Financial markets with trade on risk and return

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Abstract

In this paper, we develop a model in which risk-averse investors trade on private information regarding both a stock's expected payoff and risk. These investors may trade in the stock and a derivative whose payoff is a function of the stock's risk. We study the role played by the derivative, finding that it is used to speculate on future risk and to hedge risk uncertainty. Unlike prior rational expectation models with derivatives, its price serves a valuable informational role, communicating investors' risk information. Finally, we find that the equity risk premium is directly tied to the derivative price.

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1 Introduction

Trade in derivatives whose values depend upon their underlying's risk, such as variance swaps, is on the rise. Investors appear to trade in these derivatives to speculate on private information regarding their underlying's volatility, and, as a result, their prices play a valuable informational role in the economy, communicating investors' information about risk. For example, ETFs tracking the VIX are now among the most actively traded securities in the market; trading volume in these ETFs is converging to that in the underlying S&P index itself. Furthermore, the price of the VIX has been termed the economy's "fear gauge," serving as a measure of the market's beliefs regarding macroeconomic risk.[1] Yet, trade in and pricing of these securities are difficult to explain using traditional models of trade based on private information (e.g., Grossman and Stiglitz (1980)), which assume that investors are perfectly aware of securities' risk. As a result, these models find that derivatives play only a risk-sharing role and that their equilibrium prices do not provide investors with information (Brennan and Cao (1996), Vanden (2006)). In this paper, we demonstrate the more active role played by derivative securities in a model in which risk-averse investors trade on private information on both the expected payoff and *the risk* of a stock.

In our model, investors face uncertainty regarding both the mean and variance of an equity's payoffs and possess diverse private information on each of these components, i.e., they each possess both "mean" and "risk" information. In particular, the "mean" information received by investors informs them regarding the first moment, or location parameter, of the equity's payoffs, while their "risk" information informs them regarding the second moment, or dispersion parameter, of the equity's payoffs. In addition to trading in the equity, investors may trade in a security whose payoff is exclusively a function of the riskiness of

[1] Support for these statements is found throughout the financial press. For example, see *The Fearless Market Ignores Perils Ahead* (April 2017, Financial Times), which discusses cases in which the VIX has been used to speculate on future risk and discusses the recent uptick in trade in the VIX. See also *The Snowballing Power of the VIX, Wall Street's Fear Index* (June 2017, Wall Street Journal), which states, "Invented 24 years ago as a way to warn investors of an imminent crash, the VIX has morphed into a giant casino of its own." Finally, note that VIX open interest reached an all-time peak of close to 700,000 contracts in February 2017. See https://ycharts.com/indicators/cboe_vix_futures_open_interest.

the equity, which we refer to as a variance derivative. This security is meant to capture trade in derivatives such as variance swaps whose value increases in volatility.[2] Our goal is to examine how investors trade on their two types of information in equilibrium and the role played by price of the variance derivative.

To begin, we study where investors trade on mean and risk information when they have access to both securities. We find that uncertainty over the equity's risk affects how investors trade on their information regarding expected payoffs. Prior models of trade with known equity risk demonstrate that investors trade on their beliefs about a stock's expected payoffs in equities but not derivatives (Brennan and Cao (1996), Cao and Ou-Yang (2009)). On the other hand, in the face of risk uncertainty, the variance derivative serves as a form of insurance against fluctuations in the riskiness of the stock's payoffs. When the riskiness of the stock's payoffs is high, a risk-averse investor who holds an equity position has heightened marginal utility with respect to their wealth.[3] Therefore, they have a desire to "hedge" by purchasing a security that pays off when equity risk is high; the variance derivative fills precisely this role. As a result, investors with optimistic mean information purchase the equity and hedge their positions by holding the variance derivative. Empirically, such hedging resembles the common practice of using derivatives as portfolio insurance, that is, taking positions in derivatives such as the VIX to protect against large losses.[4]

Next, we study the information provided by the prices of the equity and derivative securities in equilibrium. The model demonstrates that there is an additional driver of trade in the variance derivative that is not associated with trade in the equity market: trade on risk information. One might expect that a risk-averse investor with private information that suggests an equity is risky would downsize their position in this equity. However, in our model,

[2] As we discuss further in the text, the derivative may also be viewed as a heurtistic approximation to a zero-delta option position such as a straddle.

[3] More specifically, prudent preferences, i.e., those characterized by a utility function with a positive third derivative, exhibit this characteristic.

[4] Evidence from The Economist Intelligence Unit, 2012 suggests that 39% of institutional investors utilize portfolio hedging strategies. Also, see *The Snowballing Power of the VIX, Wall Street's Fear Index* (June 2017, Wall Street Journal) for evidence that the VIX is used as a hedging device, which states, "'Tail risk' strategies, designed to steer clear of sudden slumps, often rely on it [the VIX]."

this is not the case: instead, they hold their equity position fixed and purchase the variance derivative. Intuitively, when the investor trades on risk information in the derivative, the only risk they face is that their information signal regarding the equity's risk is inaccurate. If the investor were to trade on risk information in the equity itself, they would face both the risk that their information is inaccurate and the risk that the equity price moves against them. In sum, there are two components to the investor's demand in the variance derivative: a risk-uncertainty hedging component and a speculative risk-information component. Thus, unlike prior rational expectations models with known risk, our model suggests the derivative price serves a valuable informational role, enabling investors to learn about the underlying's risk. This suggests, for instance, that by serving as the economy's "fear gauge," the price of the VIX may in fact guide investors' trading decisions.[5]

Finally, we study how the price of the variance derivative is related to the equity price and trading volume in the two markets. The model suggests that the price of the variance derivative directly enters the risk premium in the equity market. Intuitively, the price of this derivative reflects the cost to hedging the risk uncertainty induced by a position in the equity. Therefore, when the derivative price is higher, investors are more reluctant to hold the equity, such that its risk premium rises. Moreover, a higher derivative price also leads to a reduction in trade in the equity and derivative markets. The reason is that investors become less willing to speculate on their information regarding expected payoffs when it is more costly to hedge the risk uncertainty that results from an equity position.

By analyzing the relationship between risk uncertainty and trade in equities and derivatives in a unified, information-based framework, our model offers insight into several empirical findings. First, trading volume in individual equity options tends to predict future equity returns (Easley, O'Hara, and Srinivas (1998), Pan and Poteshman (2006)) and trading volume in options *relative* to trading volume in their underlying equities varies cross-sectionally

[5]Beyond the VIX, individual equity options also appear to aggregate investors' private information regarding risk; see Mayhew and Stivers (2003), Poon and Granger (2005), Ni, Pan, and Poteshman (2008), and Fahlenbrach and Sandås (2010).

in features such as institutional ownership and liquidity (Roll, Schwartz, and Subramanyam (2010), Ge, Lin and Pearson (2016)). Our model suggests that these correlations might in part be explained by novel risk information received by investors. Specifically, we predict that trading volume in equities is associated with higher contemporaneous equity returns and lower contemporaneous derivative returns. Intuitively, increases in investors' perceptions of the equity's risk lead not only to a larger equity risk premium, but also to a reduction in investors' willingness to trade on their information. Furthermore, our theory offers novel predictions on the relationship between trade in derivative securities and the prices of equities and derivatives, suggesting that investors trade more in a derivative when its underlying's price is high and its own price is low.

Second, our model offers insight into the empirical relationship between disagreement and security returns, for which evidence is mixed (Diether, Malloy, and Scherbina (2002), Johnson (2004) and Goetzmann and Massa (2005), Carlin, Longstaff, and Matoba (2014)). Unlike prior rational expectations models that suggest disagreement amongst investors is not priced (Banerjee (2011), Lambert, Leuz, and Verrecchia (2012)),[6] our model predicts that disagreement regarding the security's expected payoffs reduces its equity price and increases derivative prices. The intuition is as follows. Investors' desires to hedge their equity positions in the derivative rise in the *magnitude* of their equity positions: both investors who are short and investors who are long the equity are exposed to variance risk and wish to utilize the derivative to hedge this risk. Since belief dispersion creates variation in investors' equity positions, this causes an increase in the demand for, and thus the price of, the derivative. Again, as the derivative price is directly related to the risk premium in the equity market, this increase in the derivative price leads to a decrease in the equity price.

[6] The analysis in Banerjee (2011) states that belief dispersion will increase expected returns in a noisy rational expectations setting. Note, however, that this is only the case when belief dispersion is created through a change in the precision of investors' information (see Proposition 1 of his paper). That is, the analysis he considers is not a ceteris paribus modification of belief dispersion, but rather, a change in the underlying information structure that creates belief dispersion. In his model, a ceteris paribus modification of belief dispersion would have no impact on prices: he states, "investor disagreement does not affect prices (while the average beliefs do) (pg. 38)."

Finally, variance swaps (and other securities whose payoffs increase in systematic volatility, including options) are priced at a premium, termed the variance risk premium (VRP) (e.g., Carr and Wu (2009)). In our model, a VRP may arise due to investors' desire to hedge risk uncertainty in the derivative market. In line with empirical evidence that demonstrates the size of the VRP predicts future equity returns (Bollerslev, Tauchen, and Zhou (2009)), our model predicts a deterministic relation between the VRP and future equity returns, as the cost to hedging the risk uncertainty induced by an equity position rises in the VRP. Moreover, we predict that dispersion in investors' equilibrium beliefs regarding expected future cash flows increases the VRP. Finally, we predict that the variance risk premium is negatively correlated with trading volume in the stock and derivative markets.

Related Literature. Prior rational expectations models have studied trade in options (Brennan and Cao (1996), Cao (1999), Vanden (2006)). In these models, options complete the market when investors have heterogenous information quality. However, investors take *deterministic* positions in options based on the precision of their information relative to the average precision of all investors and derivative prices provide no information to investors. There are two key differences between these papers and ours: first, these models focus on the case in which the riskiness of the securities' cash flows is known and the information possessed by investors orders their posteriors in the sense of first-order stochastic dominance.[7] Second, while these papers study options, which are affected by both the expected payoff to the underlying and its risk, the derivative in our model pays off purely as a function of risk.[8] Relatedly, Chabakauri, Yuan, and Zachariadis (2016) study a rational expectations model in which investors may trade in a full set of contingent claims, again finding that derivative securities are informationally irrelevant due to the assumptions placed on the type of information possessed by investors.

[7] While some prior literature has examined rational expectations models with non-normal distributions, and hence, signals that lead to updating on moments other than the first, even in these frameworks, signals order the posterior distributions in the sense of first-order stochastic dominance (e.g., Breon-Drish (2015a), Vanden (2008)).

[8] Nevertheless, note that in these prior models, investors use options to create a quadratic position in the underlying equity payoff. Thus, options are effectively used to create a payoff that increases in risk.

Another set of models has studied trade in options when investors disagree over the mean and/or variance of future cash flows (but face no *uncertainty* over the variance) (Detemple and Selden (1991), Buraschi and Jiltsov (2006), Cao and Ou-Yang (2009), Oehmke and Zawadowski (2015)). Most similar to our model, Cao and Ou-Yang (2009) study a setting in which investors agree to disagree about the mean and precision of a signal and may trade in a stock or options. They find that disagreements about the mean lead to stock but not option trade and disagreements over precision lead to trade in both markets, the inverse of our finding. This difference may be explained by the fact that in their model, the variance of cash flows is known and information is symmetric. This eliminates updating from derivative prices and the hedging component of investors' derivative demands, such that derivatives serve a different purpose.

Prior literature has examined private-information based trade in options in strategic risk-neutral settings (Back (1993), Biais and Hillion (1994), Cherian and Jarrow (1998), Easley, O'Hara, and Srinivas (1998), Nandi (2000)). Most similar to our paper, Cherian and Jarrow (1998) and Nandi (2000) study models of strategic trade in which an investor possesses risk information and trades in options. Note that because strategic investors are risk neutral in these models, uncertainty over risk has no impact on the equity price and affects options only through its impact on their expected payoffs. Consequently, these models have no role for the derivative as a hedging instrument and, unlike our model, find no relationship between the equity and derivative prices.

Other models offer non-information related reasons for why investors trade derivatives. For example, Leland (1980) demonstrates that derivative demand may arise from the relationship between investors' risk aversion and their wealth, and Franke, Stapleton, and Subrahmanyam (1998) demonstrate that derivative demand may result from unhedgable background risks. As we abstract from these forces, it is important that these other forces be taken into consideration in testing our model. Finally, our model is related to the rational expectations literature that studies information spillovers when investors face uncertainty about multiple components of a securities' risk and may trade in correlated securities (Goldstein, Li, and Yang (2014), Goldstein and Yang (2015)). We contribute to this literature by considering the case in which one component of a securities' risk is related to its variance.

2 Baseline Model

2.1 Assumptions

The model that we analyze is a one-period model of trade, in the spirit of Hellwig (1980) and Breon-Drish (2015a). As is typical, we assume that the economy is populated by a unit continuum of informed investors indexed on $[0,1]$ with CARA utility $u(W) = -e^{-\frac{W}{\tau}}$ and with wealth normalized to zero. Investors have access to a risk-free asset with payoff normalized to one that is in unlimited supply. Furthermore, they trade in a risky asset (the stock, or equity) that pays off a one-time dividend of $\tilde{x}$ at the end of the period, with per-capita supply of $\bar{z}$. We refer to the i^{th} trader's position in the stock as D_{Si}. There are three novel assumptions in the model. First, both the mean and variance of the stock's payoffs are unknown to investors: given the realizations of two independent random variables, $\tilde{\mu}$ and $\tilde{V}$, $\tilde{x}$ is normally distributed with mean $\tilde{\mu}$ and variance $\tilde{V}$: $\tilde{x}|\tilde{\mu}, \tilde{V} \sim N\left(\tilde{\mu}, \tilde{V}\right)$. It is natural to assume that the mean parameter is also Gaussian: $\tilde{\mu} \sim N\left(m, \sigma_{\mu}^{2}\right)$. However, as $\tilde{V}$ must be non-negative, it cannot be Gaussian; we allow $\tilde{V}$ to take any distribution with a non-negative support $\Upsilon \subseteq \Re^{+}$.

