Abstract

This paper studies the corporate governance and asset pricing implications of investors owning blocks in multiple firms. Common wisdom is that multi-firm ownership weakens governance because the blockholder is spread too thinly. We show that this need not be the case. In a single-firm benchmark, the blockholder governs through exit, selling her stake if and only if the firm is underperforming. With multiple firms, the blockholder disguises the sale of one underperforming firm as being motivated by a liquidity shock, by either selling the second firm even if it is value-maximizing, or not selling the second firm even if it is underperforming. This effect weakens governance. On the other hand, governance can be stronger, because selling one firm and not the other is a powerful signal of underperformance. Common ownership creates strategic complements in managerial effort and has asset pricing implications: stock prices can be correlated even if firms’ cash flows are independent. We derive empirical predictions for the direction of correlation and for whether governance is stronger or weaker with multiple firms.

Keywords: Blockholders, corporate governance, exit, trading, correlation

JEL Classification: D72, D82, D83, G34
Most existing governance theories consider a single firm and either a single blockholder, or multiple blockholders owning stakes in the same firm. However, in reality, many institutional investors hold blocks in multiple firms (Antón and Polk (2013), Gao, Moulton, and Ng (2013)). This paper studies the implications of common ownership for corporate governance and asset pricing. In particular, we address two broad questions. First, does holding multiple blocks weaken governance by spreading a blockholder too thinly, as commonly believed? If not, under what conditions can multi-firm ownership improve governance? Second, can common ownership lead to correlation between stocks with independent fundamentals, and if so, in which direction?

In our model, the blockholder governs through “exit”, i.e. disciplining the manager by selling shares and driving down the stock price, as in Admati and Pfleiderer (2009) and Edmans (2009). Such sales reduce the pay of an equity-aligned manager ex post, thus inducing him to maximize firm value ex ante. We model governance through exit rather than “voice” (intervention) for three reasons. First, governance through exit has asset pricing implications, since it involves the blockholder trading the firm’s stock. Second, if holding multiple blocks forces the investor to hold smaller stakes in each individual firm, then she may not have sufficient control rights to intervene. In contrast, exit does not require the blockholder to have control rights. Third, McCahery, Sautner, and Starks (2011) report that exit is the main governance mechanism used by institutional investors.¹

As a benchmark against which to assess the effects of common ownership, we start with an exit model in which the blockholder owns two shares in a single firm. The manager can take an action (such as shirking, cash flow diversion, or empire building) that yields him a fixed private benefit, but reduces firm value by a random amount privately known to the manager. He follows a cutoff strategy in which he shirks if and only if the value loss is below a threshold; a lower threshold corresponds to greater efficiency and thus superior governance.

The blockholder observes the manager’s action and may choose to sell shares. As in Admati and Pfleiderer (2009), her trade is perfectly observed by the market maker, but not fully revealing of her information because she may also be forced to trade due to a privately-observed liquidity shock. We consider two variations of the model: one in which she faces large liquidity shocks that force her to sell both shares, and one in which she faces small shocks that force her to sell one share (although she may still choose to sell both). Selling either one or two shares reduces the stock price. Thus,

¹See Parrino, Sias, and Starks (2003), Bharath, Jayaraman, and Nagar (2013), and Edmans, Fang, and Zur (2013) for further evidence of governance through exit.
the threat of exit disciplines the manager and reduces the threshold value loss below which he shirks. Interestingly, the threshold is the same in both the large-shock and small-shock models, and so governance is equally strong in each model.

The core analysis is a two-firm model, where the blockholder owns a single share in each firm. Each manager privately observes the value loss from shirking in his firm before taking his action, and this variable is uncorrelated across the two firms. Each manager also has the same stock price concerns and the same private benefit from shirking. Thus, the fundamentals of the two firms are deliberately independent; the firms are only related through common ownership by the blockholder.

We first study the large-shock model, in which the blockholder may be forced to sell her share in both firms. If the blockholder observes shirking in only one firm, she has two options. First, she may sell only her stake in the underperforming firm (single exit). The disadvantage of this strategy is that selling only one stake is inconsistent with a liquidity shock, and thus fully revealing of her information. Second, she may sell her stakes in both firms (double exit), to disguise her trades as being driven by a liquidity shock. Thus, while selling the value-maximizing firm leads to losses, they may be offset by the lower price impact from selling the underperforming firm.

We show that the double exit strategy is more likely if the liquidity shock is more common, so that selling both shares provides more camouflage. Double exit is also more likely if agency problems are weak – the private benefits from shirking are lower or the manager’s stock price concerns are higher (so that he is more concerned by blockholder exit). Both of these scenarios weaken the agency problem: the threshold value loss below which the manager shirks is lower. Since less value is destroyed by shirking, the market maker sets a higher price upon observing either single or double exit. While the blockholder receives the double-exit price for both blocks that she sells, she receives the single-exit price only for the sold block. Thus, an increase in both the double-exit and single-exit prices – resulting from a weaker agency problem – make the double-exit strategy relatively more attractive.

The double-exit equilibrium is less efficient than the single-firm benchmark. Since the manager’s firm may be sold even if he works, he is more likely to shirk. In contrast, the single exit equilibrium is more efficient than the single-firm benchmark. Exit in a single-firm model only leads to a modest price drop, since it may result from a liquidity shock. However, single exit in a multi-firm model leads to a large price drop, because it is fully revealing of shirking. The blockholder can punish shirking more strongly by selling the underperforming firm and retaining the value-maximizing firm. Combined with the above comparative statics results, governance is stronger in the multi-firm
model when liquidity shocks are less frequent and the agency problem is stronger. Thus, when agency problems are strong, blockholders should be more likely to hold stakes in multiple firms, to give them the option to discipline shirking by engaging in single exit.

In both the double-exit and single-exit equilibria, the manager’s actions are strategic complements: shirking by one manager makes the other manager more likely to shirk. With double exit, shirking in one firm leads to the second firm being sold even if it is value-maximizing. Thus, the benefits of working are lower if the other manager is more likely to shirk. With single exit, working in one firm increases the punishment from shirking in the second firm, since the stock price is very low if it is the only firm sold. Thus, the benefits of working are higher if the other manager is more likely to work.

Common ownership leads to correlation between the stock prices of both firms, even though their fundamentals are independent. On the one hand, the possibility of a large liquidity shock, where both firms are sold, leads to a tendency towards positive correlation. On the other hand, single exit depresses the price of the sold firm but increases the price of the retained firm. Overall, the correlation is negative in the single-exit equilibrium and positive in the double-exit equilibrium. Thus, the frequency of liquidity shocks and the severity of the agency problem affects the direction of correlation, through determining whether the blockholder engages in single- or double-exit. Stock price correlations typically depend on the correlation between random variables, e.g. whether idiosyncratic shocks across firms are positively or negatively correlated. Here, interestingly, the correlation instead depends on non-random parameters. The model thus suggests that asset pricing regressions of correlation should control for ownership structure and agency problems.

We then move to the multi-firm, small-shock model. Again, two equilibria are possible. In the first, the blockholder never engages in double exit. Even if both managers shirk, she sells her block in only one firm (chosen at random), as she can disguise her trade as being driven by a small liquidity shock. Unlike in the large-shock model, in which shirking leads to automatic selling, here there is a “tournament”, or relative performance, element to avoiding exit: shirking does not automatically lead to selling if the other manager also shirked.

This “tournament” element may suggest that, if a firm is not sold, its price should be higher if the second firm is sold than if it is not, since the firm beats its rival in the former case. This is similar to how Gervais, Lynch, and Musto (2005) show that a mutual fund family firing one fund manager increases investors’ perceived skill of
retained managers. Interestingly, this result does not hold here. The sale of neither firm implies that both managers have worked and leads to the highest possible price for each firm. The sale of only the second firm does not reveal that the first manager worked: it could be that both managers shirked and the blockholder sold the second firm at random.

As in the large-shock model, the managers’ actions are strategic complements. Since the highest possible price is received only if both managers work, the first manager is likelier to work if the second manager is also. Moreover, shirking does not lead to automatic selling, since if both managers shirk, only one firm is sold. Thus, the first manager is likelier to work if the second manager is also, reinforcing the first effect. However, in contrast to the large-shock model with single exit, which is always more efficient than the single-firm benchmark, the small-shock model without double exit is more efficient only if agency problems are weak. There are two differences between the small-shock model without double exit and the benchmark. First, in the benchmark, a shirking manager is sold with certainty, but in the two-firm model, he is not sold if the other manager also shirks and is randomly selected. This consideration increases the incentives to shirk, particularly if agency problems are severe, as then the second manager is more likely to shirk. Second, in the benchmark, if the manager works and the blockholder suffers a liquidity shock, his firm is sold with certainty. In the two-firm model, the blockholder can satisfy her liquidity needs by selling the other firm. The price of the retained firm is decreasing in agency problems, because not being sold does not fully reveal that the manager has worked. The more severe the agency problem, the lower the price of the retained firm. This reduces the manager’s incentive to work and avoid being sold. Overall, more severe agency problems both reduce the manager’s incentive to work and increase the manager’s incentive to shirk, compared to the single-firm benchmark. When exit is relatively ineffective in governing a single firm, it is even less effective in governing two firms.

The second equilibrium involves the blockholder selling both firms if both managers have shirked. For this trade not to be fully revealing, she must sometimes sell both firms if neither manager has shirked and she faces a liquidity shock. Thus, this equilibrium is only sustainable when the agency problem is mild, as then the blockholder does not receive too low a price when she sells both shares. Since shirking now automatically leads to being sold, the disadvantage of the two-firm model (relative to the benchmark) is avoided. However, the advantage of the two-firm model is retained: a liquidity shock does not automatically lead to the sale of a value-maximizing firm, and so the incentive to work is higher. Thus, the two-firm, small-shock equilibrium with double exit is more
efficient than both the equilibrium without double exit and the single-firm benchmark. In both equilibria in the small-shock model, stock price correlations are negative despite the managers’ actions being strategic complements, because the blockholder is never forced to sell both shares due to a liquidity shock.

In general, multi-firm governance is more effective when, in equilibrium, the blockholder chooses to trade in a way that is inconsistent with liquidity shocks. Thus, her trades are more revealing of shirking, and exert stronger governance. In the large-shock model, the blockholder is less likely to engage in double-exit – i.e. imitate a large liquidity shock – when agency problems are strong. Since the market maker would infer high value destruction if the blockholder sold both shares, she would receive low prices from doing so. The blockholder thus engages in single-exit, which is inconsistent with a liquidity shock and thus imposes strong discipline on the manager. Her single-exit strategy generates negative stock price correlations; if agency problems are weak, she engages in double-exit and the correlation is positive. In the small-shock model, the blockholder’s trading is inconsistent with liquidity shocks when agency problems are weak. Selling both firms, when both managers shirk, does not lead to a large negative price impact. Thus, the blockholder finds it optimal to do so, again leading to superior governance. Stock price correlations are always negative. Our results suggest that the effect of multi-firm ownership on corporate governance, and the correlation between stock prices, depends not only on the frequency of liquidity shocks, but also their magnitude. In all cases, the managers’ actions are strategic complements.

This paper builds on a recent theoretical literature on governance through exit. Admati and Pfleiderer (2009) and Edmans (2009) study exit in a model with a single firm and single blockholder, and Levit (2013) analyzes the interaction between exit and voice in a single-firm, single-blockholder framework. Edmans and Manso (2011) model both exit and voice in a single-firm, multi-blockholder model. To our knowledge, the effectiveness of exit in a multi-firm model has not been previously studied. Our paper is related to Gervais, Lynch, and Musto (2005) who show that mutual fund families can add value by monitoring multiple managers, since firing one manager increases investors’ perceived skill of retained managers. This intuition is similar to why multi-firm governance can be more effective under single-exit. In addition to the context, there are important differences between the two models. First, our model studies moral hazard while Gervais et al. study adverse selection. This moral hazard channel is why single-exit can sometimes be less efficient than the single-firm benchmark, depending on the severity of the agency problem. Second, in Gervais et al., the fund family can commit to a firing policy; under this assumption, the fund family always creates value.
Here, the blockholder cannot commit to a trading strategy, which is why multi-firm governance can be less efficient. Vayanos and Woolley (2012) study how common ownership affects co-movement, but not governance or how the direction of co-movement depends on agency problems; we show that common ownership can sometimes lead to negative correlations.

Empirically, Gao, Moulton, and Ng (2013) document return predictability across economically unrelated stocks with common institutional ownership. Antón and Polk (2013) show that common ownership led to stocks exhibiting excess co-movement during the 2003 mutual fund trading scandal, which caused implicated funds to suffer a large liquidity shock. Jotikasthira, Lundblad, and Ramadorai (2012) find that liquidity shocks to mutual funds that invest in multiple emerging markets cause co-movement between these markets. Bartram, Griffin, and Ng (2013) find that a company’s stock return is higher when the returns to foreign stocks held by its institutional investors are high. Away from return correlation, Lou (2012) empirically analyzes a different consequence of common ownership: strong performance by one fund improves the performance of other funds holding the same stock, by inducing inflows into the first fund which cause upward price pressure. Matvos and Ostrovsky (2008) and Harford, Jenter, and Li (2011) study the extent to which common ownership of two firms affects the likelihood of them merging.

This paper is organized as follows. Section 1 presents the general model. Section 2 analyzes the case in which the blockholder faces large liquidity shocks, and Section 3 considers the small-shock model. Section 4 concludes. Appendix A gives all proofs not in the main text, and Appendix B considers a model with both large and small shocks.

1 Setup

The model consists of three periods and two public and symmetric firms, $i$ and $j$. A blockholder owns a single share in each firm; the remaining shares are owned by dispersed shareholders who play no role in the model. The blockholder represents a mutual fund, hedge fund, or other institutional investor who has superior information about firm value and thus can engage in governance through exit.

Each firm is run by a manager. At $t = 1$, manager $i$ can take action $a_i \in \{0, 1\}$ which reduces the firm’s fundamental value, but yields him a non-pecuniary private benefit $\beta > 0$, where $\beta$ is common knowledge. Examples of such actions include shirking, cash flow diversion, perquisite consumption, and empire building. For simplicity, we will refer to the choice of $a_i = 1$ as “shirking” and $a_i = 0$ as “working”. We will abuse
language slightly by using the phrase “shirking firm” to refer to a firm run by a manager
who has shirked, and “working firm” analogously, and the phrase “the manager will
be sold” to refer to the stock of the firm run by the manager being sold. The action
affects the fundamental value of the firm at \( t = 3 \), which is given by:

\[
v(\theta_i, a_i) = \bar{v} - a_i \theta_i,
\]

(1)

where \( \bar{v} \), the firm’s fundamental value if the manager works, is common knowledge. The
random variable \( \theta_i \in [0, \bar{v}] \) represents the value erosion from shirking and is privately
known to manager \( i \) (but not manager \( j \)) when he chooses his action. The probability
density function of \( \theta_i \) is given by \( f \) and its cumulative distribution function is given by
\( F \). Both are continuous and have full support. Moreover, \( \theta_i \) and \( \theta_j \) are independent of
each other. We assume that

\[
\beta \in (0, \bar{v}),
\]

(2)

which is a necessary condition for shirking to reduce total surplus: the private benefit
enjoyed by the manager is less than the maximum value erosion.

