
\left. \frac{\partial V^{R^*}(\pi, Z_t)}{\partial \pi} \right|_{\pi_t=\pi^{R^*}} > 0.
\]

Yet, then heuristically, at \( \pi_t = \pi^{R^*} \), the agent benefits from waiting another instant \( \Delta t \) and abandoning the firm afterwards, since the expected income flow is positive:

\[
(X_d - c + \sum_{z' \in \Omega_g} \pi(t) \cdot \lambda_{dg,z'}(Z) \cdot V(z', Z)) \cdot \Delta t \\
+ \lambda(Z, Z') \cdot e^{\phi(Z, Z')} \cdot v^R(\pi_t, Z')^+ \cdot \Delta t \\
= -V^R(\pi_t, Z) \frac{d\pi_t}{dt} \cdot \Delta t \\
> 0.
\]

This contradicts that \( \pi^{R^*} \) is an optimal cutoff. Since \( \left. \frac{\partial V^R(\pi, Z_t)}{\partial \pi} \right|_{\pi_t=\pi^{R^*}} > 0 \) violates optimization and since \( \left. \frac{\partial V^R(\pi, Z_t)}{\partial \pi} \right|_{\pi_t=\pi^{R*}(Z)} \) is weakly positive over the whole domain \( \pi \in [0, 1] \), it follows that \( \left. \frac{\partial V^R(\pi, Z_t)}{\partial \pi} \right|_{\pi_t=\pi^{R*}(Z)} = 0 \) must hold at the optimal cutoff \( \pi^{R*}(Z) \), given that \( \theta(Z) < 0 \).

### A.3. Proof of Proposition 3

**Learning & passage time.** The active investor’s signal arrival rate \( a(Z) \) is independent of the underlying hidden firm state \( (dg \ vs. \ dg) \) and thus does not alter the evolution of beliefs absent signals (that is, \( \theta(Z) \) is identical to the case without an active investor).
**Excess cash.** The excess cash position enters the value function linearly since the firm is free to pay out cash at any point in time, and since cash is invested at the risk-free rate, which is value-neutral.

**Informational externality.** Since trades by the active investor are effectively publicly observable and cannot generate profits, the active investor maximizes the value of her $\omega$-share of the firm’s equity by maximizing the total equity value. The active investor optimally immediately reveals information to the management and other shareholders. As the active investor has no incentive to propose policies that are not shareholder value maximizing, small investors optimally follow the active investor’s proposals. This ensures that, after bad news the firm is liquidated, and after good news, the firm is kept afloat, which is in the interest of both the active investor and small investors. As a result, small shareholders’ maximized objective is effectively a scaled version of the active investors’ maximized objective, where the scaling constant is $1/\omega$. Specifically, with an active investor, the firm’s total equity value in opaque distressed states is given by

$$V (\pi_t, Z_t, \kappa_t) = \max_{\{\tau^*, Div_{\tau}\}} \mathbb{E}_t \left[ \int_{t}^{\tau^*} \frac{\xi(Z_\tau)}{\xi(Z_t)} \left( X(z_\tau) - c + \frac{1}{\kappa_t} Div_{\tau}\right) d\tau \right] = v (\pi_t, Z_t)^+ + \omega \kappa_t.$$  

(33)

The corresponding Hamilton-Jacobi-Bellman equation yields the following set of ODEs that the function $v (\pi, Z)$ solves:

$$0 = X_d - c - r_f (Z) v (\pi, Z) + v_{\pi} (\pi, Z) \theta (Z) (1 - \pi) \pi + \sum_{z' \neq dg} \pi \lambda_{dg, z'} (Z) (V (z', Z) - v (\pi, Z))$$

$$- \sum_{z' \neq db} (1 - \pi) \lambda_{db, z'} (Z) v (\pi, Z) + a(Z) (\pi V_R (dg, Z) - v(\pi, Z))$$

$$+ \lambda (Z, Z') e^{\phi(Z, Z')} (v (\pi, Z') 1_{\{\pi \geq \pi^A (Z')\}} - v (\pi, Z)).$$

(34)

Given the above statements, the derivation of the boundary conditions is analogous to the proof of Proposition 2 in Appendix A.2.
References