The second novel assumption of the model is that investors separately possess both "mean" information and "risk" information. Clearly, private information regarding $\tilde{\mu}$ concerns the stock's expected payoff, while private information regarding $\tilde{V}$ concerns the stock's risk. All informed traders receive information signals regarding $\tilde{\mu}$ and $\tilde{V}$ and traders rationally use the stock and derivative prices as additional signals.[9] The "mean" signal received

[9] The model is easily extended to the case in which some traders do not receive a variance signal. However, as we discuss later, all traders must have homogenous information precision regarding $\tilde{\mu}$ to ensure tractability.

by investor i equals $\tilde{\varphi}_i = \tilde{\mu} + \tilde{n} + \tilde{\varepsilon}_i$ where $\tilde{n} \sim N(0, \sigma_n^2)$ and $\tilde{\varepsilon}_i \sim N(0, \sigma_\varepsilon^2)$. The "risk" signal received by investor i equals $\tilde{\eta}_i = \tilde{V} + \tilde{\upsilon} + \tilde{e}_i$ where $\tilde{\upsilon} \sim N(0, \sigma_\upsilon^2)$ and $\tilde{e}_i \sim N(0, \sigma_e^2)$.[10] The noise terms $\tilde{n}$, $\tilde{\upsilon}$, $\tilde{\varepsilon}_i$, and $\tilde{e}_i$ are independent of the other variables in the model. Note that in the standard normal prior, normal likelihood set up found throughout the rational expectations literature, the variance of cash flows falls by a deterministic quantity that depends upon investors' information quality. By allowing investors to receive a signal that directly concerns the variance of cash flows, in our model, investors' posterior variance is now a random function that depends on the realized signal $\tilde{\eta}_i$.

The final novel assumption is that investors also trade a third security that has payoffs equal to the stochastic variance, $\tilde{V}$,[11] with per-capita supply of zero. It is natural that in the presence of an additional source of risk in the stock's payoffs and heterogenous information regarding this risk, a market would develop to trade the risk. We refer to this security as a variance derivative and refer to the i^{th} trader's position in the derivative as D_{Di}. The increase in market completeness obtained by the introduction of this security allows for the construction of a closed-form equilibrium stock price and investor demands conditional on P_D, which enables the study of several applications. In its absence, investors' demand functions can only be characterized implicitly. Moreover, in the absence of the variance derivative, investors would bet on both mean and risk in a single security, the stock, causing its price to reflect two distinct pieces of information; this would lead to a complex statistical updating problem.

Our approach to modeling the derivative deviates from prior literature, which studies derivatives with option-like payoffs, or payoffs that are a quadratic or logarithmic function of returns (Brennan and Cao (1996), Vanden (2006), Cao and Ou-Yang (2009)). In contrast,

[10] Note that while signals regarding $\tilde{V}$ may be negative, which may seem to contradict the fact that variances are non-negative, a signal $\tilde{\eta}_i$ is informationally equivalent to any signal $g(\tilde{\eta}_i)$ where g is invertible. Hence, we could define instead define the signal as $\tilde{\eta}'_i = e^{\tilde{\eta}_i}$ to obtain a signal which always takes on non-negative values.

[11] It is simple to accomodate the case in which $\tilde{y}$ also pays out the fixed component of the unconditional variance of $\tilde{x}$, σ_μ^2, but this adds complexity to the expressions for price and demand while offering no additional insight.

we assume that the derivative security's pay off equals the structural variance that generates the stock's payoffs. This raises the question of what such a security represents. We offer two interpretations. Most clearly, the variance derivative may be seen as a variance swap (e.g., VIX), i.e., a security that pays off proportional to the realized variance of its underlying's returns, defined as the sum-of-squared daily returns. Intuitively, if investors periodically receive noisy information regarding future cash flows, a higher underlying cash flow risk should manifest as variance in returns. To see this in a simple framework, consider an extension of the model in which the security's dividend is equal to the sum of N i.i.d. components, $\tilde{x} = \tilde{x}_{\frac{1}{N}} + \tilde{x}_{\frac{2}{N}} + ... + \tilde{x}_1$, where $\tilde{x}_{\frac{i}{N}} \sim N\left(0, \frac{\tilde{V}}{N}\right)$ for all $i \in \{1, ..., N\}$.[12] Moreover, suppose that after the trading period studied in our model, there are N periods; in period N, investors learn $\tilde{x}_{\frac{i}{N}}$. This set up is intended to capture the notion that investors periodically learn new, albeit imperfect, information regarding the terminal cash flow $\tilde{x}$. Finally, suppose that price in each period is a linear function of investors' expectations regarding $\tilde{x}$, and let the sum-of-squared returns, SSR, equal $SSR = \sum_{i=1}^{N} \left(P_{\frac{i}{N}} - P_{\frac{i-1}{N}}\right)^2$ where $P_{\frac{i}{N}}$ is the price of the security in the N^{th} period. Then, we have that:

$$SSR \propto \sum_{i=1}^{N} \tilde{x}^2_{\frac{i}{N}}. \tag{1}$$

Taking the limit as N approaches infinity, such that investors continuously receive very small amounts of information regarding the terminal dividend, this converges to $\tilde{V}$, i.e., the payoff to a variance swap as of the initial trading date, SSR, is precisely proportional to $\tilde{V}$.

Second, the variance derivative may be viewed as a heuristic approximation to a "zero-delta" option position such as a straddle, i.e., an option position that is affected by the magnitude but not direction of the price movement. Despite the fact that their payoffs are defined as a function of realized price, the expected payoffs to positions such as straddles and strangles increase in the riskiness of the stock's payoffs, $\tilde{V}$. Hence, by modelling the derivative's payoff as simply equal to $\tilde{V}$, we capture the essential element that the expected

[12] Notice that for simplicity of exposition, the mean of $\tilde{x}$ has been set to a known constant of zero.

payout to the derivative is greater when the variance of the stock's payoffs is larger. Ni et al. (2008) offer an empirical measure that corresponds to this interpretation of the derivative, measuring volatility-based trade in option positions by controlling for their sensitivity to directional price movements in their underlyings.[13]

In order to close the model and prevent the stock price from fully revealing investors' information, we introduce noise into the model by assuming that the investors have exogenous endowments of the two stochastic components relevant to the asset, $\tilde{\mu}$ and $\tilde{V}$. This is a natural extension of the assumption made in prior work such as Wang (1994) and Schneider (2009) to the case in which there are two traded assets with two independent sources of risk. Formally, assume that the endowment of trader i in $\tilde{\mu}$ is equal to $\tilde{Z}_{\mu i} = \tilde{z}_{\mu} + \tilde{z}_{\mu i}$ where $\tilde{z}_{\mu} \sim N\left(0, \sigma^2_{z_{\mu}}\right)$ and $\tilde{z}_{\mu i} \sim N\left(0, \sigma^2_{z_{\mu i}}\right)$ and assume that the endowment of trader i in $\tilde{V}$ is $\tilde{Z}_{Vi} = \tilde{z}_V + \tilde{z}_{Vi}$ where $\tilde{z}_V \sim N\left(0, \sigma^2_{z_V}\right)$ ad $\tilde{z}_{Vi} \sim N\left(0, \sigma^2_{z_{Vi}}\right)$. Assume that the endowments $\{\tilde{z}_{\mu}, \tilde{z}_V, \tilde{z}_{\mu i}, \tilde{z}_{Vi}\}$ are independent of each other and the other variables in the model.

2.2 Equilibrium

We begin by characterizing a rational expectations equilibrium. Denote by P_S the equilibrium price of the stock and by P_D equal the equilibrium price of the derivative. Let $\Phi_i = \left\{\tilde{\varphi}_i, \tilde{\eta}_i, \tilde{Z}_{\mu i}, \tilde{Z}_{Vi}, P_S, P_D\right\}$ represent investor i's information set. We analyze the standard definition of a rational expectations equilibrium:

Definition 1 *A rational expectations equilibrium is a pair of functions P_S, P_D such that investors choose their demands to maximize their utility conditional on their information*

[13] We note that the model accomodates the case in which the derivative has both "delta" and "vega." First, note that a security with both delta and vega can be roughly approximated by taking a position in both the equity and derivative in my model. Second, suppose that the derivative payoff was instead linear in $\tilde{x}$ as well as $\tilde{V}$, i.e., its pay off was $\alpha\tilde{x} + \beta\tilde{V}$ for some $\alpha \in \Re$ and $\beta > 0$. In this case, the expression for the equity price would be materially unchanged as a result of the fact that the derivative is, on average, in zero net supply. However, trading volume in the asset would be a function of investors' risk information, as they would trade in the asset to neutralize the delta provided by a position in the derivative. The derivative price would equal $\alpha P_S + \beta P_D$ where P_S and P_D are the equity and derivative prices in our model, respectively.

set:

$$\begin{aligned} & D_{Si}\left(\Phi_i\right), D_{Di}\left(\Phi_i\right) \\ \in \; & \underset{d_{Si}, d_{Di} \in \mathbb{R}}{\arg\max} E\left[-\exp\left(-\tau^{-1}\left(d_{Si}\left(\tilde{x}-P_S\right)+d_{Di}\left(\tilde{V}-P_D\right)+\tilde{Z}_{\mu i}\tilde{\mu}+\tilde{Z}_{Vi}\tilde{V}\right)\right)|\Phi_i\right] \end{aligned} \tag{2}$$

and, in all states, markets clear:

$$\begin{aligned} \int_0^1 D_{Si}\left(\Phi_i\right) di &= \bar{z} \\ \int_0^1 D_{Di}\left(\Phi_i\right) di &= 0. \end{aligned} \tag{3}$$

To derive the equilibrium, we proceed in three steps: (i) we solve for equity demands and the equity price for a fixed derivative price; (ii) we solve for the derivative demands and derivative price for fixed equity demands; (iii) we combine the two markets to show that there exists a rational expectations equilibrium, which solves a fixed point problem.

Note that given the equilibrium definition, the investors' demands in the stock and derivative are allowed to depend on both the derivative price and the stock price. As a result, it is possible that the stock and derivative prices each contain information on both $\tilde{\mu}$ and $\tilde{V}$. We specialize slightly further in the equilibria we consider. In particular, we consider only equilibria in which the derivative price does not reveal any information incremental to the stock price regarding $\tilde{\mu}$ and the stock price does not reveal any information incremental to the derivative price regarding $\tilde{V}$. Technically, we take the following approach. Let $F_{P_S}(\cdot)$ represent the distribution function of P_S and $F_{P_D}(\cdot)$ represent the distribution function of P_D. We conjecture an equilibrium in which the derivative price is conditionally independent of $\tilde{\mu}$ given the stock price, i.e., $F_{P_D}(\cdot|P_S,\tilde{\mu}) = F_{P_D}(\cdot|P_S)$ and the stock price is conditionally independent of $\tilde{V}$ given the derivative price, i.e., $F_{P_S}\left(\cdot|P_D,\tilde{V}\right) = F_{P_S}(\cdot|P_D)$. This implies that investors use the stock price to update on expected payoffs and the derivative price to update on the riskiness of payoffs. We then show that given such a conjecture, the equilibrium stock price and derivative price indeed satisfy $F_{P_D}(\cdot|P_S,\tilde{\mu}) = F_{P_D}(\cdot|P_S)$ and $F_{P_S}\left(\cdot|P_D,\tilde{V}\right) = F_{P_S}(\cdot|P_D)$, demonstrating the existence of such an equilibrium. In fact, one needs only to conjecture that one of these two properties holds, and the other will follow in equilibrium. However, we have not been able to rule out the possibility of other equilibria.

Beginning with the stock market, we follow the standard procedure of conjecturing a linear equilibrium,

$$P_S = \tilde{\alpha}_0 + \alpha_\mu(\tilde{\mu}+\tilde{n}) + \alpha_z\tilde{z}_\mu, \tag{4}$$

where, by the conjecture that $F_{P_S}\left(\cdot|P_D,\tilde{V}\right) = F_{P_S}(\cdot|P_D)$, $\tilde{\alpha}_0$ may depend upon $\tilde{V}$ only through P_D, and hence is known to investors. The following proposition summarizes the equilibrium equity demands and equity price for a given derivative price P_D. In the appendix, we derive expressions for α_0, α_μ, and α_z.

Proposition 1 *The investors' equity demands and the equilibrium equity price given a derivative price P_D satisfy:*

$$\begin{aligned} D_{Si} &= \tau\frac{E(\tilde{x}|\Phi_i)-P_S}{P_D+Var(\tilde{\mu}|\Phi_i)} - \frac{Var(\tilde{\mu}|\Phi_i)\tilde{Z}_{\mu i}}{P_D+Var(\tilde{\mu}|\Phi_i)} \text{ and} \\ P_S &= \int_0^1 E(\tilde{x}|\Phi_i)\,di - \frac{1}{\tau}\tilde{z}_\mu Var(\tilde{\mu}|\Phi_i) - \frac{1}{\tau}\bar{z}\left(P_D+Var(\tilde{\mu}|\Phi_i)\right). \end{aligned} \tag{5}$$

To understand the expression for investors' demands, note that in the classical mean-variance framework with known variance and no derivative security, their demands would equal their expected payoff less price divided by the variance, less their endowment: $\tau\frac{E(\tilde{x}|\Phi_i)-P_S}{Var(\tilde{x}|\Phi_i)} - \tilde{Z}_{\mu i}$. In the present setting, we again have a numerator equal to the expected payoff minus price and the investors' demands are adjusted for their endowment of $\tilde{\mu}$. However, the denominator, which captures the investor's adjustment for risk, is now modified as there are two components of risk when trading in the stock: that of the uncertain mean, $\tilde{\mu}$, and the stochastic variance, $\tilde{V}$. As is the case in the classical framework, the variance of the uncertain mean component is added to the denominator since it follows a normal distribution.