The manager’s preferences are given by:

\[
u_{M,i} = v(\theta_i, a_i) + \omega p_i + a_i \beta,
\]

(3)

where \( p_i \) is firm \( i \)’s stock price at \( t = 2 \), which is set by a market maker as described
below. The variable \( \omega > 0 \) captures the manager’s concern for the stock price, which
is standard in theories of governance through exit and can stem from a number of
sources introduced in prior work. Examples include takeover threat (Stein (1988)),
termination threat (Edmans (2011)), concern for managerial reputation (Narayanan
(1985), Scharfstein and Stein (1990)), or the manager expecting to sell his own shares
at \( t = 2 \) (Stein (1989)).

Dispersed shareholders and the blockholder do not have formal control. Dispersed
shareholders are uninformed about \( a = (a_i, a_j) \) and \( \theta = (\theta_i, \theta_j) \). The blockholder
privately observes \( a \). After doing so, she can trade at \( t = 2 \) with a competitive risk-
neutral market maker who does not observe \( a \) or \( \theta \). If the blockholder is not hit by
a liquidity shock, she is free to choose whether to retain or sell her share in each
firm. With probability (“w.p.”) \( \delta_L \in [0, 1) \), she is hit by a large liquidity shock which
forces her to sell her shares in both firms, and w.p. \( \delta_S \in [0, 1) \) she is hit by a small
liquidity shock which forces her to sell her shares in one firm only (although she may
still voluntarily choose to sell both shares). We assume \( 0 < \delta_S + \delta_L < 1 \). Denote the
blockholder’s decision to sell her share in firm $i$ by $s_i = 1$, and her decision to keep it by $s_i = 0$.\footnote{For time being we assume she cannot sell half of the stake of each company. Since the blockholder is risk-neutral, she is indifferent between the two strategies.} As in the exit models of Admati and Pfleiderer (2009) and Edmans (2009), the blockholder cannot buy additional shares; those papers show that the exit governance mechanism is robust to allowing for purchases.

As in Admati and Pfleiderer (2009), the market maker observes the blockholder’s trading decisions $s = (s_i, s_j)$, but not whether she has suffered a liquidity shock. He sets the stock price $p_i(s)$ to equal the firm’s expected value. Overall, conditional on $s$, the blockholder’s utility is given by:

$$u_I(s) = \sum_i [s_ip_i(s) + (1 - s_i)v(\theta_i, a_i)].$$ \hspace{1cm} (4)

The equilibrium concept we use is Perfect Bayesian Nash Equilibrium. Here, it is defined as follows: (i) A trading strategy by the blockholder that maximizes her expected utility $u_I$ given the price-setting rule of the market maker, the strategy of each manager, and her information on $a$. (ii) A decision rule by each manager $i$ that maximizes his expected utility $u_{M,i}$ given his information on $\theta_i$, the price-setting rule of the market maker and the trading strategy of the blockholder. (iii) A price setting strategy by the market maker that allows him to break even in expectation, given the strategy of the blockholder and manager. Moreover, (iv) the market maker uses Bayes’ rule to update his beliefs from the blockholder’s trades, (v) beliefs on outcomes not observed on the equilibrium path satisfy the Cho and Kreps (1987) intuitive criterion, and (vi) all agents have rational expectations in that each player’s belief about the other players’ strategies is correct in equilibrium. Finally, we focus on symmetric equilibria in which the managers follow the same strategy and the market maker uses a symmetric pricing function.\footnote{Asymmetric equilibria may exist. We focus on symmetric equilibria since firms are symmetric. Thus, if there is an asymmetric equilibrium, there exists another equilibrium in which firms switch roles (since they are ex ante identical). These equilibria are unattractive as it is indeterminate which role each firm will play. Relatedly, in Lemma 6 in the Appendix we show that if the managers follow the same strategy and the market maker follows a symmetric pricing strategy, the blockholder’s best response is a symmetric strategy as well.}

In what follows we analyze two polar cases. In the first case, we set $\delta_S = 0$ and $\delta_L = \delta \in (0, 1)$ so the blockholder faces only large shocks. In the second case, we set $\delta_S = \delta \in (0, 1)$ and $\delta_L = 0$ so the blockholder faces only small shocks. If the blockholder faces a small shock and is indifferent between selling each share, then she sells one w.p. $\frac{1}{2}$. In each case, we use $\chi = 1$ ($\chi = 0$) to denote the case where the blockholder is (is...
subject to a liquidity shock. In Appendix B we fully characterize the case with small and large shocks simultaneously \((\delta_S > 0, \delta_L > 0)\), and show that the polar cases are the limit of this general case.

Finally, in this paper, we define efficiency as the maximization of firm value \(v\), rather than total surplus (which includes the private benefit \(\beta\)), since governance is typically focused on shareholder value maximization.

\section{Large Shocks}

In this section we consider the case of only large shocks, i.e. \(\delta_S = 0\) and \(\delta_L = \delta \in (0,1)\). Our goal is to study how the effectiveness of governance through exit changes when a blockholder holds stakes in multiple firms. Thus, we start with a benchmark where the blockholder holds shares in one firm only.

\subsection{Single-Firm Benchmark with Large Shocks}

Suppose the blockholder owns two shares in firm \(i\) and zero shares in firm \(j\), and so we drop the \(i\) subscript for brevity. The blockholder can either retain both shares, sell one share, or sell both shares. We denote the blockholder’s trading decision by \(s \in \{0,1,2\}\). Her utility function is now:

\begin{equation}
    u_I(s) = sp(s) + (2-s) v(\theta, a).
\end{equation}

The equilibrium for this case is given in Proposition 1 below.

\textbf{Proposition 1.} In any equilibrium with one firm and only large shocks:

\(i\) If \(a_i = 0\) and \(\chi = 0\), then \(s < 2\).

\(ii\) If \(a_i = 1\), then \(s = 2\) w.p. 1.

\(iii\) Prices satisfy

\begin{align*}
    p(0) &= \bar{v} \\
    p(2) &= \bar{v} - \frac{F(\theta^*)}{(1 - F(\theta^*))} \delta + F(\theta^*) \mathbb{E}[\theta|\theta < \theta^*].
\end{align*}

When \(s = 1\) is on the equilibrium path, \(p(1) = \bar{v}\).

\(iv\) The manager shirks if and only if \(\theta_i \leq \theta_B^*\), where \(\theta_B^*\) is uniquely defined by the
solution of $\psi_B(\theta^*) = \theta^*$ and

$$\psi_B(\theta^*) = \beta - \omega (1 - \delta) \frac{F(\theta^*)}{F(\theta^*) + \delta (1 - F(\theta^*))} \mathbb{E}[\theta|\theta < \theta^*]. \quad (6)$$

The proof for parts (i)-(iii) of Proposition 1 is given in Appendix A. Part (ii) of Proposition states that, if the blockholder observes shirking, she will sell both shares. One might think that she may prefer to sell only one share, as this would reduce price impact in a Kyle (1985)-type model. However, here selling two shares can be camouflaged as being driven by a liquidity shock, but selling one share cannot be.

Part (iv) states that the manager follows a threshold strategy: he shirks if and only if $\theta < \theta_B^*$, i.e. when the damage to fundamental value $\theta$ is sufficiently small. The proof of part (iv) is as follows. If the manager chooses $a = 1$, then $s = 2$ for sure and the price is $p(2)$. If $a = 0$, then w.p. $\delta$ the blockholder chooses $s = 2$, in which case the price is $p(2)$, and w.p. $(1 - \delta)$ the blockholder chooses $s < 2$, in which case $p(s) = \pi$. Thus, the manager chooses $a = 1$ if and only if

$$\pi - \theta + \omega p(2) + \beta > \pi + \omega (\delta p(2) + (1 - \delta) \pi) \iff \beta - \omega (1 - \delta) \frac{F(\theta^*)}{F(\theta^*) + \delta (1 - F(\theta^*))} \mathbb{E}[\theta|\theta < \theta^*] > \theta. \quad (7)$$

An equilibrium must satisfy expression (8) with equality. The first term on the left-hand side (“LHS”) of (8) represents the private benefits from shirking, $\beta$. The second term on the LHS represents the expected reduction in the stock price from shirking. The term on the right-hand side (“RHS”) represents the fall in fundamental value from shirking, $\theta$. We denote the LHS by $\psi_B(\theta^*)$. Since $\beta \in (0, \pi)$ (equation (2)), we have $\psi_B(0) > 0$ and $\psi_B(\pi) < \pi$. The Intermediate Value Theorem implies that a fixed point of $\psi_B(\cdot)$ always exists, and lies in the interval $(0, \beta)$. Since $\psi_B(\theta^*)$ is decreasing in $\theta^*$, a unique value of $\theta$ satisfies (8) with equality. We denote it by $\theta_B^*$.

In any equilibrium where the manager follows threshold $\theta^*$, ex-ante shareholder value is given by

$$V(\theta^*) = \pi - \text{Pr}[\theta_i < \theta^*] \mathbb{E}[\theta_i|\theta_i < \theta^*],$$

which is decreasing in $\theta^*$. Thus, a higher $\theta_B^*$ corresponds to more shirking, and thus weaker governance and a less efficient equilibrium.

Since $\psi_B(\cdot)$ increases in $\delta$, $\theta_B^*$ is increasing in $\delta$: the greater the probability of the liquidity shock, the greater the frequency of shirking. The blockholder exerts governance by exiting if the manager shirks, but if she suffers a liquidity shock, she
exits even if the manager has worked, thus reducing the incentives to do so. Put differently, with a liquidity shock, the blockholder’s trade – and thus the stock price – is less driven by firm fundamentals. Thus, the manager has a weaker incentive to improve firm fundamentals. Similarly, since \( \psi_B(\cdot) \) increases in \( \beta \) and decreases in \( \omega \), then so does \( \theta^*_B \). Where the manager’s private benefit of shirking \( \beta \) is higher, and his concern for the stock price \( \omega \) is lower, he is less likely to work.

### 2.2 Multi-Firm Governance With Large Shocks

We now move to the main analysis where the blockholder has a single share in each firm. Given our focus on symmetric equilibria, we have \( p_i(s_i, s_j) = p_j(s_j, s_i) \) for any \( s \in \{0,1\} \times \{0,1\} \). We continue to omit the subscript \( i \) whenever there is no risk of confusion. We first start with a number of useful results.

**Lemma 1.** In any equilibrium, the following hold:

(i) There exists \( \theta^* \in (0, \overline{\theta}) \) such that \( a^*_i = 1 \) if and only if \( \theta_i < \theta^* \).

(ii) If \( a_i < a_j \) then \( s_i \leq s_j \).

(iii) If \( a = (0,0) \) and \( \chi = 0 \) then \( s = (0,0) \) w.p. 1.

Part (i) of Lemma 1 states that manager \( i \) follows a threshold strategy (as in Section 2.1): he shirks if and only if \( \theta_i < \theta^* \). Part (ii) 1 implies that if manager \( j \) shirks but manager \( i \) does not shirk, in equilibrium the blockholder will never sell firm \( i \) but not \( j \). Part (iii) 1 states that, if both managers work and the blockholder does not suffer a liquidity shock, she retains both shares. In Appendix B we show that Lemma 1 applies also to the general case with both small and large shocks, and thus the small-shock model.

Lemma 2 is specific to the case with large shocks only.

**Lemma 2.** In any equilibrium with large shocks only, the following hold:

(i) \( p(1,0) < p(1,1) < \overline{v} \).

(ii) If \( s = (1,0) \) is on the equilibrium path, then \( p(1,0) = \overline{v} - \mathbb{E}[\theta|\theta < \theta^*] = \underline{v}(\theta^*) \).

(iii) If \( a = (1,1) \) then \( s = (1,1) \) w.p. 1.

Parts (i) of Lemma 2 suggests that, conditional on selling firm \( i \), its price is higher when the blockholder also sells firm \( j \) than when she does not. Part (ii) suggests that, whenever the blockholder sells only one share in equilibrium, the price of that share is \( \underline{v}(\theta^*) \), i.e. the lowest possible price that assumes that the manager has shirked with certainty. The decision to sell only one share fully reveals that the blockholder has not suffered a liquidity shock, and thus that the manager has shirked. Part (iii) states
that, if both managers shirk, the blockholder sells both shares for sure. This result also implies that if \( s = (1, 0) \), we must have \( a = (1, 0) \). Therefore, \( p(0, 1) = \tau \).

So far, we conclude that, if \( \chi = 0 \), the blockholder sells neither share if both managers have worked, and both shares if both managers have shirked. The interesting case is when \( a = (1, 0) \) and \( \chi = 0 \). We know from Part (ii) of Lemma 1 that \( s \neq (0, 1) \). Thus, the blockholder has three choices. First, she could choose \( s = (1, 0) \), i.e. sell only the shirking firm. Since the sale of one share is inconsistent with a liquidity shock, it is fully revealing that the manager has shirked and so she receives a low price of \( p_i = v(\theta^*) \). Second, the blockholder may choose \( s = (1, 1) \), i.e. sell both shares. By selling firm \( j \) as well, she can pretend that her sales are motivated by a liquidity shock rather than shirking. On the one hand, she receives a higher price for stock \( i \) since the market maker is not certain that manager \( i \) has shirked. On the other hand, she suffers a loss on her sale of firm \( j \): the true value of firm \( j \) is \( \tau \), but the price she receives is strictly lower since it incorporates the possibility that the sale was motivated by shirking. We use “double exit” to denote the cases in which the blockholder sells both shares, and “single exit” to denote the case in which the blockholder sells only one share. Finally, the blockholder may choose \( s = (0, 0) \).

Not that if the blockholder chooses, \( s = (0, 0) \), then we must have \( p(1, 0) = v(\theta^*) \), else the blockholder is strictly better off by choosing \( s = (1, 0) \). Thus, if there is an equilibrium in which the blockholder retains a shirking manager, she must be indifferent between \( s = (0, 0) \) and \( s = (1, 0) \). In Proposition 8 in Appendix A we show that if there is an equilibrium in which the blockholder chooses \( s = (0, 0) \) with a strictly positive probability when \( a = (1, 0) \) and \( \chi = 0 \), then there is another equilibrium in which she never chooses \( s = (0, 0) \) in those cases, and this equilibrium is strictly more efficient. Intuitively, by retaining a shirking manager the blockholder exerts weaker governance, reducing efficiency. Moreover, we show that equilibria in which the blockholder chooses \( s = (0, 0) \) with a strictly positive probability when \( a = (1, 0) \) and \( \chi = 0 \) are generically not robust to the inclusion of small shocks \( (\delta_S > 0) \). For all of these reasons, we henceforth focus on equilibria in which the blockholder never retains both firms when \( a = (1, 0) \) and \( \chi = 0 \).

Let \( \gamma \) be the probability that the blockholder chooses \( s = (1, 1) \) in equilibrium when \( a = (1, 0) \) and \( \chi = 0 \). Thus, she chooses \( s = (1, 0) \) w.p. \( 1 - \gamma \) in those cases. She prefers double-exit over single-exit \( (u_I(1, 1) > u_I(1, 0)) \) if and only if

\[
2p(1, 1) \geq p(1, 0) + \tau
\]  

(9)
For any $\gamma$ and $\theta^*$ we calculate the price functions. If $\gamma < 1$ then $s = (1, 0)$ is on the equilibrium path, and based on part (ii) of Lemma 2, $p(1, 0) = v(\theta^*)$. Moreover, since the blockholder never retains a shirking firm then $p(0, 0) = p(0, 1) = \overline{v}$. Last, according to Bayes’ rule, the price set by the market maker upon double-exit is

$$
p(1, 1) = \overline{v} - \left( \frac{F(\theta^*) [(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma) + \delta]}{F(\theta^*) [(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma) + \delta] + (1 - F(\theta^*)) (\gamma (1 - \delta) F(\theta^*) + \delta)} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*] \quad (10)
$$

Using (10) and the above price functions, Lemma 3 shows how the equilibrium $\gamma$ depends on the threshold $\theta^*$.

**Lemma 3.** In any equilibrium with two firms and only large shocks, $\gamma \in \gamma(\theta^*)$ where

$$
\gamma(\theta^*) = \begin{cases} 
\{1\} & \text{if } \theta^* < Y(\delta) \\
[0, 1] & \text{if } \theta^* = Y(\delta) \\
\{0\} & \text{if } \theta^* > Y(\delta), 
\end{cases} \quad (11)
$$

and $Y(\delta) \equiv F^{-1}\left(\frac{\sqrt{\delta}}{1 + \sqrt{\delta}}\right)$.

When $a = (1, 0)$ and $\chi = 0$, the blockholder sells both shares if $\theta^* < Y(\delta)$. This is intuitive. The lower $\theta^*$ is, the less value is destroyed by shirking. Thus, $p(1, 1)$ is relatively high: even if double exit arises because both managers have shirked, the value erosion is small. This relatively high price increases the blockholder’s willingness to sell both firms even if only one manager has shirked. On the other hand, if the manager only sells the shirking firm, the price that the blockholder receives ($p(1, 0)$) is also higher when $\theta^*$ is lower, which increases her incentive to sell only the shirking firm. Thus, a fall in $\theta^*$ augments both $p(1, 1)$ and $p(1, 0)$, but the effect on the blockholder’s payoff upon double exit is greater. With double exit, she receives $p(1, 1)$ for both stocks (her overall payoff is $2p(1, 1)$), but with single exit, she receives $p(1, 0)$ only for the share that she sells (her overall payoff is $\overline{v} + p(1, 0)$). Thus, a fall in $\theta^*$ increases the attractiveness of double exit relative to single exit.

In equilibrium, manager $i$ shirks ($a_i = 1$) if and only if

$$
\beta - (1 - \delta) \omega \left[ + (1 - \gamma^*) F(\theta^*) \times p(0, 1) + (1 - F(\theta^*)) \times [p(0, 0) - p(1, 0)] - [F(\theta^*) (1 - \gamma^*) + \gamma^* (1 - F(\theta^*))] \times p(1, 1) \right] > \theta_i \quad (12)
$$

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As stated in Lemma 1, the manager follows a threshold strategy. Notice that manager $i$’s decision to shirk depends on how frequently he expects manager $j$ to shirk. This is reflected above by the dependence of the LHS of (12) on $F(\theta^*)$, the probability that manager $j$ shirks.

Lemma 4 studies whether the managers’ strategies are strategic complements or substitutes. In our context, strategic complements (substitutes) arise if, given the prices and blockholder strategies, the best response (that is, the threshold) of manager $i$ increases (decreases) with the threshold of manager $j$ – i.e. the LHS of (12) is increasing (decreasing) in $\theta^*$.

Lemma 4. In the two-firm equilibrium with large shocks only, given $p(s)$ and the blockholder’s selling strategy, the managers’ decisions exhibit strategic complements.

The intuition is as follows. If the manager shirks, he is sold with certainty, regardless of the other manager’s action. The manager’s motivation to work is to reduce the stock price decline that results from being sold. If the manager works and $\chi = 1$, he is still sold, regardless of the other manager’s action. Strategic complements or substitutes arise from the case in which the blockholder does not suffer a liquidity shock. There are two sub-cases to consider.

First, if $\gamma > 0$, the blockholder sells both shares with a strictly positive probability if $\max\{a_i, a_j\} = 1$, i.e. at least one manager shirks. Under this equilibrium, even if the manager works and $\chi = 0$, there is a strictly positive probability that he is still sold if the other manager shirks. Thus, the greater the likelihood that manager $j$ shirks, the greater the likelihood that manager $i$ will be sold even if he works, and so the more likely he will be to shirk also, leading to strategic complements. Second, if $\gamma = 0$, the blockholder never sells firm $i$ if manager $i$ works, independent of $j$’s action. However, while $j$’s action does not affect the probability that firm $i$ is sold, it does affect firm $i$’s stock price if it is sold. If $j$ is more likely to work, there is a high likelihood that $i$ will be the only firm sold. In this case, $p_i$ will equal the lowest possible stock price of $\underline{v}(\theta^*)$, because the sale of a single firm reveals that a liquidity shock did not occur. In contrast, if $j$ is more likely to shirk, then it is more likely that both firms are sold if $i$ also shirks. Double exit does not lead to as low a stock price as it is consistent with a liquidity shock. Again, $i$ is more likely to work if $j$ is.

The next result characterizes the threshold $\theta_L^*$ and the probability $\gamma^*$ that arise in equilibrium.

Proposition 2. In the two-firm model with only large shocks, a symmetric equilibrium always exists, is unique, and is characterized by the following:
(i) If $a = (0, 0)$ and $\chi = 0$, the blockholder chooses $s = (0, 0)$ w.p. 1.

(ii) If $a = (1, 1)$ then $s = (1, 1)$ w.p. 1.

(iii) If $a = (1, 0)$ and $\chi = 1$, the blockholder chooses $s = (1, 0)$ w.p. $1 - \gamma^*$, where

$$
\gamma^* = \max \left\{ 0, 1 + \min \left\{ 0, \frac{1 - \beta - Y(\delta)}{\omega} \frac{2}{(1 - \delta) \mathbb{E}[\theta | \theta < Y(\delta)]} \right\} \right\}
$$

(iv) Prices are given by:

$$
p(0, 0) = \overline{v}
$$

$$
p(1, 1) = \overline{v} - \left( \frac{F(\theta^*_L)[(1 - \delta)(F(\theta^*_L) + (1 - F(\theta^*_L))\gamma^*) + \delta]}{F(\theta^*_L)[(1 - \delta)(F(\theta^*_L) + (1 - F(\theta^*_L))\gamma^*) + \delta] + (1 - F(\theta^*_L))(\gamma^* (1 - \delta) F(\theta^*_L) + \delta)} \right) \mathbb{E}[\theta | \theta < \theta^*_L]
$$

$$
p(1, 0) = \underline{v}(\theta^*_L) \quad (\text{if } \gamma^* < 1)
$$

$$
p(0, 1) = \overline{v} \quad (\text{if } \gamma^* < 1)
$$

(v) Manager $i$ shirks if and only if $\theta_i < \theta^*_L$, where $\theta^*_L$ is defined by $\psi_L(\theta^*_L, \gamma^*) = \theta^*_L$ where

$$
\psi_L(y, \gamma) = \beta - (1 - \delta) \omega \left[ \frac{F(y)(1 - \gamma) + (1 - F(y))\gamma}{1 + \frac{1 - F(y)}{F(y)} \frac{\gamma F(y) + \frac{\gamma}{\gamma + 1}}{F(y) + (1 - F(y))\gamma + \frac{\gamma}{\gamma + 1}}} + (1 - F(y))(1 - \gamma) \right] \mathbb{E}[\theta | \theta < y].
$$

Using the price functions and the blockholder’s trading strategies we derived above, for any threshold $\theta^* = y$, the LHS of expression (12) becomes $\psi_L(y, \gamma)$ in equation (14), i.e. the manager’s private benefit from shirking minus the expected price decline from doing so. A symmetric equilibrium $(\theta^*_L, \gamma^*)$ must satisfy $\psi_L(\theta^*_L, \gamma^*) = \theta^*_L$ where $\gamma^* \in \gamma(\theta^*)$: the manager’s private benefit from shirking equals the loss in fundamental value plus expected price decline.

Note that $\psi_L(y, \gamma)$ – and thus the threshold $\theta^*_L$ – increase in $\delta$ and $\beta$ and decrease in $\omega$, like $\psi_B(y)$ in the single-firm benchmark. The managers are more likely to shirk – i.e. governance is weaker – if liquidity shocks are more frequent, the private benefit from shirking is higher, and the manager’s concern for the stock price is lower. As we

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4If $\gamma^* = 1$ then $s = (1, 0)$ and $s = (0, 1)$ are off equilibrium events. In this case, we only require $2p(1, 1) \geq \overline{v} + p(1, 0)$, which is satisfied for $p(1, 0) = \underline{v}(\theta^*_L)$ when $\gamma^* = 1$. 

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explain later, the magnitude of $\gamma^*$ (the frequency with which the blockholder engages in double exit) has important implications for efficiency and comovement. We thus use (13) to derive comparative statics of $\gamma^*$ with respect to $\beta$, $\omega$ and $\delta$.

**Corollary 1.** (i) $\gamma^*$ decreases with $\beta$ and increases with $\omega$. (ii) There exists $\hat{\delta} > 0$ such that if $\delta < \hat{\delta}$ then $\gamma^* = 0$.

Intuitively, when $\beta$ is low and $\omega$ is high, the agency problem is mild. Thus, $\theta^*_L$ is lower (Proposition 2), which increases the likelihood of double exit $\gamma^*$ (Lemma 3). The comparative static of $\gamma^*$ with respect to $\delta$ is more complicated since both $\theta^*_L$ and $Y(\delta)$ increase with $\delta$. The blockholder’s incentive to sell firm $j$, even if only manager $i$ has shirked, arises from the possibility of disguising the sale of firm $i$ as being motivated by a liquidity shock. This disguise is more plausible if the frequency of liquidity shocks $\delta$ is higher, and so $Y(\delta)$ increases in $\delta$. However, $\theta^*_L$ increases with $\delta$ as well: the greater the likelihood of a liquidity shock, the more likely both managers are to shirk. It turns out that if there are few liquidity shocks ($\delta < \hat{\delta}$), then it is harder to camouflage an information-motivated sale as liquidity-motivated by engaging in double exit. Thus, the blockholder engages in single exit if $\gamma^* = 0$.

Theorem 1 specifies conditions on $\gamma^*(\beta, \omega, \delta)$ under which the single-firm benchmark is more efficient ($\theta^*_L < \theta^*_B$) and so holding stakes in multiple firms weakens governance.

**Theorem 1.** The unique equilibrium with two firms and large shocks is strictly more efficient ($\theta^*_L < \theta^*_B$) than the single-firm benchmark if and only if $\gamma^*(\beta, \omega, \delta) < \frac{\sqrt{\varphi}}{1 + \sqrt{\varphi}}$.

Theorem 1 states that multi-firm governance is more efficient if and only if double exit is sufficiently infrequent. If $\gamma^*(\beta, \omega, \delta) > \frac{\sqrt{\varphi}}{1 + \sqrt{\varphi}}$ then the blockholder engages in double exit often. Based on Corollary 1, $\gamma^*(\beta, \omega, \delta)$ is high when the agency problem is not severe (low $\beta$ and high $\omega$). Double exit leads to weak governance, because even if manager $i$ works and there is no liquidity shock, he will be sold if manager $j$ shirks. Thus, his incentives to work are lower. As a result, $\theta^*_B > \theta^*_L$, i.e. the single-firm benchmark is more efficient.

In contrast, if $\gamma^*(\beta, \omega, \delta) < \frac{\sqrt{\varphi}}{1 + \sqrt{\varphi}}$ then single exit is common, which leads to stronger governance than the single-firm benchmark. It seems that efficiency should be the same, since in both models, manager $i$ is not sold if and only if he works and $\chi = 0$. However, single exit is a greater punishment in the two-firm equilibrium than being sold in the benchmark. If the firm is sold in the benchmark, the price is not too low since the sale is consistent with a liquidity shock. If single exit occurs in the two-firm model, it is inconsistent with a liquidity shock and fully reveals shirking, leading to the lowest
possible price of $v$. Thus, the incentives to work are greater in the two-firm equilibrium. As a result, $\theta_B^* > \theta_L^*$, i.e. the two-firm equilibrium is more efficient.

Theorem 1 also implies that if the blockholder could commit to selling only underperforming firms (i.e. engaging in single exit), then the multi-firm model is always more efficient. However, the blockholder cannot commit to this strategy as she may find it optimal ex-post to sell a working manager in order to disguise the sale of another, shirking, manager as being motivated by a liquidity shock. Thus, the blockholder must have incentives to sell only one firm when one manager shirks. She has these incentives when agency problems are strong, as then selling both firms means that she receives a low stock price for both.

Proposition 3 studies the correlation between the stock prices of the two firms.

**Proposition 3.** In the unique equilibrium with two firms and only large shocks, $\rho^* < 0$ if and only if $\gamma^* (\beta, \omega, \delta) \leq \frac{\sqrt{3}}{1 + \sqrt{3}}$.