On the other hand, to account for the riskiness of the stochastic variance $\tilde{V}$, the denominator includes the price of the derivative security P_D, which in general is not simply equal to expected risk $E\left(\tilde{V}\right)$.

To provide an intuition for why investors discount the risk associated with $\tilde{V}$ at the price of the derivative security, consider investor i's expected utility conditional on $\tilde{V}$ when their demands are (D_{Si}, D_{Di}) and their outside endowments are zero:

$$\begin{aligned} & E\left\{-\exp\left[-\tau^{-1}\left(D_{Si}\left(\tilde{x}-P_S\right)+D_{Di}\left(\tilde{V}-P_D\right)\right)\right]|\Phi_i,\tilde{V}\right\} \qquad (6) \\ = & -\exp\left\{-\tau^{-1}\left[D_{Si}\left(E\left(\tilde{\mu}|\Phi_i\right)-P_S\right)+D_{Di}\left(\tilde{V}-P_D\right)-\tau^{-1}\frac{D_{Si}^2}{2}\left(Var\left(\tilde{\mu}|\Phi_i\right)+\tilde{V}\right)\right]\right\}. \end{aligned}$$

Notice that the investor's expected utility is decreasing in a linear combination of the payoff to their derivative position, $D_{Di}\left(\tilde{V}-P_D\right)$, and the riskiness of the equity position, $D_{Si}^2\tilde{V}$. As a result, the exposure to $\tilde{V}$ created by a position in the stock, $\tau^{-1}D_{Si}^2\tilde{V}$ can effectively be hedged by taking a position in the derivative, which comes at the price of P_D. When P_D rises, this exposure becomes costlier to hedge, and hence, investors will treat the stock as though it were riskier. The proof provided in the appendix demonstrates that this intuition continues to hold upon taking the expectation over $\tilde{V}$ and upon accounting for investors' random endowments.

Importantly, the equity demands D_{Si} are not directly a function of investors' risk signals $\tilde{\eta}_i$, given the conjecture that $F_{P_D}\left(\cdot|P_S,\tilde{\mu}\right)=F_{P_D}\left(\cdot|P_S\right)$. Thus, despite the fact that investors' risk signals provide them with information regarding the riskiness of the stock, they choose not to take into account these signals $\tilde{\eta}_i$ when trading in the stock market. As a result, the conjecture that the stock price is informationally redundant with respect to $\tilde{V}$ is verified. Note that the following corollary does not imply that the stock market and derivative markets function independently. Instead, the corollary only states that investors' risk information can only affect the equity price through the price of the derivative, P_D.

Corollary 1 *The stock price is informationally redundant with respect to $\tilde{V}$. That is,*

$F_{P_S}\left(\cdot|P_D,\tilde{V}\right)=F_{P_S}\left(\cdot|P_D\right)$.

With the equilibrium in the stock market established, now consider the equilibrium derivative price for fixed equity demands, $\{D_{Si}\}_{i\in[0,1]}$. As the distribution of a variance must be bounded below by zero, the distribution of the payoff of the derivative cannot be assumed normal. To make the model as general as possible, we allow for an arbitrary distribution of $\tilde{V}$. In order to derive the rational expectations equilibrium in this general case, we apply the approach of Breon-Drish (2015b), summarized below.

First, conjecture a *generalized* linear equilibrium, i.e., one in which price is a monotonic transformation of a linear function of investors' aggregate information signal $\tilde{V}+\tilde{v}$ and their aggregate endowment $\tilde{z}_V$. That is, start by conjecturing $P_D=\delta\left(l\left(\tilde{V}+\tilde{v},\tilde{z}_V\right)\right)$ where $l\left(\tilde{V}+\tilde{v},\tilde{z}_V\right)=a\left(\tilde{V}+\tilde{v}\right)+\tilde{z}_V$ for some $a\in\Re$ to be determined as part of the equilibrium and a strictly increasing function δ. Given this conjecture, investors are able to invert $l\left(\tilde{V}+\tilde{v},\tilde{z}_V\right)$ from the derivative price. Due to the fact that the additive error terms in $\tilde{\eta}_i$ and $\tilde{l}$ are normally distributed, $\tilde{\eta}_i$, $\tilde{l}$, and $\tilde{Z}_{Vi}$ are normally distributed conditional on $\tilde{V}$. This implies that the distribution of $\tilde{V}$ given $\left(\tilde{\eta}_i,\tilde{l},\tilde{Z}_{Vi}\right)$ falls into the exponential family of distributions with the following form:

$$\begin{aligned} dF_{\tilde{V}|\tilde{\eta}_i,\tilde{l}} &= \exp\left[\left(k_1(a)\,\tilde{\eta}_i+k_2(a)\,\tilde{l}+k_3(a)\,\tilde{Z}_{Vi}\right)\tilde{V}\right. \\ &\quad \left.-g\left(k_1(a)\,\tilde{\eta}_i+k_2(a)\,\tilde{l}+k_3(a)\,\tilde{Z}_{Vi}\right)\right]dH\left(\tilde{V};a\right) \end{aligned} \tag{7}$$

for some functions $k_1(a)$, $k_2(a)$, $k_3(a)$, $g(\cdot)$, and $H\left(\tilde{V};a\right)$. When investors have CARA utility and the distribution of payoffs takes this form, their demands are additively separable in their private signal $\tilde{\eta}_i$, the price signal $\tilde{l}$, their endowment $\tilde{Z}_{Vi}$, and a monotonic function of P_D. By examining the market-clearing condition, it can be seen that P_D indeed takes the generalized linear form, $\delta\left(l\left(\tilde{V}+\tilde{v},\tilde{z}_V\right)\right)$. The next proposition summarizes these results.

Proposition 2 *Suppose that investor i demands D_{Si} units of the stock. Then, investor i's* *derivative demand equals:*

$$D_{Di} = k_1 (a^*) \tilde{\eta}_i + k_2 (a^*) \tilde{l} + k_3 (a^*) \tilde{Z}_{Vi} - g'^{-1} (P_D) + \frac{1}{2\tau} D_{Si}^2, \tag{8}$$

where $k_1 (\cdot)$, $k_2 (\cdot)$, $k_3 (\cdot)$, $g (\cdot)$, and a^* are defined in the appendix. The equilibrium derivative price P_D may be written:

$$P_D = g' \left[r (a^*) \left(a^* \left(\tilde{V} + \tilde{v} \right) + \tilde{z}_V \right) + \frac{1}{2\tau} \int_0^1 D_{Si}^2 di \right]. \tag{9}$$

for a function $r(\cdot)$ defined in the appendix. P_D is positive, increasing in $\tilde{V}$ and $\tilde{v}$, and decreasing in $\tilde{z}_V$.

Critically, investors' derivative demands contain a new component that reveals an important interaction between the two markets. In particular, investor i's demand is increasing in the square of their position in the stock, $\frac{1}{2\tau} D_{Si}^2$. This arises from the investors' desire to hedge risk uncertainty: since the investors' utility functions have a positive third derivative, they prefer skewed payoff distributions (Kimball (1990)). By purchasing the derivative, an equity investor has a greater level of wealth when they face more risk, leading to payoff skewness.[14] Note that this hedging component of investors' derivative demands lead both investors who short the stock and investors who long the stock hold positions in the derivative. Moreover, it creates a link between the derivative price and investors' equity demands through the aggregate desire to hedge, $\int_0^1 D_{Si}^2 di$.

Although investors' equity demands affect the derivative price and these demands are affected by their private information and endowments, $\tilde{\varphi}_i$ and $\tilde{Z}_{\mu i}$, the derivative price is nevertheless informationally redundant with respect to $\tilde{\mu}$. Substituting the equity price in Proposition 1 into investors' equity demands, we find that their equity demands are linear

[14] See footnote 18 in Eeckhoudt and Schlesinger (2006) for a discussion of why this holds even for CARA utility, which is generally interpreted as having a preference for risk that is independent of wealth. The notion of preferences across distributions is distinct from the Arrow-Pratt measure of risk aversion, which assesses how much an investor is willing to pay to eliminate a risk at any given wealth level. The Arrow-Pratt measure also takes into account an investor's marginal utility at a given wealth level.

in the difference between their private mean signal ($\tilde{\varphi}_i$) and the average mean signal ($\tilde{\mu}$) and in their mean-zero idiosyncratic endowments $\tilde{z}_{\mu i}$. As a result, the aggregate risk-uncertainty hedging demand of informed investors for the derivative, $\int_0^1 D_{Si}^2 di$, is a function only of $\tilde{\mu}$ only through the term $\int_0^1 (\tilde{\varphi}_i - \tilde{\mu})^2 \, di$ and a function of $\tilde{Z}_{\mu i}$ only through $\int_0^1 \tilde{z}_{\mu i}^2 di$. It is easily checked that $\int_0^1 (\tilde{\varphi}_i - \tilde{\mu})^2 \, di$ and $\int_0^1 \tilde{z}_{\mu i}^2 di$ depend upon investors' information quality and endowment noise but not on $\tilde{\mu}$ or $\tilde{z}_{\mu i}$ themselves. In sum, we have the following result:

Corollary 2 *The derivative price is informationally redundant with respect to $\tilde{\mu}$. That is, $F_{P_D}(\cdot|P_S, \tilde{\mu}) = F_{P_D}(\cdot|P_S)$.*

This result also demonstrates that the derivative price would perfectly reveal investors' aggregate risk signal, $\int_0^1 \tilde{\varphi}_i di = \tilde{V} + \tilde{v}$, in the absence of noise in investors' endowments of $\tilde{V}$. One might posit that investors' risk-uncertainty hedging demands would serve as a form of noise trade in the derivative markets, obviating the need to directly introduce additional noise into the derivative market. However, this is not the case, as investors' *aggregate* hedging demands are unaffected by $\tilde{\mu}$ and $\tilde{z}_\mu$. Specifically, notice that expression (9) demonstrates that in the absence of endowment noise, $\tilde{z}_\mu$, we would have $P_D = g' \left[r(a^*) \, a^* \left(\tilde{V} + \tilde{v} \right) + \frac{1}{2\tau} \int_0^1 D_{Si}^2 di \right]$. As $\int_0^1 D_{Si}^2 di$ is independent of $\tilde{\mu}$ and $\tilde{z}_\mu$, this implies that the derivative price could be inverted to derive $\tilde{V} + \tilde{v}$.

Now that the two markets have been examined in isolation, taking the price and demands in the other market as fixed, we consider both markets in tandem and show that there exists an equilibrium.

Proposition 3 *There exists a rational expectations equilibrium P_S, P_D.*

The existence of an equilibrium boils down to the existence of a solution to a fixed-point problem; the nature of this problem is depicted in Figure 1. Expression (5) shows that the aggregate risk-uncertainty hedging demand of investors, $\int_0^1 D_{Si}^2 di$, is a decreasing function of the derivative price, P_D. Intuitively, when P_D is larger, investors are more reluctant to

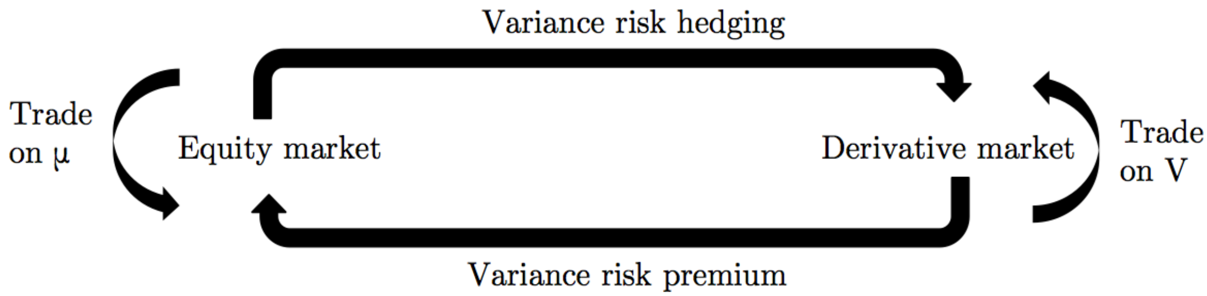

Figure 1: This figure depicts the interplay between the equity and the variance derivative markets, given that the derivative insures against fluctuations in $\tilde{V}$ induced by a position in the equity. Trade in the equity affects the derivative price through investors' desire to hedge risk uncertainty. This in turn, affects the equity market through the risk premium associated with $\tilde{V}$.

trade on their information because it is costlier to hedge the risk uncertainty that results from a directional equity position. In other words, when P_D is higher, the investor is more averse to the variance risk induced by an equity position; we thus refer to this effect in the figure as the variance risk premium (note we discuss the variance risk premium further in section 2.5). Simultaneously, the derivative price is itself a function of $\int_0^1 D_{Si}^2 di$, such that finding an equilibrium requires solving for a fixed point P_D. Note that as in Wang (1994) and Schneider (2008), the assumption that noise in the model stems from noisy endowments rather than noise trade works against equilibrium uniqueness; that is, the possibility of multiple equilibria stems purely from the standard assumptions on endowment noise, rather than risk uncertainty, the inclusion of a variance derivative, or risk information.[15]

Note that in our model, investors face two components of risk in a single security, the equity. It is insightful to compare our model to prior rational expectations models with multidimensional risk. Goldstein, Li, and Yang (2014) study a model in which investors trade in two related securities with a common component affecting the expected returns to both securities. Goldstein and Yang (2015) study a model in which investors have information on two components of a security's cash flow. These papers demonstrate that uncertainty and

[15] We have solved the model under the assumption of fixed endowments and noise trade and found a unique equilibrium. However, the required assumptions on the behavior of noise traders to ensure the existence of a tractable equilibrium are *ad hoc*.

learning regarding one factor tends have spillover effects on the other factor.