Even though firm fundamentals are independent ($\theta_i$ and $\theta_j$ are uncorrelated), their prices comove since they share a common blockholder. Proposition 3 thus suggests that asset pricing regressions of co-movement should control for ownership structure. Intuitively, it may seem that the correlation should be positive, since a liquidity shock causes the blockholder to trade both stocks in the same direction, as predicted by Vayanos and Woolley (2012). However, Proposition 3 shows that the correlation can be negative. While double exit involves the blockholder trading both stocks in the same direction, and moves the model towards a positive correlation, single exit not only reduces the price of the sold share but also increases the price of the retained share, leading to a negative correlation. Proposition 3 shows that the correlation is negative if and only if $\gamma^* (\beta, \omega, \delta)$ – the probability of double exit when $a = (1, 0)$ and $\chi = 0$ – is sufficiently low. In turn, this depends on the severity of the agency problem ($\beta$ and $\omega$) and the frequency of the liquidity shock $\delta$.

Proposition 3 delivers the interesting result that the stock price correlation depends on the severity of the agency problem. When the agency problem is weak (i.e. $\delta$ and $\beta$ are low, and $\omega$ is high), the correlation is negative. Typically, stock price correlations depend on the direction of correlation between random variables (e.g. idiosyncratic shocks) that affect each firm, but not fixed parameters: for example, changing the maximum fundamental value $\overline{v}$ will have no effect on correlation. Here, even though the parameters $\delta$, $\beta$, and $\omega$ are fixed rather than random variables, they affect the direction of correlation.
3 Small Shocks

We now move to the case of only small shocks, i.e. $\delta_S = \delta \in (0, 1)$ and $\delta_L = 0$. We again start with a benchmark where the blockholder holds shares in one firm only.

3.1 Single-Firm Benchmark with Small Shocks

The equilibrium is given in Proposition 4 below:

**Proposition 4.** In any equilibrium with one firm and only small shocks:

(i) The blockholder never sells both shares with a positive probability.

(ii) If $a_i = 1$, then $s = 1$ w.p. 1.

(iii) If $a_i = 0$, then $s = 1$ if and only if $\chi = 1$.

(iv) Prices satisfy

\[
p(0) = \overline{v} \\
p(1) = \overline{v} - \frac{F(\theta^*)}{[1 - F(\theta^*)] \delta + F(\theta^*) \mathbb{E}[\theta|\theta < \theta^*]} \\
p(2) \leq \frac{\psi(\theta^*) + p(1)}{2} \text{ (off the equilibrium path).}
\]

(v) The manager shirks if and only if $\theta < \theta_B^*$, where $\theta_B^*$ is uniquely defined by the solution of $\psi_B(\theta^*) = \theta^*$.

The manager shirks if and only if

\[
\overline{v} - \theta + \omega p(1) + \beta > \overline{v} + \omega (\delta p(1) + (1 - \delta) p(0)),
\]

the same inequality as in the large-shock benchmark (equation (7)), except that $p(2)$ is replaced with $p(1)$. The price $p(1)$ in the small-shock benchmark is identical to $p(2)$ in the large-shock benchmark. Intuitively, $s = 2$ in the large-shock case and $s = 1$ in the small-shock case are both consistent with either shirking or a liquidity shock. Thus, both lead to the same price, and so the equilibrium has the same definition: $\psi_B(\theta^*) = \theta^*$. Note that, with small shocks, the blockholder never sells both shares in equilibrium. Intuitively, the sale of both shares cannot be attributed to a liquidity shock. Thus, if the manager has shirked, the blockholder is better off selling only one share and disguising the sale as being driven by a small liquidity shock.

This single-firm model extends the exit model of Admati and Pfleiderer (2009) to the case in which partial exit is possible. In Admati and Pfleiderer (2009), the
blockholder’s liquidity shock always forces her to sell her entire stake, and thus when she observes shirking, she sells her entire stake. Here, we allow for small liquidity shocks, and so there are equilibria in which the blockholder voluntarily sells only part of her stake. While the equilibria and proofs are more complex, this analysis shows that the conclusions of Admati and Pfleiderer (2009) model – the disciplinary effect of governance through exit – are robust to allowing for partial exit.\footnote{Partial exit sometimes occurs in the exit model of Edmans (2009). That model is different in that it features liquidity traders, and so the blockholder may choose to sell less than her entire stake to camouflage with them. In Admati and Pfleiderer (2009), as in this model, the blockholder’s trade is fully observed, and so there are no partial sales.}

### 3.2 Multi-Firm Governance With Small Shocks

Suppose now the blockholder has a single share in each firm. As mentioned above, Lemma 1 continues to hold. In particular, in any equilibrium, if \(a = (0, 0)\) and \(\chi = 0\), then \(s = (0, 0)\) for sure. The difference from the large-shock model comes from cases where \(a \neq (0, 0)\). In particular, here it is possible to have equilibria in which the blockholder never sells both shares. Proposition 5 characterizes such an equilibrium.

**Proposition 5.** In the two-firm model with only small shocks, an equilibrium in which \(s = (1, 1)\) is not on the equilibrium path always exists and is characterized by the following:

(i) If \(a = (0, 0)\) and \(\chi = 0\), the blockholder chooses \(s = (0, 0)\) w.p. 1.

(ii) If \(a = (1, 0)\), the blockholder chooses \(s = (1, 0)\) w.p. 1.

(iii) If \(a = (1, 1)\), or \(a = (0, 0)\) and \(\chi = 1\), the blockholder chooses \(s = (1, 0)\) and \(s = (0, 1)\) with equal probability.

(iv) Prices are given by:

\[
\begin{align*}
p(0, 0) &= \bar{v} \\
p(1, 0) &= \bar{v} - \frac{1}{2} F(\theta^*)^2 + \frac{F(\theta^*) (1 - F(\theta^*))}{2} \mathbb{E} [\theta_i | \theta_i < \theta^*] \\
p(0, 1) &= \bar{v} - \frac{1}{2} F(\theta^*)^2 + \frac{F(\theta^*) (1 - F(\theta^*))}{2} \mathbb{E} [\theta_i | \theta_i < \theta^*] \\
p(1, 1) &\leq \frac{\bar{v}(\theta^*) + p(1, 0)}{2} \quad \text{(off-equilibrium)}
\end{align*}
\]

(v) Manager \(i\) shirks if and only if \(\theta_i < \theta^* = \theta^*_{S,N,D}\), where \(\theta^*_{S,N,D}\) is defined by
\[ \psi_{S,ND}(\theta_{S,ND}) = \theta_{S,ND}^* \text{ where} \]
\[
\psi_{S,ND}(y) = \beta - \omega (2 - \delta) \frac{F(y) (1 - F(y))}{1 - (1 - \delta)(1 - F(y))^2} \mathbb{E} [\theta | \theta < y]. \quad (15)
\]

In this equilibrium, the price that the blockholder receives if she sells both shares, \( p(1,1) \), is so low that she prefers to sell only one share at random if both managers have shirked. Since we only have small liquidity shocks, \( s = (1,1) \) is unlikely to have resulted from a liquidity shock. She similarly sells one share at random if both managers have worked and she suffers a liquidity shock. Both \( p_i(1,0) \) and \( p_i(0,1) \) increase in \( \delta \) (holding \( F(\theta^*) \) constant). Intuitively, \( p_i(1,0) \) is increasing in \( \delta \) as the sale of \( i \) is more likely to result from a liquidity shock rather than manager \( i \) shirking. \( p_i(0,1) \) is increasing in \( \delta \) as the sale of \( j \) is more likely to result from both managers working and the blockholder suffering a liquidity shock, rather than both managers shirking and the blockholder selling \( j \) at random.

In the large shock model \( p_i(0,0) = p_i(0,1) = \varpi \): if firm \( i \) is not sold, it fully reveals that manager \( i \) has worked, regardless of the blockholder’s trade in stock \( j \). This result no longer holds in the small shock model. With small shocks, it remains the case that \( p_i(0,0) = \varpi \) as the decision to retain both stocks reveals that both managers worked and \( \chi = 0 \). However, \( s = (0,1) \) may arise because both managers shirked and the blockholder sold \( j \) at random. Since the blockholder never sells both stocks in the equilibrium of Proposition 5, there is a “tournament”, or relative performance, element to avoiding disciplinary exit: shirking does not automatically lead to exit if the other manager also shirked. In the large shock model, shirking led to automatic selling.

Note that we have \( p(0,0) > p(0,1) > p(1,0) \). One might think that the relative performance component should lead to \( p(0,0) < p(0,1) \): if stock \( i \) is not sold, its price should be higher if stock \( j \) is sold since firm \( i \) “beats” its rival. However, \( p(0,0) > p(0,1) \) as \( s = (0,0) \) signals that both managers worked for sure, whereas \( s = (0,1) \) may occur if both managers shirked and the blockholder sold \( j \) at random.

Proposition 6 characterizes an equilibrium in which \( s = (1,1) \) is on the equilibrium path.

**Proposition 6.** In the two-firm model with only small shocks, an equilibrium in which \( s = (1,1) \) is on the equilibrium path exists if and only if

\[
\beta - \omega \left( 1 - \frac{\sqrt{\delta}}{2} \right) \mathbb{E} [\theta | \theta < Y(\delta)] \leq Y(\delta). \quad (16)
\]
There exist $0 < \varphi^* \leq 1$ and $0 \leq \gamma^* \leq \varphi^*$, where $\varphi^* = 1 - \frac{(1 - F(\theta^*_{S,D}))^2 \delta - F(\theta^*_{S,D})}{(1 - F(\theta^*_{S,D}))^2} (1 - \gamma^*)$, such that in any equilibrium of this type, the following hold:

(i) If $a = (1, 1)$, the blockholder chooses $s = (1, 1)$ w.p. 1.

(ii) If $a = (0, 0)$ and $\chi = 0$, the blockholder chooses $s = (0, 0)$ w.p. 1.

(iii) If $a = (1, 0)$, the blockholder chooses $s = (1, 1)$ w.p. $\gamma^*$ and $s = (1, 0)$ otherwise.

(iv) If $a = (0, 0)$ and $\chi = 1$, the blockholder chooses $s = (1, 1)$ w.p. $\varphi^*$. W.p. $1 - \varphi^*$ she chooses between $s = (1, 0)$ and $s = (0, 1)$ at random.

(v) Prices are given by:

\[
p(0, 0) = p(0, 1) = \overline{v} - \frac{F(\theta^*) (1 - \gamma^*)}{F(\theta^*) (1 - \gamma^*) + (1 - F(\theta^*)) \frac{1}{2} (1 - \varphi^*)} \mathbb{E} [\theta | \theta < \theta^*]
\]

\[
p(1, 0) = \overline{v} - \frac{F(\theta^*) [F(\theta^*) + (1 - F(\theta^*)) \gamma^*]}{F(\theta^*) [F(\theta^*) + (1 - F(\theta^*)) \gamma^*] + (1 - F(\theta^*)) [(1 - F(\theta^*)) \delta \varphi^* + F(\theta^*) \gamma^*]} \mathbb{E} [\theta | \theta < \theta^*]
\]

(vi) Manager $i$ shirks if and only if $\theta_i < \theta^* = \theta^*_{S,D}$.

Finally, an equilibrium with $\gamma^* = 0$ always exists if condition (16) is satisfied.

Equilibria where $s = (1, 1)$ is on the equilibrium path exist only if $p(1, 1)$ is relatively high. As in the double exit equilibrium in the large-shock model, $p(1, 1)$ is high when $\beta$ is low and $\omega$ is high, i.e. the agency problem is mild so that shirking destroys little value.\(^6\)

**Lemma 5.** In any two-firm equilibrium with small shocks only, given $p(s)$ and the blockholder’s selling strategy, the managers’ decisions exhibit strategic complements.

As in the large-shock model, here we also have strategic complements. The intuition for strategic complements in the equilibrium with double exit (Proposition 6) is similar to the large-shock model. However, the intuition for strategic complements in the equilibrium with no double exit (Proposition 5) is different. Here, if $i$ works, his utility

\(^6\)Note that there are two differences with the double exit equilibrium in the large-shock model. First, in the small-shock model, the blockholder’s decision of whether to sell both shares concerns the case in which both managers shirk. In the large-shock model, it concerns the case in which only one manager shirks, since she always sells both shares if both managers shirk. Second, in Proposition 6, $s = (1, 1)$ is only on the equilibrium path if the blockholder sells both shares with a positive probability when $a_i = 0$, and in particular when $a = (0, 0)$. Only if $s = (1, 1)$ is consistent with one manager working will $p(1, 1)$ be sufficiently high to encourage the blockholder to sell both shares if both managers shirk. In contrast, in the large-shock model, the blockholder never voluntarily sells both shares when $a = (0, 0)$.  

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is higher if $j$ also works, because $s = (0, 0)$ is fully revealing that $i$ has worked. If $i$ works and $j$ shirks, then we have $s = (0, 1)$, which leads to a lower price. Not only does $j$ working increase the payoff to $i$ working, but it also reduces the payoff to $i$ shirking. If $i$ shirks, and $j$ works, $i$ is sold for sure. However, if $j$ also shirks and the blockholder sells $j$ at random, $i$ avoids being sold despite shirking.

We now compare the efficiency of the two-firm, small-shock model with the single-firm benchmark.

**Theorem 2.** (i) There are $\bar{\omega} > 0$ and $\bar{\beta} < (0, \pi)$ such that: if $\omega < (0, \bar{\omega})$ and $\beta < (0, \bar{\beta})$ then any two-firm equilibrium with only small shocks in which $s = (1, 1)$ is off the equilibrium path is more (less) efficient than the single-firm benchmark.

(ii) The most efficient two-firm equilibrium with only small shocks, in which $s = (1, 1)$ is on the equilibrium path, involves $\gamma^* = 0$ and $\varphi^* < 1$. This equilibrium is more efficient than the single-firm benchmark and any two-firm equilibrium in which $s = (1, 1)$ is off the equilibrium path.

There are two differences between the benchmark and the two-firm equilibrium without double exit, which work in opposite directions. On the one hand, in the benchmark, a shirking manager is sold with certainty. In the two-firm equilibrium without double exit, if manager $i$ shirks, he is not sold if $j$ also shirks and the blockholder randomly sells firm $j$ (which occurs w.p. $\frac{1}{2} F(\theta^*)$). This consideration increases the incentives to shirk relative to the benchmark – particularly if $\theta^*$ is high, as then manager $j$ is more likely to shirk. On the other hand, in the benchmark, if the manager works and the blockholder suffers a liquidity shock (w.p. $\delta$), he is sold with certainty. In the two-firm model without double exit, if the blockholder suffers a liquidity shock, a working manager is not automatically sold, as the blockholder may choose to sell the other firm. The price of the retained firm is decreasing in $\theta^*$. Being retained does not fully reveal that the manager has worked, as it could be that both managers shirked. The higher $\theta^*$ is, the more value is destroyed if both managers have indeed shirked, and thus the lower the price of the retained firm. This reduces the manager’s incentive to work and be retained.