Unlike these papers, the two components of uncertainty in our model affect different moments of the cash flow distribution. This creates new economic forces that do not appear in the set up in which each component affects expected payoffs: the realization of investors' information regarding risk affects both the risk premium and investors' trading intensities in the second security. Thus, while Goldstein, Li, and Yang (2014) document that uncertainty regarding one component of cash flows has spillover effects on investors' trading intensities in both markets, our model suggests that the *price* of the variance derivative itself affects how intensely investors trade on their information and causes the derivative price to enter the equity's risk premium (i.e., cost of capital). These forces are crucial to the applications studied below, leading to relationships between price changes and trading volume in both markets.

2.3 Trading volume and price changes

In this section, we analyze the relationships between trading volume and price changes within and across equity and derivative markets. To summarize the results, note that Propositions 1 and 2 reveal three key relationships: i) the risk premium in the equity market increases in the derivative price, ii) investors' willingness to trade on their mean-based information in the equity decreases in the derivative price, and iii) when investors trade more in the equity, they also trade more in the derivative to hedge risk uncertainty. Analyzing these effects jointly, increases in the derivative price and reductions in the equity price are negatively associated with trading volume in the equity and derivative.

Critically, these results are founded on 1) investor risk aversion and 2) a derivative security that pays off purely as a function of its underlyings' volatility. Therefore, they apply most directly to trade in the VIX (and similar securities), which possesses risk that is primarily systematic in nature and pays off purely as a function of realized volatility. However, note that our results also provide insight into option trade that is driven by a demand for exposure to volatility; again this has been measured by controlling for option "delta" (Ni et al. (2008)). An empirical literature has studied trade in options versus stocks, arguing that the relative trading volume in each security tends to be driven by concerns for leverage and liquidity (e.g., Easley et al. (1998), Pan and Poteshman (2006), Ni et al. (2008), Fahlenbrach and Sandås (2010), Roll et al. (2010), and Johnson and So (2012)). Our model suggests that the correlations discovered in these papers might in part reflect innovations to risk and risk information, which jointly impact trading volume and prices.

We begin by characterizing trading volume in the two markets. Formally, we define trading volume in the stock and derivative as the total differences between investors' equilibrium demands and the average endowments. That is, let trading volume (double counted) in the stock equal $Vol_S \equiv \int_0^1 \left| D_{Si} - \int_0^1 D_{Si} di \right| di$ and trading volume in the derivative equal $Vol_D \equiv \int_0^1 \left| D_{Di} - \int_0^1 D_{Di} di \right| di$. The next proposition formally characterizes volume in the two markets.

Proposition 4 *Trading volume in the stock market is equal to:*

$$Vol_S = \tau \int_0^1 \left| \frac{E(\tilde{x}|\Phi_i) - \int_0^1 E(\tilde{x}|\Phi_i) di - \tau^{-1} Var(\tilde{\mu}|\Phi_i) \tilde{z}_{\mu i}}{P_D + Var(\tilde{\mu}|\Phi_i)} \right| di \tag{10}$$

and trading volume in the derivative market is equal to:

$$Vol_D = \int_0^1 \left| \tau k_1(a^*) \tilde{e}_i + \frac{1}{2\tau} \left(D_{Si}^2 - \int_0^1 D_{Si}^2 di \right) + (\tau k_3(a^*) - 1) \tilde{z}_{Vi} \right| di. \tag{11}$$

Volume in the stock market has two components, a component related to speculation on investors' beliefs regarding $\tilde{\mu}$, $\int_0^1 \left| E(\tilde{\mu}|\Phi_i) - \int_0^1 E(\tilde{\mu}|\Phi_i) di \right| di$, and a risk-sharing component, $\tau^{-1} Var(\tilde{\mu}|\Phi_i) \tilde{z}_{\mu i}$. Importantly, note that investors' willingness to trade on their information is disciplined by their assessment of the riskiness of the stock's payoffs, as measured by $P_D + Var(\tilde{\mu}|\Phi_i)$. This reveals an interaction between the stock and derivative markets: any force that raises the derivative price also leads to a reduction in trade in the stock market. For instance, innovations to $\tilde{V}$, $\tilde{v}$, or $\tilde{z}_V$ reduce trade in the stock market. Likewise, changes in investors' risk-information quality or changes in the distribution of $\tilde{V}$ that cause P_D to increase also reduce trade in the stock market.

Next, Proposition 4 shows that trading volume in the variance derivative has three components, a risk-information component, $\tau k_1(a^*)\tilde{e}_i$, a risk-uncertainty hedging component, $\frac{1}{2\tau}\left(D_{Si}^2 - \int_0^1 D_{Si}^2 di\right)$, and a risk-sharing component, $(\tau k_3(a^*) - 1)\tilde{z}_{Vi}$. Risk-information related trade is driven by the deviation between the signal received by investor i, $\tilde{\eta}_i$, and the average signal received by investors, $\int_0^1 \tilde{\eta}_i di = \tilde{\upsilon}$, that is, $\tilde{e}_i$. Risk-uncertainty hedging related trade results from differences in investors' equilibrium beliefs regarding $\tilde{\mu}$. Such differences in beliefs lead investors to hold disparate positions in the stock. Investors who hold large positions in the stock hedge the resulting variance uncertainty by purchasing the derivative from investors with smaller positions in the stock. Finally, variance-risk sharing trade simply captures investors' wish to trade away their idiosyncratic outside endowments of $\tilde{V}$.

Note that because the derivative price impacts trading volume in the equity, it also has a spillover effect on trading volume in the derivative market. Since investors' desires to hedge risk uncertainty are driven by their equity positions, a reduction in trade in the equity market also leads to a reduction in the risk-uncertainty hedging component of derivative trade. Therefore, the derivative price P_D is *negatively* associated with trade in both markets. Finally, because the derivative price enters the equity *risk premium*, this suggests that the equity price is *positively* associated with trade in both markets. In sum, we have the following corollary.

Corollary 3 *i) Increases in the derivative price driven by greater $\tilde{V}$ or $\tilde{\upsilon}$, or smaller $\tilde{z}_V$, changes in the parameters $\sigma_{z_V}^2$, $\sigma_{z_{Vi}}^2$, σ_{υ}^2, or σ_e^2, or in the distribution of $\tilde{V}$ lead to a decline in trade in the stock.*
ii) Increases in the derivative price driven by greater $\tilde{V}$ or $\tilde{\upsilon}$, or smaller $\tilde{z}_V$ lead to a decline in trade in the variance derivative.

The first part of the corollary states that any change in the underlying information structure of the model or the realized level of risk and noise in the derivative security decreases trading volume in the equity market. The second part of the corollary is weaker: it states only that changes in the derivative price driven by the shocks $\tilde{V}$, $\tilde{v}$, and $\tilde{z}_V$ decrease trade in the derivative. Intuitively, the information structure affects trade in the derivative market in two, potentially opposing ways: directly, through the risk-information component of trade, and indirectly, through its impact on investors' willingness to trade on their mean information. This renders comparative statics of derivative trade with respect to the information structure difficult. In contrast, increases in the derivative price driven by $\tilde{V}$ or $\tilde{z}_V$ affect the risk-uncertainty hedging component of trading volume only, and hence, definitively lead to a reduction in derivative volume.

This corollary has immediate implications for the covariance between trading volume in equities and derivatives and price changes in the two markets. Formally, suppose that prior to trade, there exist initial prices $P_{S,0}$ and $P_{D,0}$ and define $\Delta P_S \equiv P_S - P_{S,0}$ and $\Delta P_D \equiv P_D - P_{D,0}$. Then, we have the following result.

Corollary 4 *The statistical relationship between contemporaneous returns in the stock and derivative and trading volume in the two markets can be summarized as follows:*

i) $Cov\left(\Delta P_S, Vol_S\right) > 0$*,*

ii) $Cov\left(\Delta P_S, Vol_D\right) > 0$*,*

iii) $Cov\left(\Delta P_D, Vol_S\right) < 0$*,*

iv) $Cov\left(\Delta P_D, Vol_D\right) < 0$*.*

2.4 Belief Dispersion and Prices

A well-documented result is that in a perfectly competitive rational expectations equilibrium, only the average expectation of payoffs across investors and the average precision of investors' beliefs affect expected returns (e.g., Banerjee (2011), Lambert, Leuz, and Verrecchia (2012)). The thought experiment posed by these studies is as follows. Consider a change in the underlying parameters of the model, such as the quality of investors' private information and the extent of noise trade, that causes investors' beliefs and equilibrium demands to diverge, but leads to no change in the average quality of their information. These studies show that under the standard utility and distributional assumptions made in noisy rational expectations models, the risk premium is purely a function of the precision of investors' posteriors, and hence, prices will not change, on average. Our model suggests that in the presence of uncertainty over a security's risk, greater differences in investors' equilibrium beliefs lead to decreases in its equity price and increases in the price of the variance derivative.

Our results have implications for the empirical literature that studies the effect of disagreement on individual equity prices, which documents mixed results on the relationship between disagreement and equity prices depending upon the empirical proxy used to capture disagreement (Diether, Malloy, and Scherbina (2002), Johnson (2004) and Goetzmann and Massa (2005), Carlin, Longstaff, and Matoba (2014)). Our model suggests that investors' risk preferences alone may lead to a positive relationship between disagreement and future returns both in equity markets and derivative markets. Furthermore, empirically, our results suggest that the relationship between disagreement and future returns to a security should rise as investors face more uncertainty regarding that securities' risk.

Expression (9) demonstrates that the price of the variance derivative (and thus the risk premium in the stock market) increases in the aggregate *squared* demands of investors in the equity market, $\int_0^1 D_{Si}^2 di$. This term reflects the aggregate desire of investors to hedge the risk uncertainty created by their positions in the stock. Define the dispersion in investors' beliefs regarding the expectation of $\tilde{x}$ as follows (e.g., Banerjee (2011)):

$$BeliefDispersion \equiv \int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - \int_0^1 E\left(\tilde{x}|\Phi_i\right) di \right)^2 di. \tag{12}$$

Then, upon simplifying $\int_0^1 D_{Si}^2 di$, we find that:

$$\int_0^1 D_{Si}^2 di = \left(\frac{\tau}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \right)^2 BeliefDispersion + Var\left(\tilde{\mu}|\Phi_i\right)^2 \sigma_{z_{\mu i}}^2 + \bar{z}^2, \tag{13}$$

i.e., $\int_0^1 D_{Si}^2 di$ increases in the dispersion in investors' equilibrium beliefs. Intuitively, when investors' beliefs grow disparate, optimistic investors increase their positions in the stock market and pessimistic investors decrease their positions. As investors' desire to hedge risk uncertainty is a function of their squared demand for the stock, both types of investors have a greater desire to hedge, and thus, the derivative price increases. Since this raises the cost to hedging the risk uncertainty that accompanies a position in the equity, this increases the equity risk premium. Note that in equilibrium, variation in investors' beliefs holding fixed their average precision may stem from variation in the underlying parameters σ_{μ}^2, σ_{ε}^2, $\sigma_{z_{\mu}}^2$, and $\sigma_{z_{\mu i}}^2$. In sum we have the following proposition:[16]

Proposition 5 *Holding fixed the precision of investors' beliefs regarding $\tilde{\mu}$, the derivative price P_D increases and the expected stock price $E(P_S)$ decreases in the dispersion in investors' equilibrium beliefs regarding the mean of $\tilde{x}$, $\int_0^1 \left(E(\tilde{x}|\Phi_i) - \int_0^1 E(\tilde{x}|\Phi_i)\, di \right)^2 di$.*

2.5 Variance Risk Premium

A large body of empirical evidence demonstrates that, at both the index and individual equity level, variance swaps have prices that overshoot investors' expectations of the underlying return variance. The size of this premium has been termed the variance risk premium (VRP) (e.g., Bakshi and Kapadia (2003), Bollerslev, Tauchen, and Zhou (2009), Carr and Wu (2009)). Existing theory argues that the index-level VRP results from variance swaps' negative betas: when realized market variance is high, returns tend to be low (Carr and Wu (2009)). Alternatively, Bakshi and Madan (2006) argue that the VRP may arise from investors' preferences for higher moments. In line with the latter explanation, in our model, a variance risk premium can arise due to investors' preference for skewness; the variance derivative pays off when risk is high, such that holding the derivative alongside the equity creates positive skewness in the investors' wealth distribution. In our model, the size of this premium is related to the dispersion in investors' beliefs regarding $\tilde{\mu}$, trading volume in

[16] Note that we have also solved the model in the case in which investors have exogenous variation in their prior beliefs regarding the mean of $\tilde{x}$. Variation in prior beliefs also is priced, unlike in prior models. These results are available upon request.

the equity and derivative markets, and the equity price. Note that our model speaks most strongly to index-level variance risk premia, where investors' risk aversion plays a greater role.

Specifically, we define the VRP as -1 times the average over investors' expectations of the dollar returns to the variance derivative, i.e.,

$$VRP \equiv \int_0^1 E\left(P_D - \tilde{V}|\Phi_i\right) di. \tag{14}$$

This definition captures the difference between how investors price the variance derivative $\tilde{V}$ and their average expectation of the future variance. As discussed in section 2.1, the variance derivative's price may be seen as the price of a variance swap. Moreover, the average investor belief regarding $\tilde{V}$, $\int_0^1 E\left(\tilde{V}|\Phi_i\right) di$, should, in expectation, approximate the ex-post realized variance $\tilde{V}$. Thus, the definition corresponds closely to the empirical measures found in Carr and Wu (2009) and Bollerslev, Tauchen, and Zhou (2009), which equal the price of the variance swap less the ex-post realized return variance. Note that under this definition, a higher VRP corresponds to a greater *excess valuation* of the variance derivative. That is, unlike the risk premium in the equity market, which tends to be negative, the VRP is generally positive.

First, note that section 2.4 suggests the price of the variance derivative is related to investors' disagreement over $\tilde{\mu}$ in equilibrium, $\int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - \int_0^1 E\left(\tilde{x}|\Phi_i\right) di\right)^2 di$. Therefore, the VRP increases in investors' equilibrium disagreement over $\tilde{\mu}$. Interestingly, this suggests that despite the fact that $\tilde{\mu}$ has no direct impact on investors' beliefs regarding $\tilde{V}$, the VRP is affected by investors' private information quality regarding $\tilde{\mu}$ (σ_n^2 and σ_ε^2), the amount of noise in investors' endowments of $\tilde{\eta}$ ($\sigma_{z_\mu}^2$ and $\sigma_{z_{\mu i}}^2$), and prior uncertainty over $\tilde{\mu}$ (σ_μ^2). Intuitively, disagreement regarding $\tilde{\mu}$ indirectly lead them to hold different quantities of $\tilde{V}$ in equilibrium, which affects the VRP.

Second, the model implies a negative relationship between the variance risk premium and trading volume in the two markets. In particular, Corollary 4 implies that when the derivative price is higher (corresponding to a higher VRP), investors are more averse to the risk uncertainty that results from a position in the stock market. This makes them less willing to trade on their information regarding $\tilde{\mu}$, such that trading volumes in both the stock and derivative are lower.

Finally, the model also suggests a connection between the variance risk premium and future equity returns, which has been empirically validated (Bollerslev, Tauchen, and Zhou (2009)). This follows trivially from expression (5), which shows that the risk premium in the stock market increases with P_D. Effectively, both the VRP and the equity price capture investors' aversion to variance uncertainty. In summary, we have the following proposition.

Proposition 6 *i) The variance risk premium increases in investors' belief dispersion regarding* $\tilde{\mu}$, $\int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - \int_0^1 E\left(\tilde{x}|\Phi_i\right) di\right)^2 di$.
ii) The variance risk premium is negatively correlated with trading volume in the stock and derivative markets.
iii) The equilibrium equity price decreases one-for-one with increases in the variance risk premium.

As a final point, note that the one-for-one relationship between the VRP and the equity price implies that even when driven by noise, increases in the derivative price lead to an increase in the equity risk premium. When investors' noisy endowments of $\tilde{V}$ are greater, they must hold more of $\tilde{V}$ in equilibrium, which causes them to demand a greater VRP. If the investors had mean-variance preferences rather than CARA utility, their demands for the stock would be unaffected by their endowments in $\tilde{V}$, because the stock has a covariance of zero with $\tilde{V}$: $Cov\left(\tilde{x}, \tilde{V}\right) = 0$.[17]

[17] To see this, note that, applying $E\left(\tilde{x}\right) = E\left(\tilde{\mu}\right)$ and the independence of $\tilde{\mu}$ and $\tilde{V}$,

$$\begin{aligned} E\left(\tilde{x}\tilde{V}\right) &= E\left(E\left(\tilde{x}\tilde{V}|\tilde{\mu}\right)\right) \\ &= E\left(\tilde{\mu}\tilde{V}\right) \\ &= E\left(\tilde{x}\right)E\left(\tilde{V}\right). \end{aligned} \tag{15}$$

3 Conclusion

This article develops a noisy rational expectations model in which risk-averse investors possess information not only on a stock's expected payoffs, but also the risk of these payoffs. Investors in the model can trade in a stock or a derivative security whose value increases in the riskiness of the stock's payoffs. In the equilibrium studied in the model, the stock price serves as an aggregator of investors' mean information regarding and the derivative price serves as an aggregator of investors' risk information. Investors trade on information regarding expected payoffs in both the stock and derivative markets, as the derivative serves as insurance against adverse fluctuations in the risk of the stock's payoffs. On the other hand, investors trade on risk information in the derivative only. The model has implications for relationship between trading volume in stock and derivative markets and the respective prices in these two markets. Moreover, it suggests that belief dispersion impacts expected stock returns and derivative prices. Finally, it justifies the empirically documented negative relationship between the variance risk premium and returns in the stock market and offers predictions on the association between trading volume, information quality, and variance risk premia.

In the current set up of the model, investors have homogenous information quality, which leads to a derivative price that is uninformative regarding investors' information regarding expected equity payoffs. Preliminary investigation suggests that this will not be the case when investors have heterogenous information quality, since, in this case, the dispersion in their beliefs will be a function of the fundamental $\tilde{\mu}$. It may be interesting, but technically challenging, to study the value of the derivative price to investors when it aggregates both information on the mean of future cash flows and their risk. Furthermore, a weakness of the model is that because the variance distribution is fully general, it is difficult to offer much intuition into how the parameters of the variance distribution impact the derivative price. It may be interesting to study more specific distributions of the variance $\tilde{V}$ in order to offer more definitive comparative statics on the drivers of the derivative price in the model.

4 Appendix

Proof of Proposition 1. Investor i's first-order conditions with respect to their equity and derivative demands are equal to:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial D_{Si}} E\left\{-\exp\left[-\frac{1}{\tau}\left(D_{Si}\left(\tilde{x}-P_S\right)+D_{Si}\left(\tilde{V}-P_D\right)+\tilde{Z}_{\mu i}\tilde{\mu}+\tilde{Z}_{Vi}\tilde{V}\right)\right]|\Phi_i\right\} \quad (16)\\
0 &= \frac{\partial}{\partial D_{Di}} E\left\{-\exp\left[-\frac{1}{\tau}\left(D_{Si}\left(\tilde{x}-P_S\right)+D_{Si}\left(\tilde{V}-P_D\right)+\tilde{Z}_{\mu i}\tilde{\mu}+\tilde{Z}_{Vi}\tilde{V}\right)\right]|\Phi_i\right\}. \quad (17)
\end{aligned}
$$

Under the conjecture that $F_{P_D}(\cdot|P_S,\tilde{\mu}) = F_{P_D}(\cdot|P_S)$, the derivative price serves no role in updating on $\tilde{\mu}$, and hence, its distribution is irrelevant in determining the posterior distribution of $\tilde{\mu}$ given the investors' information. Consequently, upon conditioning on the uncertain variance $\tilde{V}$, due to the linearity of P_S and $\tilde{\varphi}_i$ in $\tilde{\mu}$, the investors' belief regarding $\tilde{x}$ is normally distributed: $\tilde{x}|\tilde{V},\Phi_i \sim N\left(E\left(\tilde{x}|\Phi_i\right), Var\left(\tilde{\mu}|\Phi_i\right)+\tilde{V}\right)$. Hence, evaluating the expectations in expressions 16 and 17 conditional on $\tilde{V}$ yields the following simplified first-order conditions:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial D_{Si}} E_V\left\{-\exp\left[-\frac{1}{\tau}W\left(D_{Si},D_{Di},\tilde{V}\right)\right]|\Phi_i\right\} \quad (18)\\
0 &= \frac{\partial}{\partial D_{Di}} E_V\left\{-\exp\left[-\frac{1}{\tau}W\left(D_{Si},D_{Di},\tilde{V}\right)\right]|\Phi_i\right\},
\end{aligned}
$$

where:

$$
\begin{aligned}
& W\left(D_{Si},D_{Di},V\right) \quad (19)\\
\equiv\ & \left(D_{Si}+\tilde{Z}_{\mu i}\right)E\left(\tilde{\mu}|\Phi_i\right)-D_{Si}P_S-D_{Di}P_D\\
& -\frac{1}{2\tau}\left(D_{Si}+\tilde{Z}_{\mu i}\right)^2 Var\left(\tilde{\mu}|\Phi_i\right)-\left(\frac{1}{2\tau}D_{Si}^2-D_{Di}-\tilde{Z}_{Vi}\right)\tilde{V}.
\end{aligned}
$$

As we show in the proof of Proposition 2, $\tilde{V}|\Phi_i$ lies in the exponential family and thus has a moment-generating function that is defined on the reals, i.e., $\forall t \in \Re,\ E\left(-\exp\left(t\tilde{V}\right)|\Phi_i\right) < \infty$. This implies that the order of differentiation and expectation can be interchanged in these two conditions,[18][19] which yields the following:

$$\begin{aligned} 0 &= E_V\left\{\left[E\left(\tilde{\mu}|\Phi_i\right) - P_S - \frac{1}{\tau}\left(D_{Si} + \tilde{Z}_{\mu i}\right) Var\left(\tilde{\mu}|\Phi_i\right) - \frac{1}{\tau} D_{Si}\tilde{V}\right] \exp\left[-\frac{1}{\tau} W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right] |\Phi_i\right\} \\ 0 &= E_V\left\{\left(P_D - \tilde{V}\right)\exp\left[-\frac{1}{\tau}W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right]|\Phi_i\right\}. \end{aligned} \tag{20}$$

Given that P_D is known conditional on Φ_i, the second condition implies that:

$$E_V\left\{\exp\left[-\frac{1}{\tau}W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right]|\Phi_i\right\} P_D = E_V\left\{\tilde{V}\exp\left[-\frac{1}{\tau}W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right]|\Phi_i\right\}. \tag{21}$$

Substituting this result into the first equation in expression (20) and rearranging, we find:

$$\begin{aligned} &\left[E\left(\tilde{\mu}|\Phi_i\right) - P_S - \frac{1}{\tau}\left(D_{Si} + \tilde{Z}_{\mu i}\right) Var\left(\tilde{\mu}|\Phi_i\right)\right] E_V\left\{\exp\left[-\frac{1}{\tau}W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right]|\Phi_i\right\} \\ = &\ \frac{1}{\tau} D_{Si} P_D E_V\left\{\exp\left[-\frac{1}{\tau}W\left(D_{Si}, D_{Di}, \tilde{V}\right)\right]|\Phi_i\right\}. \end{aligned} \tag{22}$$

---

[18] In particular, let $M(t) \equiv E\left(-\exp\left(t\tilde{V}\right)|\Phi_i\right)$. We wish to show that $\frac{d}{dt}M(t) = E\left(-\tilde{V}\exp\left(t\tilde{V}\right)|\Phi_i\right)$. In order to do so, we apply the dominated convergence theorem. We show that, for any sequence $\{t_n\}$ with $t_n \to t$, there exists a function $\kappa(V)$ such that:

$$\left|\frac{-\exp(t_n V) - \exp(tV)}{t_n - t}\right| < \kappa(V)$$

and $E(\kappa(V)) < \infty$. To find such a $\kappa(V)$, note that the mean value theorem implies that there exists a ξ between t_n and t such that:

$$\begin{aligned} \left|\frac{-\exp(t_n V) - \exp(tV)}{t_n - t}\right| &= \left|-\xi V \exp(\xi V)\right| \\ &= \xi V \exp(\xi V) \end{aligned}$$

Now, using the fact that $V \leq \sum_{j=0}^{\infty} \frac{1}{j!}V^j = \exp(V)$, we get that $\xi V \exp(\xi V) < \exp((\xi+1)V)$. Letting $\kappa(V) = \exp((\xi+1)V)$, and using the fact that the MGF exists for all reals, we have the result.

[19] Since we may change the order of differentiation and expectation, and the utility function is concave, the second order condition holds.