Overall, increases in $\theta^*$ reduce the manager’s incentive to work and increase the manager’s incentive to shirk compared to the single-firm benchmark. When exit is relatively ineffective in governing a single firm, it is even less effective in governing two firms. Note that this result differs from the large-shock model, in which multi-firm governance with single exit is always more efficient than single-firm governance. Here, it is more efficient only if the agency problem is mild.
We now turn to part (ii). If one firm shirks, it is sold with certainty (regardless of $j$’s actions), just as in the benchmark. Thus, the disadvantage of the two-firm model in part (i) does not apply here. Moreover, the punishment for being sold is greatest with $\gamma^* = 0$. $\gamma$ is the probability that a shirking firm is not the only firm sold. Since the shirking firm receives a higher stock price if both firms are sold, the punishment is stronger when $\gamma$ is lower. On the other hand, the advantage of the two-firm model in part (i) continues to apply here: if manager $i$ works and the blockholder suffers a small liquidity shock, she may not be sold. Thus, relative both to the benchmark and the two-firm equilibrium without double exit, the two-firm equilibrium with double exit provides more incentives to work. As in Section 2.2, we henceforth focus on the most efficient equilibria and so, when $s = (1, 1)$ is on the equilibrium path, we consider equilibria in which $\gamma^* = 0$ and $\varphi^* < 1$.

Note that the efficient equilibrium with double exit is only possible when (16) holds, i.e. agency problems are weak. Thus, in the presence of small shocks, weak agency problems mean that multi-firm governance is more effective than single-firm governance. This is because weak agency problems encourage double exit if both managers shirk, which reduces the incentive to shirk. In contrast, with large shocks, multi-firm governance is less effective if agency problems are weak. This is because weak agency problems encourage double exit if only one manager shirks, which reduces the incentive to work.

We finally consider the stock price correlations.

**Proposition 7.** In an equilibrium with two firms and only small shocks, $\rho^* < 0$ when $s = (1, 1)$ is off the equilibrium path, and $\rho^* < 0$ in an efficient equilibrium in which $s = (1, 1)$ is on the equilibrium path.

In the large shock model, the correlation can be positive overall, since the blockholder is sometimes forced to sell both shares even when only one manager shirks. Thus, the prices of both firms are the same even though their fundamentals are different. This is the only source of positive correlation in the large shock model, and is absent from the small shock model since the blockholder is never forced to sell both firms. On the other hand, the small shock model contains a source of negative correlation, since single exit reduces the price of the sold share and increases the price of the retained share. Thus, the correlation is always negative in the small shock model.\(^7\)

\(^7\)In unreported analysis of inefficient equilibria in which both firms are sold even if only one manager shirks, we show that the correlation can sometimes be positive, because the source of positive correlation is now reintroduced.
4 Conclusion

This paper has studied how multi-firm ownership affects the effectiveness of corporate governance and leads to stock price correlations between economically unrelated firms. Compared to a single-firm benchmark, multi-firm ownership may worsen governance because it weakens the link between managerial effort and the blockholder’s exit decision. If one manager shirks, the blockholder will typically sell her stake in that firm. However, she may also sell her stake in the other firm, even if it is value-maximizing, to disguise her sale in the first firm as driven by a large liquidity shock. This is the case in the double-exit equilibrium of the large-shock model. Alternatively, she may retain her stake in the other firm, even if it is underperforming, to disguise her sale in the first firm as driven by small liquidity shock. This behavior occurs in the equilibrium of the small-shock model without double exit.

On the other hand, multi-firm ownership may strengthen governance. In a single-firm model, shirking only leads to a modest decline in the stock price, since the resulting sale can be disguised as being driven by liquidity needs. In a multi-firm model with large shocks, selling one firm can be a particularly strong signal of underperformance if the blockholder also retains her stake in the second firm. Single exit cannot result from a liquidity shock, and thus fully reveals managerial shirking. In a multi-firm model with small shocks, if the blockholder suffers a liquidity shock, she may be able to retain her stake in the value-maximizing firm by selling the other firm. Thus, the manager is more likely to be rewarded for effort, strengthening the link between his action and the blockholder’s exit decision.

The overall effect of multi-firm ownership on governance depends intricately on a number of parameters. The greater the frequency of liquidity shocks and the weaker the agency problem (i.e., the smaller the private benefits from shirking, and the greater the manager’s stock price concerns), the more likely the blockholder is to engage in double exit in the large-shock model. Since underperformance of one firm leads to the sale of the other firm even if it is value-maximizing, the incentives to maximize value are lower and so governance is weaker. If these parameters move in the opposite direction, the blockholder is more likely to engage in single exit, which provides greater discipline to a shirking manager and improves governance. By affecting the blockholder’s optimal strategy, these parameters also drive the direction of correlation between firms’ stock prices. When the blockholder engages in single exit, the correlation is negative as single exit reduces (increases) the price of the sold (retained) firm. In contrast, double exit leads to a positive correlation.
Governance is most efficient in a multi-firm model with small shocks, in which the blockholder sells both firms if both are underperforming. Here, shirking automatically leads to exit, as in the single-firm model and the multi-firm model with large shocks. However, there is an important advantage of the multi-firm model with small shocks that is not shared by these other two models: the existence of the second firm gives the blockholder another channel to satisfy her liquidity needs, and so she need not sell her stake in a value-maximizing firm. Stock price correlations are always negative in the most efficient equilibrium, regardless of agency and liquidity parameters.

Over and above its specific results, the paper has broader implications for governance and co-movement. First, allowing a blockholder to own stakes in multiple firms need not weaken governance by spreading the blockholder too thinly, as commonly argued. Second, common ownership between stocks can lead to negative correlation by introducing a “tournament” aspect in the blockholder’s exit decision, rather than only the positive correlation commonly believed. Third, the magnitude of liquidity shocks has a quite different effect from their frequency, and empirical analyses should attempt to identify these components separately. Fourth, asset pricing studies of comovement between stocks should control for common ownership and the extent of agency problems, since they will affect the correlation.
References


A Proofs

This Appendix first starts with Lemma which justifies our focus on symmetric equilibria (see footnote 3). It then moves to proofs for the large-shock model, and finally proofs for the small-shock model.

Lemma 6. If the managers follow the same strategy and the market maker follows a symmetric pricing strategy, the blockholder’s best response is a symmetric strategy as well.

**Proof of Lemma 6.** To show that it is weakly optimal for the blockholder to respond with a symmetric strategy, it is sufficient to show that

\[ E[u_I(s_i, s_j) | (a_i, a_j) = (x_i, x_j)] = E[u_I(s_j, s_i) | (a_i, a_j) = (x_j, x_i)] \]  

(17)

Since both managers follow the same strategy, \[ E[v(θ_j, a_j) | a_j = x_j] = E[v(θ_i, a_i) | a_i = x_j] \] and \[ E[v(θ_i, a_i) | a_i = x_i] = E[v(θ_j, a_j) | a_j = x_i] \]. Since the market maker follows a symmetric strategy, \[ p_i(s_i, s_j) = p_j(s_j, s_i) \]. Thus, (17) holds \( \forall s \in \{0, 1\} \times \{0, 1\} \). □

A.1 Proofs for Large Shocks

Proof of Proposition 1. Based on (7) the manager must follow a cutoff rule where \( a = 1 \) if and only if \( θ \) is below some threshold, denoted by \( θ^* \). Consider part (i). Suppose \( a = 0 \) and \( χ = 0 \). By retaining both shares, the blockholder’s payoff is \( 2v \). Thus, she will choose \( s > 0 \) only if \( p(s) = v \). Since \( δ > 0 \), \( s = 2 \) can arise if the blockholder was forced to sell both shares even though \( a = 0 \) (in which case \( v = v \)) or if the blockholder was forced to sell both shares and \( a = 1 \) (in which case \( v = v(θ^*) \)), plus cases in which the blockholder chooses to sell voluntarily. Since \( p(2) \) must take into account these cases, we must have \( p(2) ∈ (v(θ^*), v) \). Since \( p(2) < v \), the blockholder never chooses \( s = 2 \) when \( a = 0 \). Moving to part (ii), suppose that \( a = 1 \). The blockholder will not choose \( s = 0 \) as his payoff will be \( 2v(θ^*) \). Since \( p(2) > v(θ^*) \), the blockholder is strictly better off choosing \( s = 2 \) over \( s = 0 \). Suppose in equilibrium the blockholder chooses \( s = 2 \) w.p. \( γ \) and \( s = 1 \) w.p. \( 1 − γ \) when \( a = 1 \). We wish to prove that \( γ = 1 \). Assume on the contrary that \( γ < 1 \). Then \( p(1) < v \). Recall a blockholder who observes \( a = 0 \) will choose \( s = 1 \) with a strictly positive probability only if \( p(1) = v \) (which requires the blockholder to never choose \( s = 1 \) when \( a = 1 \)). Since \( p(1) < v \), a blockholder who observes \( a = 0 \) never chooses \( s = 1 \). Note that,
since \( s = 1 \) fully reveals \( \chi = 0 \), it has to be that \( s = 1 \) fully reveals \( a = 1 \). We thus have \( p(1) = v(\theta^*) \). Then the blockholder who observes \( a = 1 \) and \( \chi = 0 \) prefers \( s = 2 \) over \( s = 1 \) if and only if

\[
2p(2) > v(\theta^*) + p(1) \iff p(2) > v(\theta^*),
\]

which always holds. This contradicts \( \gamma < 1 \). The prices in part (iii) follow automatically from Bayes’ rule and parts (i) and (ii). Can \( s = 1 \) be on the equilibrium path? This equilibrium requires \( p(1) = \pi \). Note that the blockholder does not find it optimal to deviate from \( s = 2 \) to \( s = 1 \) if and only if

\[
2p(2) > v(\theta^*) + p(1) \iff \frac{\delta}{1 + \delta} > F(\theta^*)
\]

so, an equilibrium in which \( s = 1 \) is on the equilibrium path is possible if and only if \( \frac{\delta}{1 + \delta} \geq F(\theta^*) \). If \( \frac{\delta}{1 + \delta} < F(\theta^*) \) then \( s = 1 \) is not on the equilibrium path. Note that this equilibrium survives the intuitive criterion since if upon deviation the market maker believes that \( a = 1 \) (type \( a = 0 \) has weak incentives not choose \( s = 1 \) regardless of \( p(1) \)) the blockholder has no incentives to deviate from \( s = 2 \) to \( s = 1 \) when \( a = 1 \). 

**Proof of Lemma 1.** Consider part (i). If manager \( i \) chooses \( a_i = 0 \), his utility is 

\[
\pi + \omega E[p_i(s_i, s_j)|a_i = 0],
\]

which is independent of \( \theta \). If manager \( i \) chooses \( a_i = 1 \), his utility is 

\[
\pi - \theta_i + \omega E[p_i(s_i, s_j)|a_i = 1] + \beta,
\]

which is decreasing in \( \theta_i \). Thus, he chooses \( a_i = 1 \) if and only if

\[
\beta - \omega (E[p_i(s_i, s_j)|a_i = 0] - E[p_i(s_i, s_j)|a_i = 1]) \geq \theta_i
\]

thus, the manager chooses \( a_i = 1 \) if and only if \( \theta_i \) is below a certain threshold \( \theta_i^* \). We argue that \( \theta_i^* \in (0, \pi) \). Suppose on the contrary that \( \theta_i^* = 0 \) (\( \theta_i^* = \pi \)). The market maker knows \( a_i = 0 \) (\( a_i = 1 \)) for sure and hence

\[
E[p_i(s_i, s_j)|a_i = 0] = E[p_i(s_i, s_j)|a_i = 1]
\]

Thus, the market maker does not learn about \( a_i \) from \( s \). It follows that the manager has incentives to choose \( a_i = 1 \) whenever \( \beta \geq \theta_i \) and \( a_i = 0 \) whenever \( \beta < \theta_i \). From (2), \( \beta > 0 \) and \( \beta < v_i \), which yields a contradiction. We conclude if an equilibrium exists, then \( \theta_i^* \in (0, \pi) \). Consider part (ii). Suppose \( a_i = 0 < 1 = a_j \) and on the contrary the
blockholder weakly prefers \( s = (1, 0) \) over \( \hat{s} \neq (1, 0) \). A necessary condition is \( u_I (1, 0) \geq u_I (0, 1) \), i.e. \( p_i (1, 0) + \mathbb{E}[v (\theta_j, a_j) | a] < p_j (0, 1) + \mathbb{E}[v (\theta_i, a_i) | a] \). Since \( a_i < a_j \) then \( \mathbb{E}[v (\theta_j, a_j) | a] < \mathbb{E}[v (\theta_i, a_i) | a] \), and since \( p_j (0, 1) = p_i (1, 0) \) then \( u_I (1, 0) < u_I (0, 1) \). This is a contradiction. Consider part (iii). Suppose \( a = (0, 0) \) and \( \chi = 0 \). Then \( u_I (0, 0) = 2\overline{v}, u_I (1, 1) = 2p (1, 1), u_I (0, 1) = u_I (1, 0) = \overline{v} + p_i (1, 0) \). Clearly, \( u_I (0, 0) \) is weakly optimal. To show that it is strictly optimal, suppose that on the contrary, there is a strictly positive probability that \( s = (0, 0) \) is not chosen. This requires \( \max \{ p_i (1, 1), p_i (1, 0) \} = \overline{v} \). If \( p_i (1, 0) \leq p_i (1, 1) = \overline{v} \) then, whenever \( a = (1, 1) \), the blockholder strictly prefers \( s = (1, 1) \), yielding a contradiction. If \( p_i (1, 1) < p_i (1, 0) = \overline{v} \) then, whenever \( a_i = 1 \) and \( a_j = 0 \), the blockholder strictly prefers \( s = (1, 0) \), yielding a contradiction.

\[ \Box \]