Solving for D_{Si} yields:

$$D_{Si} = \tau \frac{E\left(\tilde{\mu}|\Phi_i\right) - P_S - \frac{1}{\tau} Var\left(\tilde{\mu}|\Phi_i\right) \tilde{Z}_{\mu i}}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)}. \tag{23}$$

The condition for market clearing requires that $\int_0^1 D_{Si} di = \bar{z}$, i.e.,

$$\int_0^1 \tau \frac{E\left(\tilde{\mu}|\Phi_i\right) - P_S - \frac{1}{\tau} Var\left(\tilde{\mu}|\Phi_i\right) \tilde{Z}_{\mu i}}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} di = \bar{z}. \tag{24}$$

Solving for P_S and using the fact that $E\left(\tilde{\mu}|\Phi_i\right) = E\left(\tilde{x}|\Phi_i\right)$, we find:

$$P_S = \int_0^1 E\left(\tilde{x}|\Phi_i\right) di - \frac{1}{\tau} \tilde{z}_\mu Var\left(\tilde{\mu}|\Phi_i\right) - \frac{1}{\tau} \bar{z} \left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right). \tag{25}$$

In the proof of Proposition 3, we show that there is a unique linear equilibrium price that satisfies this equation. ■

Proof of Corollary 1. Note that there is no dependence of $E\left(\tilde{x}|\Phi_i\right)$ on $\{\tilde{\eta}_i\}_{i\in[0,1]}$ and $-\frac{1}{\tau}\tilde{z}_\mu Var\left(\tilde{\mu}|\Phi_i\right) - \frac{1}{\tau}\bar{z}\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)$ depends upon $\tilde{V}$ only through P_D. Thus, we have that $F_{P_S}\left(\cdot|P_D, \tilde{V}\right) = F_{P_S}\left(\cdot|P_D\right)$. ■

Proof of Proposition 2. We start by conjecturing a generalized linear equilibrium, as in Breon-Drish (2015b). Specifically, conjecture that price satisfies:

$$\begin{aligned} P_D\left(\tilde{V} + \tilde{v}, \tilde{z}_V\right) &= \delta\left(l\left(\tilde{V} + \tilde{v}, \tilde{z}_V\right)\right) \\ \text{where } \delta' &> 0 \\ \text{and } l\left(\tilde{V} + \tilde{v}, \tilde{z}_V\right) &= a\left(\tilde{V} + \tilde{v}\right) + \tilde{z}_V, \end{aligned} \tag{26}$$

for a constant a to be determined as part of the equilibrium. Given that $F_{P_S}\left(\cdot|P_D, \tilde{V}\right) = F_{P_S}\left(\cdot|P_D\right)$, we have that $F_{\tilde{V}|P_D, P_S, \tilde{\eta}_i, \tilde{Z}_{Vi}} = F_{\tilde{V}|P_D, \tilde{\eta}_i, \tilde{Z}_{Vi}}$. As $\delta' > 0$, investors can invert the linear statistic $l\left(\tilde{V} + \tilde{v}, \tilde{z}_V\right)$ from price, and hence, the information in P_D is equivalent to $\tilde{l}\left(\tilde{V} + \tilde{v}, \tilde{z}_V\right)$: $F_{\tilde{V}|P_D, \tilde{\eta}_i, \tilde{Z}_{Vi}} = F_{\tilde{V}|\tilde{l}, \tilde{\eta}_i, \tilde{Z}_{Vi}}$. Note that $\tilde{z}_V|\tilde{Z}_{Vi} \sim N\left(\frac{\sigma^2_{z_V} Z_{Vi}}{\sigma^2_{z_V} + \sigma^2_{z_{Vi}}}, \frac{\sigma^2_{z_V}\sigma^2_{z_{Vi}}}{\sigma^2_{z_V} + \sigma^2_{z_{Vi}}}\right)$.

Therefore, conditional on Φ_i, investor i's belief distribution regarding $\tilde{V}$ satisfies (where differential notation simply indicates that the distribution of $\tilde{V}$ may exhibit discontinuities):

$$\begin{aligned}
& dF_{\tilde{V}|\tilde{l},\tilde{\eta}_i,\tilde{Z}_{Vi}}(y) \qquad (27) \\
\propto\ & dF_{\tilde{V}}(y)\, dF_{\tilde{Z}_{Vi}}(Z_{Vi})\, dF_{\tilde{l},\tilde{\eta}_i|\tilde{V},\tilde{Z}_{Vi}}(l,\eta_i) \\
\propto\ & dF_{\tilde{V}}(y)\, dF_{\tilde{l},\tilde{\eta}_i|\tilde{V},\tilde{Z}_{Vi}}(l,\eta_i) \\
\propto\ & \exp\left[-\frac{1}{2}\frac{(\sigma_\upsilon^2+\sigma_e^2)\left(l-ay-\frac{\sigma_{z_V}^2 Z_{Vi}}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)^2-2a\sigma_\upsilon^2\left(l-ay-\frac{\sigma_{z_V}^2 Z_{Vi}}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)(\eta_i-y)}{(\sigma_\upsilon^2+\sigma_e^2)\left(a^2\sigma_\upsilon^2+\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)-a^2\sigma_\upsilon^4}\right. \\
& \left.-\frac{1}{2}\frac{\left(a^2\sigma_\upsilon^2+\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)(\eta_i-y)^2}{(\sigma_\upsilon^2+\sigma_e^2)\left(a^2\sigma_\upsilon^2+\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)-a^2\sigma_\upsilon^4}\right] dl d\eta_i dF_{\tilde{V}}(y).
\end{aligned}$$

Tedious algebra yields that this is proportional to:

$$\exp\left[\frac{-\frac{1}{2}\left(a^2\sigma_e^2+\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)y^2+\left(\left(\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}\right)\eta_i-a\frac{\sigma_{z_V}^2\sigma_e^2}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}Z_{Vi}+a\sigma_e^2 l\right)y}{a^2\sigma_\upsilon^2\sigma_e^2+\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2(\sigma_\upsilon^2+\sigma_e^2)}{\sigma_{z_V}^2+\sigma_{z_{Vi}}^2}}\right] dl d\eta_i dF_{\tilde{V}}(y). \tag{28}$$

Defining this final expression as $J(y;l,\eta_i,Z_{Vi})$, we may write:

$$dF_{\tilde{V}|\tilde{l},\tilde{\eta}_i,\tilde{Z}_{Vi}}(y)=\frac{J(y;l,\eta_i,Z_{Vi})}{\int_\Upsilon J(x;l,\eta_i,Z_{Vi})\,dx}. \tag{29}$$

This distribution belongs to the exponential family, i.e., it may be written in the form

$\exp\left\{\left(k_1(a)\,\tilde{\eta}_i + k_2(a)\,\tilde{l} + k_3(a)\,\tilde{Z}_{Vi}\right) y - g\left(k_1(a)\,\tilde{\eta}_i + k_2(a)\,\tilde{l} + k_3(a)\,\tilde{Z}_{Vi}\right)\right\} H(y;a)$ where:

$$
\begin{aligned}
k_1(a) &= \frac{\sigma_{z_V}^2 \sigma_{z_{Vi}}^2}{a^2\sigma_\upsilon^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_\upsilon^2+\sigma_e^2\right)} \\
k_2(a) &= \frac{a\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)}{a^2\sigma_\upsilon^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_\upsilon^2+\sigma_e^2\right)} \\
k_3(a) &= \frac{-a\sigma_{z_V}^2\sigma_e^2}{a^2\sigma_\upsilon^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_\upsilon^2+\sigma_e^2\right)} \\
g(\xi) &= \log\left[\int_\Upsilon \exp\left(\xi x - \frac{1}{2}\frac{a^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{a^2\sigma_\upsilon^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_\upsilon^2+\sigma_e^2\right)}x^2\right) dF_V(x)\right] \text{ and} \\
H(y;a) &= \exp\left[-\frac{1}{2}\frac{a^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{a^2\sigma_\upsilon^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_\upsilon^2+\sigma_e^2\right)}y^2\right] dF_V(y).
\end{aligned} \tag{30}
$$

This implies that $\tilde{V}|\tilde{\eta}_i, \tilde{l}, \tilde{Z}_{Vi}$ has moment-generating function:

$$
\begin{aligned}
E\left[\exp\left(t\tilde{V}\right)|\tilde{\eta}_i, \tilde{l}, \tilde{Z}_{Vi}\right] &= \exp\Big\{g\left(k_1(a)\,\tilde{\eta}_i + k_2(a)\,\tilde{l} + k_3(a)\,\tilde{Z}_{Vi} + t\right) \\
&\quad -g\left(k_1(a)\,\tilde{\eta}_i + k_2(a)\,\tilde{l} + k_3(a)\,\tilde{Z}_{Vi}\right)\Big\}.
\end{aligned} \tag{31}
$$

Evaluating the expectation in the investors' first-order condition with respect to D_{Di} from expression (20) and simplifying yields:

$$\begin{aligned}
0 &= P_D E_V\left\{\exp\left[-\frac{1}{\tau}W\left(D_{Si},D_{Di},\tilde{V}\right)\right]|\Phi_i\right\} - E_V\left\{\tilde{V}\exp\left[-\frac{1}{\tau}W\left(D_{Si},D_{Di},\tilde{V}\right)\right]|\Phi_i\right\} \\
&= P_D \exp\left\{g\left[k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi} + \frac{1}{\tau}\left(\frac{1}{2\tau}D_{Si}^2 - D_{Di} - \tilde{Z}_{Vi}\right)\right]\right. \qquad (32) \\
&\quad \left. -g\left[k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi}\right] + \frac{1}{\tau}D_{Di}P_D\right\} \\
&\quad -\frac{\partial}{\partial t}\left[\exp\left\{g\left(k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi} + t\right)\right.\right. \\
&\quad \left.\left. - g\left(k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi}\right) + \frac{1}{\tau}D_{Di}P_D\right\}\right]_{t=\frac{1}{\tau}\left(\frac{1}{2\tau}D_{Si}^2 - D_{Di} - \tilde{Z}_{Vi}\right)} \\
&= \left\{P_D - g'\left[k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi} + \frac{1}{\tau}\left(\frac{1}{2\tau}D_{Si}^2 - D_{Di} - \tilde{Z}_{Vi}\right)\right]\right\} \\
&\quad * \exp\left\{g\left[k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi} - \frac{1}{\tau}W\left(D_{Si},D_{Di},V\right)\right]\right. \\
&\quad \left. -g\left[k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi}\right] + \frac{1}{\tau}D_{Di}P_D\right\},
\end{aligned}$$

which yields:

$$D_{Di} = \tau\left(k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi}\right) - \tilde{Z}_{Vi} - \tau g'^{-1}(P_D) + \frac{1}{2\tau}D_{Si}^2.$$

The market-clearing condition is:

$$\int_0^1 \left[\tau\left(k_1(a)\tilde{\eta}_i + k_2(a)\tilde{l} + k_3(a)\tilde{Z}_{Vi}\right) - \tau g'^{-1}(P_D) + \frac{1}{2\tau}D_{Si}^2\right]di - \int_0^1 \tilde{Z}_{Vi}di = 0. \qquad (33)$$

Applying the law-of-large numbers and simplifying yields:

$$P_D = g'\left\{\frac{1}{\tau}\left[\tau k_1(a)\left(\tilde{V} + \tilde{v}\right) + (\tau k_3(a) - 1)\tilde{z}_V + \tau k_2(a)\tilde{l} + \frac{1}{2\tau}\int_0^1 D_{Si}^2 di\right]\right\}. \qquad (34)$$

In order for this to satisfy our conjecture that price depends on $\tilde{V}$ and $\tilde{z}_V$ only through the linear statistic $l\left(\tilde{V}+\tilde{v},\tilde{z}_V\right) = a\left(\tilde{V}+\tilde{v}\right)+\tilde{z}_V$, we must have that $\tilde{l} = \frac{\tau k_1(a)}{\tau k_3(a)-1}\left(\tilde{V}+\tilde{v}\right)+\tilde{z}_V$. This is equivalent to $\frac{\tau k_1(a)}{\tau k_3(a)-1} = a$. Substituting $k_1(a)$ and $k_3(a)$ and simplifying yields the following equilibrium condition:

$$
\begin{aligned}
a &= \frac{\tau\frac{\sigma_{z_V}^2\sigma_{z_{Vi}}^2}{a^2\sigma_v^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_v^2+\sigma_e^2\right)}}{-\tau\frac{a\sigma_{z_V}^2\sigma_e^2}{a^2\sigma_v^2\sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\left(\sigma_v^2+\sigma_e^2\right)}-1} \qquad (35)\\
0 &= \left(\sigma_{z_V}^2\sigma_e^2\sigma_v^2+\sigma_{z_{Vi}}^2\sigma_e^2\sigma_v^2\right)a^3+\tau\sigma_{z_V}^2\sigma_e^2a^2+\left(\sigma_{z_V}^2\sigma_{z_{Vi}}^2\sigma_e^2+\sigma_{z_V}^2\sigma_{z_{Vi}}^2\sigma_v^2\right)a+\tau\sigma_{z_V}^2\sigma_{z_{Vi}}^2.
\end{aligned}
$$

As this is a cubic equation with positive third-order and constant terms, it has at least one negative solution. Hence, an equilibrium P_D is defined by:

$$
\begin{aligned}
P_D &= g'\left\{\frac{1}{\tau}\left[\left(\tau k_3\left(a^*\right)-1\right)\left(\frac{\tau k_1\left(a^*\right)}{\tau k_3\left(a^*\right)-1}\left(\tilde{V}+\tilde{v}\right)+\tilde{z}_V\right)+\tau k_2\left(a^*\right)\tilde{l}+\frac{1}{2\tau}\int_0^1 D_{Si}^2di\right]\right\}\\
&= g'\left\{\frac{1}{\tau}\left[\left(\tau\left(k_2\left(a^*\right)+k_3\left(a^*\right)\right)-1\right)\tilde{l}+\frac{1}{2\tau}\int_0^1 D_{Si}^2di\right]\right\}, \qquad (36)
\end{aligned}
$$

where a^* is a solution to equation (35). Letting $r\left(a^*\right)=\tau\left(k_2\left(a^*\right)+k_3\left(a^*\right)\right)-1$, we have the result in the statement of the proof. We prove that P_D increases in $\tilde{V}$ and $\tilde{v}$ and decreases in $\tilde{z}_V$ in the proof of Proposition 3. ■