**Proof of Lemma 2.** Consider parts (i) and (ii). If \( \chi = 1 \), the blockholder sells both shares regardless of \( a \). Therefore, in any equilibrium with large shocks only, \( p (1, 1) \in (\underline{v} (\theta^*), \overline{v}) \). If \( s \neq (1, 1) \), the market maker infers \( \chi = 0 \). From Lemma 1 Part (iii), if \( a = (0, 0) \), the blockholder has strict incentives not to choose \( s = (1, 0) \). If \( a = (0, 1) \) then \( u_I (1, 0) = \underline{v} (\theta^*) + p (1, 0) \) and \( u_I (0, 1) = \overline{v} + p (1, 0) \). We thus have \( u_I (1, 0) < u_I (0, 1) \), and so the blockholder has strict incentives not to choose \( s = (1, 0) \). Thus, \( s = (1, 0) \) is inconsistent with \( a_i = 0 \). Hence, if \( s = (1, 0) \) is on the equilibrium path, upon observing \( s = (1, 0) \) the market maker infers \( \chi = 0 \) and \( a_i = 1 \) for sure. This implies \( p (1, 0) = \underline{v} (\theta^*) < p (1, 1) (\in (\underline{v} (\theta^*), \overline{v})) \). If \( s = (1, 0) \) is not on the equilibrium path, a necessary condition is that when \( a = (1, 0) \) and \( \chi = 0 \), the blockholder does not have strict incentives to choose \( s = (1, 0) \). It follows that:

\[ \max \{ 2p (1, 1), \overline{v} + \underline{v} (\theta^*) \} \geq \overline{v} + p (1, 0) . \tag{18} \]

There are two cases to consider. First, if \( 2p (1, 1) \geq \overline{v} + \underline{v} (\theta^*) \), then (18) implies \( 2p (1, 1) \geq \overline{v} + p (1, 0) \). Since \( p (1, 1) < \overline{v} \), this requires \( p (1, 0) < p (1, 1) \). Second, if \( 2p (1, 1) < \overline{v} + \underline{v} (\theta^*) \), then (18) implies \( \underline{v} (\theta^*) \geq p (1, 0) \). Since \( p (1, 1) > \underline{v} (\theta^*) \), we have \( p (1, 0) < p (1, 1) \) as required. Consider part (iii). If \( a = (1, 1) \) then \( u_I (0, 0) = 2\underline{v} (\theta^*), u_I (1, 0) = u_I (0, 1) = \underline{v} (\theta^*) + p (1, 0) \) and \( u_I (1, 1) = 2p (1, 1) \). Based on parts (i) and (ii), \( p (1, 1) > p (1, 0) \geq \underline{v} \). Therefore, \( u_I (1, 1) > \max \{ u_I (1, 0), u_I (0, 0) \} \) as required.

\[ \Box \]
Proposition 8. If there is an equilibrium in which the blockholder chooses \( s = (0, 0) \) with a strictly positive probability when \( a = (1, 0) \) and \( \chi = 0 \), then there is another equilibrium in which the blockholder never chooses \( s = (0, 0) \) in those cases, and this equilibrium is strictly more efficient.

Proof of Proposition 8. Let \( \eta \in [0, 1 - \gamma] \) be the probability that the Blockholder chooses \( s = (0, 0) \) when \( a = (1, 0) \) and \( \chi = 0 \). Note that in this case the manager chooses \( s = (1, 0) \) with probability \( 1 - \gamma - \eta \). By Bayes’ rule we have:

\[
p(1, 1) = \nu - \frac{F(\theta^*) [(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma) + \delta]}{F(\theta^*) [(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma) + \delta] + (1 - F(\theta^*)) (\gamma (1 - \delta) F(\theta^*) + \delta)} \mathbb{E} [\theta_i | \theta_i < \theta^*],
\]

\[
p(0, 0) = \nu - \frac{\nu F(\theta^*) (1 - F(\theta^*))}{(1 - F(\theta^*))^2 + 2 \nu F(\theta^*) (1 - F(\theta^*))} \mathbb{E} [\theta_i | \theta_i < \theta^*],
\]

Manager \( i \) chooses \( a_i = 1 \) if and only if

\[
\nu - \theta_i + \omega \left[ (\delta + (1 - \delta) [F(\theta^*) + (1 - F(\theta^*)) \gamma]) p(1, 1) + (1 - \delta) (1 - F(\theta^*)) (1 - \gamma - \eta) p(1, 0) + (1 - \delta) (1 - F(\theta^*)) \eta p(0, 0) \right] + \beta \]

\[
> \nu + \omega \left[ (\delta + (1 - \delta) F(\theta^*) \gamma) p(1, 1) + (1 - \delta) F(\theta^*) (1 - \gamma - \eta) p(0, 1) + [(1 - \delta) (F(\theta^*) \eta + 1 - F(\theta^*))] p(0, 0) \right]
\]

This condition holds if and only if

\[
\beta - \omega (1 - \delta) \left[ \frac{F(\theta^*) (1 - \gamma - \eta) p(0, 1)}{+ ((1 - F(\theta^*)) (1 - \eta) + F(\theta^*) \eta) p(0, 0) - [F(\theta^*) (1 - \gamma) + (1 - F(\theta^*)) \gamma] p(1, 1)} - (1 - F(\theta^*)) (1 - \gamma - \eta) p(1, 0) \right] > \theta_i \tag{19}
\]

Using the expressions for \( p(s) \), this condition holds if and only if \( \psi_L (\theta^*, \gamma, \eta) > \theta_i \)

where

\[
\psi_L (y, \gamma, \eta) = \beta - (1 - \delta) \omega \left[ \frac{F(y)(1-\gamma)+(1-F(y))\gamma}{1+\frac{1-F(y)}{F(y)+\frac{\gamma F(y)+(1-\gamma)\gamma}{1-\eta \frac{1-F(y)+(1-\gamma)\gamma}{1-\eta}}} + (1 - F(y)) (1 - \gamma) - \eta \frac{1-F(y)+(1-\gamma)\gamma}{1-\eta \frac{1-F(y)+(1-\gamma)\gamma}{1-\eta}} \right] \mathbb{E} [\theta | \theta < y] \tag{20}
\]

Note that if \( (\theta^*, \gamma^*, \eta^*) \) is an equilibrium then \( \psi_L (\theta^*, \gamma^*, \eta^*) = \theta^* \). Suppose there is an
equilibrium with $\eta^* > 0$ and threshold $\theta^*$. According to Proposition 2 an equilibrium with $\eta^* = 0$ always exists. Thus, there is another equilibrium with $\hat{\theta} = 0$ and threshold $\hat{\theta}$. We argue that $\hat{\theta} < \theta^*$. Note that since $\eta^* > 0$, then $u_I(0,0) \geq u_I(1,0) \Leftrightarrow \overline{\sigma} + \underbar{\tau}(\theta^*) \geq \overline{\sigma} + p(1,0)$. Since $p(1,0) \geq \underbar{\tau}(\theta^*)$ then it has to be $p(1,0) = \underbar{\tau}(\theta^*)$. Also note that $\eta^* > 0$ implies $\gamma^* < 1$. If $\gamma^* < 1$ then

$$2p(1,1) \leq p(1,0) + \overline{\sigma} \Leftrightarrow$$

$$2 \left[ \overline{\sigma} - \frac{F(\theta^*) \frac{(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma^*) + \delta}{F(\theta^*) \frac{(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma^*) + \delta}} \right] \leq \underbar{\tau}(\theta^*) + \overline{\sigma} \Leftrightarrow$$

$$2 - \frac{F(\theta^*) \frac{(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma^*) + \delta}{F(\theta^*) \frac{(1 - \delta) (F(\theta^*) + (1 - F(\theta^*)) \gamma^*) + \delta}} \geq 1 \Leftrightarrow$$

$$Y(\overline{\sigma}) \leq \theta^*$$

If $\hat{\theta} < Y(\delta)$, then $\hat{\theta} < \theta^*$ automatically. Thus, we restrict attention to $\hat{\theta} > Y(\delta)$. Suppose $\hat{\theta} > Y(\delta)$. Based on Lemma 3, $\hat{\gamma}$, the $\gamma$ that corresponds to this alternative equilibrium, is zero. Note that $\eta^* \frac{1 - F(\theta^*) + \eta F(\theta^*)}{1 - F(\theta^*) + 2 \eta F(\theta^*)}$ decreases with $\theta^*$ and hence $\psi_L(y,0,\eta)$ decreases with $y \forall y \geq Y(\delta)$ and $\eta \in [0,1]$. Moreover, note that $\eta^* \frac{1 - F(\theta^*) + \eta F(\theta^*)}{1 - F(\theta^*) + 2 \eta F(\theta^*)}$ increases with $\eta$. This means that $\psi_L(y,\gamma,\eta)$ strictly increases in $\eta \forall \gamma$ and $y \geq Y(\delta)$. Therefore, if $\gamma^* = 0$ then $\theta^* > \hat{\theta}$. Suppose $\gamma^* \in (0,1)$. This implies $2p(1,1) = p(1,0) + \overline{\sigma}$. Based on the above arguments, this also implies $\theta^* = Y(\delta)$, and hence, $Y(\delta) = \psi_L(Y(\delta),\gamma^*,\eta^*)$. Since $\hat{\theta} > Y(\delta)$ and $\psi_L(\hat{\theta},0,0) = \hat{\theta}$, then $\psi_L(\hat{\theta},0,0) > Y(\delta)$. Since $\psi_L(y,0,0)$ is decreasing when $y \geq Y(\delta)$, then $\psi_L(Y(\delta),0,0) > Y(\delta)$. Since $\psi_L(y,\gamma,\eta)$ strictly increases in $\eta \forall \gamma$ and $y \geq Y(\delta)$ then $\psi_L(Y(\delta),0,\eta^*) > Y(\delta)$. Finally, note that $\psi_L(Y(\delta),\gamma,\eta)$ is increasing in $\gamma \forall \eta$. Therefore, $\psi_L(Y(\delta),\gamma^*,\eta^*) > Y(\delta)$, which contradicts $Y(\delta) = \psi_L(Y(\delta),\gamma^*,\eta^*)$.

Proof of Lemma 3. Note that based on (10),

$$2p(1,1) \leq \overline{\sigma} + \underbar{\tau}(\theta^*) \Leftrightarrow$$

$$F(\theta^*)^2 + \frac{2 \delta}{1 - \delta} F(\theta^*) - \frac{\delta}{1 - \delta} \geq 0 \Leftrightarrow$$

$$\theta^* \geq Y(\delta)$$
We start by arguing that if $\gamma < 1$ then $p(1, 0) = \psi(\theta^*)$. Since $\gamma < 1$ then $s = (1, 0)$ is chosen with a strictly positive probability, and according to Lemma 2 part (ii), $p(1, 0) = \psi(\theta^*)$. Suppose $a_i = 1$ (and $a_j = 0$) and $\chi = 0$. In this case, $u_I(1, 1) = 2p(1, 1)$, $u_I(0, 0) = \sigma + \psi(\theta^*)$ and $u_I(1, 0) = \sigma + p(1, 0)$. Thus, if $\gamma < 1$ then $u_I(1, 0) = u_I(0, 0)$.

We proceed in two steps. First, suppose $\theta^* < Y(\delta)$. This implies $u_I(1, 1) > u_I(0, 0)$. We wish to prove that $\gamma = 1$. Assume on the contrary that $\gamma < 1$. Then, $s = (1, 0)$ is on the equilibrium path, and by revealed preferences, $u_I(1, 0) \geq u_I(1, 1)$. This implies that $u_I(1, 0) > u_I(0, 0)$, and so $p(1, 0) > \psi(\theta^*)$. This contradicts our earlier result that, if $\gamma < 1$, then $p(1, 0) = \psi(\theta^*)$. Therefore, if $\theta^* < Y(\delta)$ then $\gamma(\theta^*) = \{1\}$. Second, suppose $\theta^* > Y(\delta)$. This implies $u_I(1, 1) < u_I(0, 0)$. We wish to prove that $\gamma = 0$.

Assume on the contrary that $\gamma > 0$. By revealed preferences, $u_I(1, 1) \geq u_I(1, 0)$, and hence, $u_I(0, 0) > u_I(1, 0)$. This implies $\psi(\theta^*) > p(1, 0)$, a contradiction. We conclude that if $\theta^* > Y(\delta)$ then $\gamma(\theta^*) = \{0\}. \quad \square$

**Proof of Lemma 4.** Recall $p(0, 1) = \sigma$ and if $\gamma < 1$ then $p(1, 0) = \psi(\theta^*)$. The derivative of the term in square brackets of (12) with respect to $F(\theta^*)$ is

$$\Delta(\gamma) \equiv (1 - 2\gamma)p(1, 1) - (1 - \gamma)(p(1, 0) + p(0, 1)) + p(0, 0).$$

If $\gamma = 1$ then $\Delta(1) = p(0, 0) - p(1, 1)$, which is positive from Lemma 2 part (i). If $\gamma \in (0, 1)$ (or $\gamma = 0$ and $\sigma + \psi(\theta^*) = 2p(1, 1)$), then $p(1, 0) = \psi(\theta^*)$ and $2p(1, 1) = \sigma + \psi(\theta^*)$ implying $p(1, 1) = \frac{p(1, 0) + p(0, 1)}{2}$. In this case, $\Delta(\gamma) = (p(0, 0) - p(1, 1)) > 0$. If $\gamma = 0$ and $\sigma + \psi(\theta^*) > 2p(1, 1)$ then

$$\Delta(0) = p(1, 1) - (p(1, 0) + p(0, 1)) + p(0, 0)$$

We have $p(0, 1) = p(0, 0) = \sigma$ and so $\Delta(0) = p(1, 1) - p(1, 0)$, which is positive since $p(1, 1) > p(1, 0) = \psi(\theta^*)$. \quad \square

**Proof of Proposition 2.** Part (i) follows from Lemma 1 part (iii), and part (ii) follows from Lemma 2 part (iii). Part (iv) follows directly from Lemma 2 Part (ii) and the discussion in the main text that precedes Proposition 2. Consider part (v). Given
\( \gamma \), manager \( i \) chooses \( a_i = 1 \) if and only if

\[
\overline{v} - \theta_i + \omega \left[ (\delta + (1 - \delta) [F(\theta^*) + (1 - F(\theta^*)) \gamma]) p(1, 1) + (1 - \delta) (1 - F(\theta^*)) (1 - \gamma) p(1, 0) \right] + \beta > \overline{v} + \omega \left[ (\delta + (1 - \delta) F(\theta^*) \gamma) p(1, 1) + (1 - \delta) F(\theta^*) (1 - \gamma) p(0, 1) + (1 - \delta) (1 - F(\theta^*)) p(0, 0) \right]
\]

This condition holds if and only if (12) holds. Using the explicit expressions for \( p(s) \) (in particular: \( p(1,1) \) is given by (10), \( p(0,0) = p(0,1) = \overline{v} \), and \( p(1,0) = \overline{v}(\theta^*) \)), the LHS of (12) yields (14). Note that the threshold equilibrium is a solution of \( \psi_L(\theta^*, \gamma^*) = \theta^* \) where \( \gamma^* \) satisfies (11). Note that \( \delta \in (0, 1) \) implies \( Y(\delta) \in (0, \overline{v}) \). We proceed in several steps. First, note that \( \psi_L(0, \gamma) = \beta \) and \( \psi_L(y, \gamma) \leq \beta \ \forall \ \gamma \in [0, 1] \) and \( y \leq \beta \). Thus, a solution for \( \psi_L(y, \gamma) = y \) exists, and any solution is smaller than \( \beta \). From (2), any solution of \( \psi_L(y, \gamma) = y \) is also in \((0, \overline{v})\). Second, note that \( \psi_L(y, 1) \) decreases in \( y \) when \( y < Y(\delta) \) and \( \psi_L(y, 0) \) decreases in \( y \) when \( y > Y(\delta) \). Moreover, note that \( \psi_L(y, 0) < \psi_L(y, 1) \ \forall \ y \). Recall that according to (11) if \( \gamma^* \in (0, 1) \) then \( \theta^* = Y(\delta) \). It follows, if \( \psi_L(Y(\delta), 1) \leq Y(\delta) \) then the unique equilibrium satisfies \( \gamma^* = 1 \). If \( Y(\delta) \leq \psi_L(Y(\delta), 0) \) then the unique equilibrium satisfies \( \gamma^* = 0 \). Suppose \( \psi_L(Y(\delta), 0) < Y(\delta) < \psi_L(Y(\delta), 1) \). Note that \( \forall \ \gamma \in [0, 1] \),

\[
\psi_L(Y(\delta), \gamma) = \beta - (1 - \delta) \omega \mathbb{E}[\theta_i | \theta_i < Y(\delta)] \left[ 1 - \frac{F(Y(\delta)) + \gamma}{2} \right],
\]

which is strictly increasing and continuous in \( \gamma \). The intermediate value theorem implies that there exists \( \gamma^{**} \in (0, 1) \) such that \( \psi_L(Y(\delta), \gamma^{**}) = Y(\delta) \) and note that because \( \psi_L(Y(\delta), \gamma) \) is strictly increasing then \( \gamma^{**} \in (0, 1) \) is unique. Thus, the unique equilibrium satisfies \( \gamma^* = \gamma^{**} \). Overall, an equilibrium always exists and is unique. Finally, consider part (iii). Note that

\[
\psi_L(Y(\delta), \gamma) \leq Y(\delta) \iff \gamma \leq 1 + \frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \frac{2}{(1 - \delta) \mathbb{E}[\theta_i | \theta_i < Y(\delta)]}
\]

Thus, if \( \psi_L(Y(\delta), 1) \leq Y(\delta) \) then \( 1 \leq 1 + \frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \frac{2}{(1 - \delta) \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \) and \( \gamma^* = 1 \). If \( \psi_L(Y(\delta), 0) \geq Y(\delta) \) then \( 0 \geq 1 + \frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \frac{2}{(1 - \delta) \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \) and \( \gamma^* = 0 \). Otherwise, \( \gamma^* = 1 + \frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \frac{2}{(1 - \delta) \mathbb{E}[\theta_i | \theta_i < Y(\delta)]} \in (0, 1) \). This explains expression (13). Note that by construction, expression (13) implies that if \( \gamma^* = 0 \) then \( \theta^*_L \geq Y(\delta) \), if \( \gamma^* = 1 \)
then \( \theta^*_L \leq Y(\delta) \), and if \( \gamma^* \in (0,1) \) then \( \theta^*_L = Y(\delta) \). Also, based on the proof of 3,

\[
2p(1,1) \leq \nu_{+\nu}(\theta^*) \Leftrightarrow \theta^* \geq Y(\delta)
\]

Suppose \( \gamma^* = 0 \). Then \( \theta^*_L \geq Y(\delta) \), and based on Lemma 2, \( p(1,0) = \nu(\theta^*) \). Therefore, \( 2p(1,1) \leq \nu_{+}\nu(1,0) \) and BH indeed finds it optimal to choose \( s = (1,0) \) w.p.1 when \( a_i = 1 \) (and \( a_j = 0 \)) and \( \chi = 0 \). Suppose \( \gamma^* \in (0,1) \). Then \( \theta^*_L = Y(\delta) \), and based on Lemma 2, \( p(1,0) = \nu(\theta^*) \). Therefore, \( 2p(1,1) = \nu_{+}\nu(1,0) \) and BH indeed finds it optimal to choose \( s = (1,0) \) w.p. strictly between zero and one when \( a_i = 1 \) (and \( a_j = 0 \)) and \( \chi = 0 \). Last, suppose \( \gamma^* = 1 \). Then \( \theta^*_L \leq Y(\delta) \). If the off equilibrium price of \( s = (1,0) \) satisfies \( 2p(1,1) \geq \nu_{+}\nu(1,0) \), then BH indeed finds it optimal to choose \( s = (1,0) \) w.p.0. Since \( 2p(1,1) \geq \nu_{+}\nu(\theta^*) \) then off equilibrium price \( p(1,0) = \nu(\theta^*) \) always satisfies this requirement. Note that if \( \theta^*_L \leq Y(\delta) \) then \( 2p(1,1) \geq \nu_{+}\nu(\theta^*) \), and since \( 2\nu > p(1,1) \), then \( s = (1,0) \) is an equilibrium dominated strategy for type \( a = (1,1) \) but not for types \( a \neq (1,1) \). Thus, the intuitive criterion does not impose restrictions on the off-equilibrium beliefs in this case.

\[\square\]

**Proof of Corollary 1.** Part (i) follows directly from (13). Consider part (ii). Note that

\[
\lim_{\delta \to 0} \frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega} \frac{2}{(1 - \delta)\mathbb{E}[\theta|\theta < Y(\delta)]} = -\infty
\]

Therefore, there is \( \delta > 0 \) such that if \( \delta \in (0,\delta) \) then

\[
\frac{1}{1 + \sqrt{\delta}} - \frac{\beta - Y(\delta)}{\omega} \frac{2}{(1 - \delta)\mathbb{E}[\theta|\theta < Y(\delta)]} < -1.
\]

Based on (13), if \( \delta \in (0,\delta) \) then \( \gamma^* = 0 \).

\[\square\]

**Proof of Theorem 1.** We first note that, \( \forall y \), tedious algebra shows us that

\[
\psi_L(y,0) < \psi_B(y) < \psi_L(y,1).
\]

Moreover, recall all three functions are decreasing in \( \theta^* \) and \( \psi_L(\theta^*_L,0) = \theta^*_L \) and \( \psi_B(\theta^*_B) = \theta^*_B \). Note that

\[
\psi_B(Y(\delta)) < \psi_L(Y(\delta),\gamma) \Leftrightarrow \gamma^* > \frac{\sqrt{\delta}}{1 + \sqrt{\delta}}
\]

Suppose \( \theta^*_L = \theta^*_B \). Given (21) it has to be \( \gamma \in (0,1) \) and hence \( \theta^*_L = \theta^*_B = Y(\delta) \). Therefore, \( \psi_B(Y(\delta)) = \psi_L(Y(\delta),\gamma) \). Based on (22) it has to be \( \gamma^* = \frac{\sqrt{\delta}}{1 + \sqrt{\delta}} \). Suppose
\( \theta^*_L > \theta^*_B \) but on the contrary \( \gamma^* \leq \frac{\sqrt{3}}{1 + \sqrt{3}} \). Suppose \( \gamma^* = 0 \). Since \( \psi_L(y, \gamma) \) is decreasing function of \( y \) then \( \psi_L(\theta^*_L, 0) < \psi_L(\theta^*_B, 0) \). Given (21) \( \psi_L(\theta^*_B, 0) < \psi_B(\theta^*_B) \). Therefore, \( \psi_L(\theta^*_L, 0) < \psi_B(\theta^*_B) \), implying \( \theta^*_L < \theta^*_B \), a contradiction. Suppose \( \gamma^* \in \left(0, \frac{\sqrt{3}}{1 + \sqrt{3}}\right) \).

Based on (11), \( \theta^*_L = Y(\delta) \). Since \( \gamma^* \leq \frac{\sqrt{3}}{1 + \sqrt{3}} \), based on (22) \( \psi_L(Y(\delta), 0) \leq \psi_B(Y(\delta)) \).

Since \( Y(\delta) = \theta^*_L > \theta^*_B \) and \( \psi_B(y) \) is a decreasing function then \( \psi_B(Y(\delta)) < \psi_B(\theta^*_B) \).

It follows, \( \theta^*_L \leq \theta^*_B \), a contradiction. We conclude that if \( \theta^*_L > \theta^*_B \) then \( \gamma^* > \frac{\sqrt{3}}{1 + \sqrt{3}} \).

Suppose \( \theta^*_L < \theta^*_B \) but on the contrary \( \gamma^* \geq \frac{\sqrt{3}}{1 + \sqrt{3}} \). Suppose \( \gamma^* = 1 \). Since \( \psi_L(y, \gamma) \) is decreasing function of \( y \) then \( \psi_L(\theta^*_B, 1) < \psi_L(\theta^*_L, 1) \). Given (21) \( \psi_B(\theta^*_B) < \psi_L(\theta^*_B, 1) \). Therefore, \( \psi_B(\theta^*_B) < \psi_L(\theta^*_L, 1) \), implying \( \theta^*_L < \theta^*_B \), a contradiction. Suppose \( \gamma^* \in \left[\frac{\sqrt{3}}{1 + \sqrt{3}}, 1\right) \). Based on (11), \( \theta^*_L = Y(\delta) \). Since \( \gamma^* \geq \frac{\sqrt{3}}{1 + \sqrt{3}} \), based on (22) \( \psi_B(Y(\delta)) \leq \psi_L(Y(\delta), 0) \).

Since \( Y(\delta) = \theta^*_L < \theta^*_B \) and \( \psi_B(y) \) is a decreasing function then \( \psi_B(\theta^*_B) < \psi_B(Y(\delta)) \). It follows, \( \theta^*_B < \theta^*_L \), a contradiction. We conclude that if \( \theta^*_L < \theta^*_B \) then \( \gamma^* < \frac{\sqrt{3}}{1 + \sqrt{3}} \).

**Proof of Proposition 3.** The correlation of two prices is positive if and only if the covariance is positive. The covariance in turn is given by:

\[
\sigma_{ij} = \mathbb{E}[p_i \times p_j] - \mathbb{E}[p_i] \times \mathbb{E}[p_j],
\]

where \( \mathbb{E}[p_i] = \mathbb{E}[p_j] = \bar{v} - F(\theta^*) \mathbb{E}[\theta_i | \theta_i < \theta^*] \), and

\[
\mathbb{E}[p_i \times p_j] = \delta p(1,1)^2 + (1 - \delta) \left[ \frac{F(\theta^*)^2 p(1,1)^2 + (1 - F(\theta^*))^2 p(0,0)^2 + \gamma^* 2 F(\theta^*) (1 - F(\theta^*)) p(1,1)^2 + (1 - \gamma^*) 2 F(\theta^*) (1 - F(\theta^*)) p(0,1) p(1,0)}{(1 - \delta) 2 F(\theta^*) (1 - F(\theta^*)) p(0,1) p(1,0)} \right].
\]

This yields

\[
\sigma_{ij} = -\delta (1 - F(\theta^*)) \left[ (\bar{v}^2 - \bar{u}^2) - F(\theta^*) (\bar{v} - \bar{u})^2 \right] + \left[ (1 - \delta) F(\theta^*)^2 + \delta \right] \left[ p(1,1)^2 - \bar{v}^2 \right] + (1 - \delta) 2 F(\theta^*) (1 - F(\theta^*)) \gamma^* \left[ p(1,1)^2 - \bar{v} \times \bar{u} \right]
\]

Therefore, \( \sigma_{ij} > 0 \iff \gamma^* \left[ p(1,1)^2 - \bar{v} \times \bar{u} \right] > \frac{\delta (1 - F(\theta^*)) \left[ (\bar{v}^2 - \bar{u}^2) - F(\theta^*) (\bar{v} - \bar{u})^2 \right] - \left[ (1 - \delta) F(\theta^*)^2 + \delta \right] \left[ p(1,1)^2 - \bar{v}^2 \right]}{(1 - \delta) 2 F(\theta^*) (1 - F(\theta^*))} \) \quad (23)

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Note that \((\frac{\bar{v} + v}{2})^2 > \bar{v} \times v\) and recall that: (i) if \(\gamma \in (0, 1)\) then \(\theta^* = Y(\delta)\) and \(p(1, 1) = \frac{\bar{v} + v}{2}\), (ii) if \(\gamma = 1\) then \(\theta^* < Y(\delta)\) and \(p(1, 1) > \frac{\bar{v} + v}{2}\), (iii) if \(\gamma = 0\) then \(\theta^* > Y(\delta)\) and \(p(1, 1) < \frac{\bar{v} + v}{2}\). Suppose \(\gamma \in (0, 1)\). A direct calculation shows that (23) holds if and only if \(\gamma^* > \frac{\sqrt{2}}{1 + \sqrt{3}}\). Suppose \(\gamma^* = 0\) and note that \(p(1, 1) = \bar{v} + \frac{\delta(1 - F(\theta^*))}{(1 - \delta)F(\theta^*)^2 + \delta} (\bar{v} - v)\). (23) holds if and only if
\[
p(1, 1)^2 > \frac{\delta(1 - F(\theta^*))}{(1 - \delta)F(\theta^*)^2 + \delta} [(\bar{v}^2 - v^2) - F(\theta^*) (\bar{v} - v)^2]
\]
Substituting the expression of \(p(1, 1)\) proves that this term never holds, and so \(\sigma_{ij} < 0\) which yields \(\rho < 0\). Suppose \(\gamma^* = 1\), then (23) becomes:
\[
p(1, 1)^2 > \bar{v}^2 - \frac{F(\theta^*) (\bar{v} - v)}{1 - (1 - \delta)(1 - F(\theta^*))^2} ((2 - F(\theta^*)) \bar{v} + F(\theta^*) v)
\]
Substituting \(p(1, 1) = \bar{v} - \frac{F(\theta^*)}{1 - (1 - \delta)(1 - F(\theta^*))^2} (\bar{v} - v)\) for the case \(\gamma^* = 1\) confirms that the above condition always holds.

\[\square\]

**A.2 Proofs for Small Shocks**

**Proof of Proposition 4.** We start by proving two facts that will be useful for the proofs of the individual parts of the Proposition. We first prove that if \(a = 1\) then \(s > 0\) for sure. Suppose on the contrary the blockholder chooses \(s = 0\) with positive probability. Since \(a = 1\) implies \(v = v(\theta^*)\), this requires
\[
2 (v(\theta^*)) \geq \max \{2p(2), p(1) + v(\theta^*)\}.
\]
Since \(p(s) \geq v(\theta^*) \forall s \in \{0, 1, 2\}\), it follows that
\[
p(2) = p(1) = v(\theta^*).
\]
However, since \(\delta > 0\), there is a strictly positive probability that \(s > 0\) when \(a = 0\). This implies
\[
\max \{p(2), p(1)\} > v(\theta^*),
\]
a contradiction. Second, we show that if \(a = 0\) then \(s < 2\) for sure. Suppose on the contrary the blockholder chooses \(s = 2\) with positive probability. Since \(a = 0\) implies \(v = \bar{v}\), this requires
\[
2p(2) \geq \min \{2\bar{v}, p(1) + \bar{v}\}.
\]