Proof of Corollary 2. Expression (9) shows the only dependence of P_D on $\tilde{\mu}$ is through the term, $\int_0^1 D_{Si}^2di$. Moreover, note that:

$$
\begin{aligned}
\int_0^1 D_{Si}^2di &= \int_0^1\left[\tau\frac{E\left(\tilde{x}|\Phi_i\right)-P_S-\tau^{-1}Var\left(\tilde{\mu}|\Phi_i\right)\tilde{Z}_{\mu i}}{P_D+Var\left(\tilde{\mu}|\Phi_i\right)}\right]^2di \qquad (37)\\
&= \int_0^1\left[\tau\frac{E\left(\tilde{x}|\Phi_i\right)-\int_0^1E\left(\tilde{x}|\Phi_i\right)di-\tau^{-1}\tilde{z}_{\mu i}Var\left(\tilde{\mu}|\Phi_i\right)+\tau^{-1}\bar{z}\left(P_D+Var\left(\tilde{\mu}|\Phi_i\right)\right)}{P_D+Var\left(\tilde{\mu}|\Phi_i\right)}\right]^2di\\
&= \left(\frac{1}{P_D+Var\left(\tilde{\mu}|\Phi_i\right)}\right)^2\int_0^1\left[\tau\left(E\left(\tilde{x}|\Phi_i\right)-\int_0^1E\left(\tilde{x}|\Phi_i\right)di\right)-\tilde{z}_{\mu i}Var\left(\tilde{\mu}|\Phi_i\right)\right]^2di+\bar{z}^2.
\end{aligned}
$$

The term $\int_0^1\left[\tau\left(E\left(\tilde{x}|\Phi_i\right)-\int_0^1E\left(\tilde{x}|\Phi_i\right)di\right)-\tilde{z}_{\mu i}Var\left(\tilde{\mu}|\Phi_i\right)\right]^2di$ does not depend on $\tilde{\mu}$. To see this, note that given the conjectured linear equilibrium and normality of the terms $\tilde{\mu}$, $\tilde{\varepsilon}$, $\tilde{z}_{\mu i}$, and $\tilde{z}_\mu$, $E\left(\tilde{x}|\Phi_i\right)-\int_0^1E\left(\tilde{x}|\Phi_i\right)di$ is linear in P_S, $\tilde{Z}_{\mu i}$ and $\tilde{\varphi}_i$. Moreover, because investors' information precisions are homogenous, they apply equal linear weights to each of P_S, $\tilde{Z}_{\mu i}$

and $\tilde{\varphi}_i$. Let $E(\tilde{x}|\Phi_i) = D_0 + D_1\tilde{\varphi}_i + D_2\tilde{Z}_{\mu i} + D_3 P_S$. We have:

$$\begin{aligned}
& \int_0^1 \left[E(\tilde{x}|\Phi_i) - \int_0^1 E(\tilde{x}|\Phi_i)\, di - \tilde{z}_{\mu i} Var(\tilde{\mu}|\Phi_i)\right]^2 di \qquad (38) \\
= & \int_0^1 \Big[D_0 + D_1\tilde{\varphi}_i + D_2\tilde{Z}_{\mu i} + D_3 P_S \\
& - \int_0^1 \left(D_0 + D_1\tilde{\varphi}_i + D_2\tilde{Z}_{\mu i} + D_3 P_S\right) di - \tilde{z}_{\mu i} Var(\tilde{\mu}|\Phi_i)^2\Big] di \\
= & \int_0^1 \left[D_1\tilde{\varepsilon}_i + (D_2 - Var(\tilde{\mu}|\Phi_i))\, \tilde{z}_{\mu i}\right]^2 di \\
= & D_1^2 \int_0^1 \tilde{\varepsilon}_i^2 di + (D_2 - Var(\tilde{\mu}|\Phi_i))^2 \int_0^1 \tilde{z}_{\mu i}^2 di \\
= & D_1^2 \sigma_\varepsilon^2 + (D_2 - Var(\tilde{\mu}|\Phi_i))^2 \sigma_{z_{\mu i}}^2 .
\end{aligned}$$

Since this is not a function of $\tilde{\mu}$, the conjecture has been verified. ■

Proof of Proposition 3. Using the results from Proposition 1 and Proposition 2, a rational expectations equilibrium must simultaneously satisfy the following three conditions:

$$(1)\ D_{Si} = \tau \frac{E(\tilde{x}|\Phi_i) - P_S - \tau^{-1}\tilde{Z}_{\mu i} Var(\tilde{\mu}|\Phi_i)}{P_D + Var(\tilde{\mu}|\Phi_i)} \qquad (39)$$

(2) a^* solves equation 35, and (40)

$$(3)\ P_D = g'\left\{\frac{1}{2\tau}\left[(\tau(k_2(a^*) + k_3(a^*)) - 1)\,\tilde{l} + \frac{1}{2\tau}\int_0^1 D_{Si}^2 di\right]\right\}. \qquad (41)$$

Note that the first condition itself contains a fixed-point problem in the sense that $E(\tilde{x}|\Phi_i)$ is a function of P_S, which is impacted by investors' demands D_{Si}. We first show that fixing P_D, there is a unique solution to this fixed-point problem, i.e., there is a unique equilibrium in the equity market given P_D. To begin, we derive $E(\tilde{x}|\Phi_i)$ and $Var(\tilde{\mu}|\Phi_i)$. Let $A \equiv \frac{Cov\left(\tilde{x}, P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right)}{Var\left(P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right)}$. Note that:

$$\begin{aligned}
E(\tilde{x}|\Phi_i) &= E\left(\tilde{x}|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) + A\left(P_S - E\left(P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right)\right), \qquad (42) \\
\text{and } Var(\mu|\Phi_i) &= Var\left(\mu|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) - A\, Cov(\tilde{x}, P_S|\varphi_i, Z_{\mu i}, P_D).
\end{aligned}$$

The following facts may be easily verified by utilizing the variance-covariance matrix of $\tilde{x}, P_S, \tilde{\varphi}_i, \tilde{Z}_{\mu i}$ and applying the properties of the multivariate-normal distribution:

$$\begin{aligned}
E\left(\tilde{x}|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) &= \frac{\left(\sigma_n^2 + \sigma_\varepsilon^2\right) m + \sigma_\mu^2 \tilde{\varphi}_i}{\sigma_\mu^2 + \sigma_n^2 + \sigma_\varepsilon^2}, \qquad (43)\\
Cov\left(\tilde{x}, P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) &= \frac{\alpha_\mu \sigma_\mu^2 \sigma_\varepsilon^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2},\\
Var\left(P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) &= \sigma_{z_\mu}^2 \alpha_z^2 + \alpha_\mu^2\left(\sigma_n^2 + \sigma_\mu^2\right) - \frac{\alpha_z^2 \sigma_{z_\mu}^4}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2} - \frac{\alpha_\mu^2\left(\sigma_n^2 + \sigma_\mu^2\right)^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2}, \text{ and}\\
E\left(P_S|\tilde{\varphi}_i, \tilde{Z}_{\mu i}, P_D\right) &= \alpha_0 + \alpha_\mu \frac{\tilde{\varphi}_i\left(\sigma_n^2 + \sigma_\mu^2\right) + m\sigma_\varepsilon^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2} + \frac{\alpha_z \sigma_{z_\mu}^2}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2}\tilde{Z}_{\mu i}.
\end{aligned}$$

Substituting into expressions (42) and (40) and simplifying, we find:

$$\begin{aligned}
P_S &= \frac{1}{1-A}\left(\frac{\left(\sigma_n^2 + \sigma_\varepsilon^2\right) m + \sigma_\mu^2\left(\mu + n\right)}{\sigma_\mu^2 + \sigma_n^2 + \sigma_\varepsilon^2} - A\left(\alpha_0 + \alpha_\mu \frac{\left(\mu + n\right)\left(\sigma_n^2 + \sigma_\mu^2\right) + m\sigma_\varepsilon^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2} + \frac{\alpha_z \sigma_{z_\mu}^2}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2}\tilde{Z}_{\mu i}\right)\right)\\
&\quad -\frac{1}{\tau}\frac{1}{1-A}\left(\left(\tilde{z}_\mu + \bar{z}\right)\left(\sigma_\mu^2 \frac{\sigma_n^2 + \sigma_\varepsilon^2\left(1 - A\alpha_\mu\right)}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2}\right) + \bar{z}P_D\right). \qquad (44)
\end{aligned}$$

Comparing coefficients with the conjectured form for price, this implies that:

$$\begin{aligned}
\alpha_\mu &= \frac{1}{1-A}\frac{\sigma_\mu^2 - A\alpha_\mu\left(\sigma_n^2 + \sigma_\mu^2\right)}{\sigma_\mu^2 + \sigma_n^2 + \sigma_\varepsilon^2} \qquad (45)\\
&= \frac{\sigma_\mu^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2\left(1 - A\right)}, \text{ and}\\
\alpha_z &= -\frac{1}{\tau}\frac{1}{1-A}\left(\sigma_\mu^2 \frac{\sigma_n^2 + \sigma_\varepsilon^2\left(1 - A\alpha_\mu\right)}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2}\right) - \frac{A}{1-A}\frac{\alpha_z \sigma_{z_\mu}^2}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2} \qquad (46)\\
&= -\frac{1}{\tau}\sigma_\mu^2 \frac{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2\left(1 - A\right)}\frac{\sigma_n^2 + \sigma_\varepsilon^2\left(1 - A\right)}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2\left(1 - A\right)}.
\end{aligned}$$

Hence, for there to exist an equilibrium, there must exist an $A^* \in (0, 1)$ that satisfies the above conditions as well as:

$$A^* = \frac{\frac{\alpha_\mu \sigma_\mu^2 \sigma_\varepsilon^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2}}{\sigma_{z_\mu}^2 \alpha_z^2 + \alpha_\mu^2\left(\sigma_n^2 + \sigma_\mu^2\right) - \frac{\alpha_z^2 \sigma_{z_\mu}^4}{\sigma_{z_\mu}^2 + \sigma_{z_{\mu i}}^2} - \frac{\alpha_\mu^2\left(\sigma_n^2 + \sigma_\mu^2\right)^2}{\sigma_n^2 + \sigma_\mu^2 + \sigma_\varepsilon^2}}. \qquad (47)$$

Substituting, we arrive at a cubic equation in A^* that is positive when $A^* = 1$ and negative when $A^* = 0$. This implies the existence of a solution $A^* \in (0,1)$ and hence the existence of a solution to equilibrium condition 1. Next, consider the second and third equilibrium conditions. Notice that:

$$\begin{aligned}
\int_0^1 D_{Si}^2 di &= \tau^2 \int_0^1 \left(\frac{E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right)}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \right)^2 di \qquad (48) \\
&= \frac{B\left(A^*\right)}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2}, \\
\text{where } B\left(A^*\right) &\equiv \tau^2 \left(\int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right) \right)^2 di \right).
\end{aligned}$$

Thus, we have:

$$\begin{aligned}
P_D &= g' \left\{ \frac{1}{\tau} \left[\left(\tau \left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \int_0^1 D_{Si}^2 di \right] \right\} \qquad (49) \\
&= g' \left\{ \frac{1}{\tau} \left[\left(\tau \left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B\left(A^*\right)}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right\}.
\end{aligned}$$

Let $h\left(P_D; \tilde{l}\right) = P_D - g' \left\{ \frac{1}{2\tau} \left[\left(\tau \left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B(A^*)}{\left(P_D + Var(\tilde{\mu}|\Phi_i)\right)^2} \right] \right\}$. Note that if $h\left(\cdot; \tilde{l}\right)$ has a unique zero, then, by equilibrium condition 2, we can solve for the unique P_D. Hence, we will have proven that there exists a unique rational expectations equilibrium for each solution a^* to equation (35) and solution A^* to equation (47). To show that this is the case, note first that g' is continuous. Thus, noting that $B\left(A^*\right)$ is unaffected by P_D, we have

$$\begin{aligned}
&\lim_{P_D \to 0} h\left(P_D; \tilde{l}\right) \qquad (50) \\
&= -g' \left(\lim_{P_D \to 0} \frac{1}{\tau} \left[\left(\tau \left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B\left(A^*\right)}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right) \\
&= -g' \left(\frac{1}{\tau} \left[\left(\tau \left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B\left(A^*\right)}{Var\left(\tilde{\mu}|\Phi_i\right)^2} \right] \right) < 0,
\end{aligned}$$

since g' is always positive (see Breon-Drish (2015a)). Moreover,

$$\begin{aligned}
& \lim_{P_D \to \infty} h\left(P_D; \tilde{l}\right) && (51) \\
= & \lim_{P_D \to \infty} \left\{ P_D - g'\left(\lim_{P_D \to \infty} \frac{1}{2\tau} \left[\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B\left(A^*\right)}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right) \right\} \\
= & \lim_{P_D \to \infty} P_D - g'\left(\frac{1}{2\tau} \left[\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} \right] \right) = \infty.
\end{aligned}$$

This ensures a positive solution P_D exists. To see that it is unique, note that:

$$\begin{aligned}
& \frac{\partial}{\partial P_D} h\left(P_D; \tilde{l}\right) && (52) \\
= & \frac{\partial}{\partial P_D} \left(P_D - g'\left(\frac{1}{2\tau} \left[\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B\left(A^*\right)}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right) \right) \\
= & 1 + 2g'' * \frac{B\left(A^*\right)}{\tau\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^3},
\end{aligned}$$

which is positive given that $g'' > 0$.[20] Finally, we demonstrate that $\frac{dP_D}{dz_V} < 0$ and $\frac{dP_D}{dV} > 0$. Applying the implicit-function theorem, since $\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial P_D} > 0$, $\frac{dP_D}{dz_V}$ has the sign of $-\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial z_V}$ and $\frac{dP_D}{dV}$ has the sign of $-\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial V}$. Given that $\tilde{l} = a^*\tilde{V} + \tilde{z}_V$,

$$\begin{aligned}
& \frac{\partial h\left(P_D; \tilde{l}\right)}{\partial z_V} && (53) \\
= & \left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) g''\left(\frac{1}{2\tau} \left[\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right), \text{ and} \\
& \frac{\partial h\left(P_D; \tilde{l}\right)}{\partial V} && (54) \\
= & a^*\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) g''\left(\frac{1}{2\tau} \left[\left(\tau\left(k_2\left(a^*\right) + k_3\left(a^*\right)\right) - 1\right) \tilde{l} + \frac{1}{2\tau} \frac{B}{\left(P_D + Var\left(\tilde{\mu}|\Phi_i\right)\right)^2} \right] \right).
\end{aligned}$$

[20] For a proof, see Lemma A7 in Breon-Drish (2015a).

Now,

$$\begin{aligned}
& \tau\left(k_2\left(a^*\right)+k_3\left(a^*\right)\right)-1 \qquad (55)\\
= & \tau\left(\frac{a^* \sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)}{a^{*2} \sigma_v^2 \sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2 \sigma_{z_{Vi}}^2\left(\sigma_v^2+\sigma_e^2\right)}-\frac{a^* \sigma_{z_V}^2 \sigma_e^2}{a^{*2} \sigma_v^2 \sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2 \sigma_{z_{Vi}}^2\left(\sigma_v^2+\sigma_e^2\right)}\right)-1 \\
= & \tau\left(\frac{a^* \sigma_e^2 \sigma_{z_{Vi}}^2}{a^{*2} \sigma_v^2 \sigma_e^2\left(\sigma_{z_V}^2+\sigma_{z_{Vi}}^2\right)+\sigma_{z_V}^2 \sigma_{z_{Vi}}^2\left(\sigma_v^2+\sigma_e^2\right)}\right)-1<0,
\end{aligned}$$

since $a^*<0$. Hence, $\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial z_V}<0$ and $\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial V}>0$. This completes the proof. ■

Proof of Proposition 4. Simplifying the definition of trading volume in the stock, we find:

$$\begin{aligned}
Vol_S= & \int_0^1\left|\tau \frac{E\left(\tilde{x} \mid \Phi_i\right)-P_S-\tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu} \mid \Phi_i\right)}{P_D+Var\left(\tilde{\mu} \mid \Phi_i\right)}-\int_0^1 D_{Si} di\right| di \qquad (56)\\
= & \tau \int_0^1\left|\frac{E\left(\tilde{x} \mid \Phi_i\right)-\int_0^1 E\left(\tilde{x} \mid \Phi_i\right) di-\tau^{-1} Var\left(\tilde{\mu} \mid \Phi_i\right) \tilde{z}_{\mu i}}{P_D+Var\left(\tilde{\mu} \mid \Phi_i\right)}\right| di .
\end{aligned}$$

Likewise, applying expression (34) to the definition of trading volume in the derivative, we find:

$$\begin{aligned}
& Vol_D \qquad (57)\\
= & \int_0^1\left|\tau\left(k_1\left(a^*\right) \tilde{\eta}_i+k_2\left(a^*\right) \tilde{l}+k_3\left(a^*\right) \tilde{Z}_{Vi}\right)-\tilde{Z}_{Vi}-\tau g^{\prime-1}\left(P_D\right)+\frac{1}{2 \tau} D_{Si}^2\right| di \\
= & \int_0^1 \left\lvert\, \tau\left(k_1\left(a^*\right) \tilde{\eta}_i+k_2\left(a^*\right) \tilde{l}+k_3\left(a^*\right) \tilde{Z}_{Vi}\right)-\tilde{Z}_{Vi}-\right. \\
& \left.\left(\tau k_1(a)\left(\tilde{V}+\tilde{v}\right)+\tau k_2(a) \tilde{l}+\left(\tau k_3(a)-1\right) \tilde{z}_V+\frac{1}{2 \tau} \int_0^1 D_{Si}^2 di\right)+\frac{1}{2 \tau} D_{Si}^2 \right\rvert\, di \\
= & \int_0^1\left|\tau k_1\left(a^*\right) \tilde{e}_i+\left(\tau k_3\left(a^*\right)-1\right) \tilde{z}_{Vi}+\frac{1}{2 \tau}\left(D_{Si}^2-\int_0^1 D_{Si}^2 di\right)\right| di .
\end{aligned}$$

■

Proof of Corollary 3. i) Note that since $E\left(\tilde{x} \mid \Phi_i\right)$ and $Var\left(\tilde{\mu} \mid \Phi_i\right)$ are independent of P_D

by the proof of Proposition 3,

$$Vol_S = \frac{\tau}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \int_0^1 \left| E\left(\tilde{x}|\Phi_i\right) - \int_0^1 E\left(\tilde{x}|\Phi_i\right) di - \tau^{-1} Var\left(\tilde{\mu}|\Phi_i\right) \tilde{z}_{\mu i} \right| di \tag{58}$$

is trivially decreasing in P_D. Applying the chain rule and the fact that P_D increases in $\tilde{V}$ and $\tilde{v}$ and declines in $\tilde{z}_V$, the result follows.

ii) Note that:

$$Vol_D = \int_0^1 \left| \tau k_1(a)\, \tilde{e}_i + \left(\tau k_3(a) - 1\right) \tilde{z}_{Vi} + \frac{1}{2\tau} \left(D_{Si}^2 - \int_0^1 D_{Si}^2 di \right) \right| di \tag{59}$$

is only affected by $\tilde{V}$, $\tilde{v}$, and $\tilde{z}_V$ through $D_{Si}^2 - \int_0^1 D_{Si}^2$. We have:

$$\begin{aligned} D_{Si}^2 - \int_0^1 D_{Si}^2 di &= \tau^2 \left[\left(\frac{E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right)}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \right)^2 \right. \\ &\quad \left. - \int_0^1 \left(\frac{E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right)}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \right)^2 di \right] \\ &= \left(\frac{\tau}{P_D + Var\left(\tilde{\mu}|\Phi_i\right)} \right)^2 \left[\left(E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right) \right)^2 \right. \\ &\quad \left. - \int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right) \right)^2 di \right]. \end{aligned} \tag{60}$$

Thus, Vol_D is affected by $\tilde{V}$, $\tilde{v}$, and $\tilde{z}_V$ through P_D. Define $B_i \equiv \left(E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right) \right)^2 - \int_0^1 \left(E\left(\tilde{x}|\Phi_i\right) - P_S - \tau^{-1} \tilde{Z}_{\mu i} Var\left(\tilde{\mu}|\Phi_i\right) \right)^2 di$, $\Psi \equiv \frac{\tau}{2} \left(P_D + Var\left(\tilde{\mu}|\Phi_i\right) \right)^2$, and $Y = \tau k_1(a)\, \tilde{e}_i + \left(\tau k_3(a) - 1\right) \tilde{z}_{Vi}$. Note we can write:

$$\frac{dVol_D}{dP_D} = \frac{d}{dP_D} \int_0^1 \left| Y + \frac{B_i}{\Psi} \right| di. \tag{61}$$

Applying a similar technique as in the proof of Corollary 2, it can be seen that B_i is not affected by P_D. Furthermore, it is clearly the case that $\frac{\partial \Psi}{\partial P_D} > 0$. Therefore,

$$\begin{aligned} & \frac{d}{dP_D} \int_0^1 \left| Y + \frac{B_i}{\Psi} \right| di \\ = & \frac{\partial \Psi}{\partial P_D} \frac{\partial}{\partial \Psi} \int_0^1 \left| Y + \frac{B_i}{\Psi} \right| di \\ \propto & \frac{\partial}{\partial \Psi} \int_0^1 \left| Y + \frac{B_i}{\Psi} \right| di. \end{aligned} \tag{62}$$

Simplifying,

$$\begin{aligned} & \frac{\partial}{\partial \Psi} \int_0^1 \left| Y + \frac{B_i}{\Psi} \right| di \\ = & \frac{\partial}{\partial \Psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left| Y + \frac{B_i}{\Psi} \right| dF_{B_i} dF_Y \\ = & \frac{\partial}{\partial \Psi} \int_{-\infty}^{\infty} \left[\int_{-\Psi Y}^{\infty} \left(Y + \frac{B_i}{\Psi} \right) dF_{B_i} - \int_{-\infty}^{-\Psi Y} \left(Y + \frac{B_i}{\Psi} \right) dF_{B_i} \right] dF_Y \\ = & -\frac{1}{\Psi^2} \int_{-\infty}^{\infty} \left[\int_{-\Psi Y}^{\infty} B_i dF_{B_i} - \int_{-\infty}^{-\Psi Y} B_i dF_{B_i} \right] dF_Y. \end{aligned} \tag{63}$$

Note that $\int_0^1 B_i di = 0$, such that:

$$\begin{aligned} \int_{-\Psi Y}^{\infty} B_i dF_{B_i} + \int_{-\infty}^{-\Psi Y} B_i dF_{B_i} \quad &= \quad 0 \\ &\Longrightarrow \quad \int_{-\Psi Y}^{\infty} B_i dF_{B_i} = -\int_{-\infty}^{-\Psi Y} B_i dF_{B_i} \\ &\Longrightarrow \quad \int_{-\Psi Y}^{\infty} B_i dF_{B_i} - \int_{-\infty}^{-\Psi Y} B_i dF_{B_i} = -2 \int_{-\infty}^{-\Psi Y} B_i dF_{B_i}. \end{aligned} \tag{64}$$

We thus have:

$$\begin{aligned} & -\frac{1}{\Psi^2} \int_{-\infty}^{\infty} \left[\int_{-\Psi Y}^{\infty} B_i dF_{B_i} - \int_{-\infty}^{-\Psi Y} B_i dF_{B_i} \right] dF_Y \\ = & \frac{2}{\Psi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{-\Psi Y} B_i dF_{B_i} dF_Y < 0 \end{aligned} \tag{65}$$

since $\int_{-\infty}^{-\Psi Y} B_i dF_{B_i} < \int_{-\infty}^{\infty} B_i dF_{B_i} = E(B_i) = 0$. Again applying the chain rule and the fact that P_D increases in $\tilde{V}$ and $\tilde{\upsilon}$ and declines in $\tilde{z}_V$ completes the proof. ■

Proof of Corollary 4. The proof follows by considering the directional effect of a change in each of the random parameters of the model, $\tilde{z}_\mu$, $\tilde{z}_V$, $\tilde{V}$, and $\tilde{\mu}$, on ΔP_S, ΔP_D, Vol_S, and Vol_D. By examining the comparative statics that are implied by the results of propositions 1, 2, and 4, we find that any change in the underlying random variables that causes the stock price to increase causes volume in both markets to either remain constant or increase. Likewise, any change in an underlying random variable that causes the derivative price to increase leads volume in both markets to remain constant or decrease. ■

Proof of Proposition 5.

The proof is an application of the implicit-function theorem to the equilibrium condition $h\left(P_D;\tilde{l}\right)=0$. Since $\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial P_D}>0$,

$$\begin{aligned} & \frac{dP_D}{dBeliefDispersion} \\ \propto\ & -\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial\int_0^1 D_{Si}^2 di}\frac{\partial\int_0^1 D_{Si}^2 di}{BeliefDispersion}. \end{aligned} \tag{66}$$

From the proof of Corollary 2, we have that:

$$\begin{aligned} & \int_0^1 D_{Si}^2 di \\ = & \left(\frac{1}{P_D+Var\left(\tilde{\mu}|\Phi_i\right)}\right)^2\int_0^1\left[\tau\left(E\left(\tilde{x}|\Phi_i\right)-\int_0^1 E\left(\tilde{x}|\Phi_i\right)di\right)-\tilde{z}_{\mu i}Var\left(\tilde{\mu}|\Phi_i\right)\right]^2 di+\bar{z}^2 \\ = & \left(\frac{\tau}{P_D+Var\left(\tilde{\mu}|\Phi_i\right)}\right)^2\left[BeliefDispersion-\int_0^1\left(\tilde{z}_{\mu i}Var\left(\tilde{\mu}|\Phi_i\right)\right)^2 di\right]+\bar{z}^2, \end{aligned} \tag{67}$$

which clearly increases in $BeliefDispersion$. Now, given that $\frac{\partial h\left(P_D;\tilde{l}\right)}{\partial\int_0^1 D_{Si}^2 di}<0$, $\frac{dP_D}{dBeliefDispersion}$ is positive. Next, note that:

$$\begin{aligned} E\left(P_S\right) &= E\left[\int_0^1 E\left(\tilde{x}|\Phi_i\right)di-\frac{1}{\tau}\tilde{z}_\mu Var\left(\tilde{\mu}|\Phi_i\right)-\frac{1}{\tau}\bar{z}\left(P_D+Var\left(\tilde{\mu}|\Phi_i\right)\right)\right] \\ &= E\left(\tilde{x}\right)-\frac{1}{\tau}\bar{z}\left(E\left(P_D\right)+Var\left(\tilde{\mu}|\Phi_i\right)\right). \end{aligned} \tag{68}$$

Hence, an increase in $BeliefDispersion$ that leaves $Var(\tilde{\mu}|\Phi_i)$ unchanged only affects $E(P_S)$ through P_D; given that P_D increases in $BeliefDispersion$, $E(P_S)$ decreases in $BeliefDispersion$. ■

Proof of Proposition 6. i) This follows from the proof of Proposition 5.

ii) Letting $P_D = \int_0^1 E\left(\tilde{V}|\Phi_i\right) di + VRP$, the proof follows by replicating the proof of Corollary 3 holding fixed $\int_0^1 E\left(\tilde{V}|\Phi_i\right) di$.

iii) This follows from an examination of expression (5). ■

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