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Since \( p(s) \leq \tau \) \( \forall s \in \{0, 1, 2\} \), it follows that
\[
2p(2) \geq p(1) + \tau. \tag{24}
\]

When \( a = 1 \), the blockholder’s payoff is \( 2\theta^* \) if \( s = 0 \), \( p(1) + \theta^* \) if \( s = 1 \), and \( 2p(2) \) if \( s = 2 \). Equation (24) means that her payoff is strictly highest if \( s = 2 \), and so she chooses \( s = 2 \) for sure. Thus, either \( s = 1 \) implies \( a = 0 \) for sure or \( s = 1 \) is an off-equilibrium event. In the former case, \( p(1) = \tau \), and so \( p(2) = \tau \). However, this cannot be an equilibrium, since we proved that if \( a = 1 \) then \( s > 0 \), and so \( \min \{p(2), p(1)\} \leq \tau \), a contradiction. Consider the latter case where \( s = 1 \) is an off-equilibrium event. We argue that this equilibrium does not survive the intuitive criterion. In this equilibrium, when \( a = 1 \) the blockholder chooses \( s = 2 \) for sure, and therefore, \( p(2) < \tau \). Moreover, equation (24) has two implications. First, since \( p(2) < \tau \), we must have \( p(2) > p(1) \), and so \( \tau > p(1) \). Second, since \( p(1) \geq \theta^* \) then
\[
2p(2) \geq \theta^* + \tau.
\]

Since any possible price \( p(1)^8 \) must be less than \( \tau \), we have
\[
2p(2) > \theta^* + p(1).
\]

When \( a = 1 \), the blockholder strictly prefers \( s = 2 \) over deviation to \( s = 1 \), regardless of the off-equilibrium beliefs. Thus, a blockholder who sees \( a = 1 \) will not deviate. In contrast, a blockholder who sees \( a = 0 \) will deviate from \( s = 2^9 \) to \( s = 1 \), because her payoff will rise from \( u_I(2) = 2p(2) < 2\tau \) to \( u_I(1) = p(1) + \tau = 2\tau \). Thus, a blockholder who observes \( a = 1 \) will not deviate to \( s = 1 \), but a blockholder who observes \( a = 0 \) will deviate. Thus, we must have \( p(1) = \tau \) off-equilibrium. This contradicts \( p(1) < \tau \), and so the equilibrium does not hold. Overall, in both cases, there is no equilibrium where, if \( a = 0 \), \( s = 2 \) with positive probability. Thus, if \( a = 0 \), then \( s < 2 \) for sure.

We now prove part (i) of the Proposition. Since the blockholder never chooses \( s = 2 \) when \( a = 0 \), in any equilibrium in which \( s = 2 \) is on the equilibrium path, the trade must arise from a blockholder who has observed \( a = 1 \), and so \( p(2) = \theta^* \). Since a blockholder who has observed \( a = 0 \) is sometimes forced to choose \( s = 1 \), \( p(1) > p(2) \) and so a blockholder who has observed \( a = 1 \) is strictly better off choosing \( s = 1 \). This

\(^8\)We abuse notation slightly here in that \( p(1) \) refers to any possible price for \( s = 1 \), rather than the price in the candidate equilibrium.

\(^9\)Recall that we are considering equilibria where the investor chooses \( s = 2 \) with positive probability.
implies that in any equilibrium, \( s = 2 \) is played w.p. zero. We now move to part (ii). Since we have proven that, if \( a = 1 \) then \( s > 0 \) for sure, and part (i) implies that \( s < 2 \) always, \( a = 1 \) must imply \( s = 1 \). We finally turn to part (iv). The expressions for \( p(0) \) and \( p(1) \) follows directly from parts (i), (ii), and (iii). It is left to derive a bound on \( p(2) \) so that deviation to \( s = 2 \) is unprofitable, and show that the equilibrium satisfies the intuitive criterion with this off-equilibrium price. When \( a = 1 \), deviation from \( s = 1 \) to \( s = 2 \) is unprofitable if:

\[
2p(2) \leq v(\theta^*) + p(1),
\]

which yields the inequality in the Proposition. To show that the equilibrium satisfies the intuitive criterion, it is sufficient to note that a blockholder who sees \( a = 0 \) never (strictly) wants to deviate to \( s = 2 \). Thus, upon deviation to \( s = 2 \), the market maker should infer \( a_i = 1 \). Since \( p(1) > v(\theta^*) \), a blockholder who observes \( a = 1 \) will not deviate to \( s = 2 \) if \( p(2) = v(\theta^*) \). Since \( p(2) > v(\theta^*) \), the equilibrium does satisfy the intuitive criterion.

\[
\text{Lemma 7. In any equilibrium, } p(1,1) \notin \left( \frac{\nu(\theta^*)+p(1,0)}{2}, \frac{\nu(\theta^*)+p(1,0)}{2} \right). \text{ Moreover, } s = (1,1) \text{ is on the equilibrium path if and only if } p(1,1) \geq \frac{\nu(\theta^*)+p(1,0)}{2}. 
\]

\[
\text{Proof of Lemma 7. Suppose on the contrary } \frac{\nu(\theta^*)+p(1,0)}{2} < p(1,1) < \frac{\nu(\theta^*)+p(1,0)}{2}. \text{ The blockholder chooses } s = (1,1) \text{ if and only if } a = (1,1). \text{ Therefore, } s = (1,1) \text{ implies } a = (1,1) \text{ for sure and } p(1,1) = v(\theta^*). \text{ Since } p(1,0) \geq v(\theta^*) \text{ implies } \frac{\nu(\theta^*)+p(1,0)}{2} \geq v(\theta^*), \frac{\nu(\theta^*)+p(1,0)}{2} < p(1,1) = v(\theta^*) \text{ can never hold, hence a contradiction. If } \frac{\nu(\theta^*)+p(1,0)}{2} \leq p(1,1) \text{ then whenever } a = (1,1) \text{ the blockholder has strict incentives to choose } s = (1,1) \text{ and it is on the equilibrium path. If } p(1,1) < \frac{\nu(\theta^*)+p(1,0)}{2} \text{ then even when } a = (1,1) \text{ the blockholder has no incentives to choose } s = (1,1). \text{ Therefore, } s = (1,1) \text{ has to be off-equilibrium. If } p(1,1) = \frac{\nu(\theta^*)+p(1,0)}{2} \text{ then the blockholder has incentives to choose } s = (1,1) \text{ only if } a = (1,1). \text{ If on the contrary } s = (1,1) \text{ is on the equilibrium path then } p(1,1) = v(\theta^*), \text{ implying } p(1,0) = v(\theta^*). \text{ However, since } \delta > 0 \text{ and } 2p(1,1) = v(\theta^*) + p(1,0) < \nu + p(1,0), \text{ the blockholder picks } s = (1,0) \text{ when she faces a liquidity shock and } a = (0,0) \text{ with a positive probability, so } p(1,0) > v(\theta^*), \text{ yielding a contradiction. Therefore, } s = (1,1) \text{ would be off-equilibrium in this case.} \]

\[
\text{Proof of Proposition 5. Consider an equilibrium where } s = (1,1) \text{ is off the equilibrium path. Part (i) follows from Lemma 1 part (iii). Consider part (iii). If } s = (1,1) \text{ then }
\]
is off the equilibrium path and \( \chi = 1 \) then \( s \in \{(0, 1), (1, 0)\} \) with probability one. By assumption, if the blockholder is indifferent between \( s = (0, 1) \) and \( s = (1, 0) \), she chooses each with equal probability. This concludes part (iii) when \( a = (0, 0) \) and \( \chi = 1 \). Consider part (ii). According to part (iii), the blockholder chooses \( s = (1, 0) \) with a strictly positive probability when \( a = (0, 0) \). This implies \( p(1, 0) > \nu(\theta^*) \).

When \( a = (1, 0) \) the blockholder never chooses \( s = (0, 1) \). The blockholder strictly prefers \( s = (1, 0) \) over \( s = (0, 0) \) if and only if \( p(1, 0) + \overline{v} > \nu(\theta^*) + \overline{v} \), which always holds. Thus, the blockholder chooses \( s = (1, 0) \) for sure. Consider part (iii) when \( a = (1, 1) \). Since \( p(1, 0) > \nu(\theta^*) \) the blockholder strictly prefers \( s \in \{(0, 1), (1, 0)\} \) over \( s = (0, 0) \). The blockholder strictly prefers \( s = (1, 0) \) over \( s = (1, 1) \) if and only if \( p(1, 1) \leq \frac{\nu(\theta^*) + p(1, 0)}{2} \). Note that this condition can always be met when \( p(1, 1) = \nu(\theta^*) \).

Turning to the prices in part (iv), if \( s = (1, 1) \) is not on the equilibrium path, we must have \( p(1, 1) \leq \frac{\nu(\theta^*) + p(1, 0)}{2} \). Since \( p(1, 1) \leq \frac{\nu(\theta^*) + p(1, 0)}{2} \) then the blockholder chooses \( s = (1, 0) \) unless \( a = (0, 0) \) and \( \chi = 0 \), in which case, the blockholder chooses \( s = (0, 0) \) for sure. Thus, \( s = (1, 0) \) can arise in three cases. First, both managers shirk (w.p. \( F(\theta^*)^2 \)) and \( i \) is sold (w.p. \( \frac{1}{2} \)). Second, \( j \) works (w.p. \( 1 - F(\theta^*) \)) and \( i \) shirks (w.p. \( F(\theta^*) \)). Third, both managers work (w.p. \( (1 - F(\theta^*))^2 \)), the blockholder suffers a liquidity shock (w.p. \( \delta \)), and \( i \) is sold (w.p. \( \frac{1}{2} \)). This leads to the expression for \( p(1, 0) \). \( p(0, 1) \) similarly follows from Bayes’ rule. From Lemma 7, we must have \( p(1, 1) \leq \frac{\nu(\theta^*) + p(1, 0)}{2} \). This is satisfied by \( p(1, 1) = \nu(\theta^*) \). \( p(0, 0) \) is automatic. Moving to part (v), if the manager shirks, his payoff is

\[
\beta + \omega \left[ \frac{1}{2} F(\theta^*) p(0, 1) + \left( 1 - \frac{1}{2} F(\theta^*) \right) p(1, 0) \right] + \overline{v} - \theta_i
\]

\( p_i(0, 1) \) occurs if \( j \) shirks (w.p. \( F(\theta^*) \)) and \( j \) is sold (w.p. \( \frac{1}{2} \)). \( p_i(1, 0) \) occurs in all other cases. If the manager works, his payoff is

\[
\omega \left[ \left( 1 - F(\theta^*) \right) (1 - \delta) p(0, 0) + \left( 1 - F(\theta^*) \right) \frac{\delta}{2} F(\theta^*) \right] + \overline{v}
\]

\( p(0, 0) \) occurs if \( j \) also works \( (1 - F(\theta^*)) \) and there is no shock \( (1 - \delta) \). \( p(0, 1) \) occurs if \( j \) also works \( (1 - F(\theta^*)) \), there is a shock \( (\delta) \), and \( j \) is sold \( (\frac{1}{2}) \), or if \( j \) shirks. \( p(1, 0) \) occurs if \( j \) also works, there is a shock, and \( i \) is sold. Thus, the manager shirks if and
only if
\[
\beta + \omega \left[ p(0,1) \frac{1}{2} F(\theta^*) + p(1,0) \left[ 1 - \frac{1}{2} F(\theta^*) \right] \right] + \bar{v} - \theta_i \\
> \omega \left[ p(0,0) \left[ (1 - F(\theta^*)) (1 - \delta) \right] + p(0,1) \left[ (1 - F(\theta^*)) \frac{\delta}{2} + F(\theta^*) \right] \\
+ p(1,0) \left[ (1 - F(\theta^*)) \frac{\delta}{2} \right] \right] + \bar{v}
\]
which simplifies to
\[
\beta + \omega \left[ p(1,0) \left[ (1 - \frac{\delta}{2}) (1 - F(\theta^*)) + \frac{1}{2} F(\theta^*) \right] + p(0,1) \left[ -\frac{\delta}{2} (1 - F(\theta^*)) - \frac{1}{2} F(\theta^*) \right] - \right] > \theta_i. \tag{25}
\]
Plugging in for \( p(s) \) from part (iv) yields:
\[
\begin{align*}
\beta + \omega & \left[ \bar{v} - F \frac{\frac{1}{2} F^2}{F - \frac{\delta}{2} (1 - F)^2} \mathbb{E} [\theta_i | \theta_i < \theta^*] \left[ (1 - \frac{\delta}{2}) (1 - F) + \frac{1}{2} F \right] + \right] \\
& > \theta_i \\
\beta - \omega & \frac{\bar{v} (1 - \delta) (1 - F)}{F - \frac{1}{2} F^2 + \frac{\delta}{2} (1 - F)^2} \mathbb{E} [\theta_i | \theta_i < \theta^*] \left[ -\frac{\delta}{2} (1 - F) - \frac{1}{2} F \right] - \theta_i
\end{align*}
\]
Hence, the equilibrium is defined by:
\[
\beta - \omega \left( 2 - \delta \right) \frac{F \left( \theta^* \right) \left( 1 - F \left( \theta^* \right) \right)}{1 - (1 - \delta) \left( 1 - F \left( \theta^* \right) \right)^2} \mathbb{E} [\theta | \theta < \theta^*] = \theta^*,
\]
which leads to part (v). To prove that an equilibrium always exists, we need to prove two results. First, we show that \( \psi_{S,ND} (\theta^*) \) has a fixed point. This follows from the observation that \( \psi_{S,ND} (0) = \beta = \psi_{S,ND} (\bar{v}) \). Second, we need to show that equilibrium with \( p(1,1) \leq \frac{\psi(\theta^*) + p(1,0)}{2} \) survives the intuitive criterion. Note that if \( p(1,1) \) is sufficiently high then each type of blockholder has incentives to deviate to \( s = (1,1) \). The lack of an equilibrium-dominated deviation ensures that the intuitive criterion is satisfied.

\textbf{Proof of Proposition 6.} According to Lemma 7, if \( s = (1,1) \) is on the equilibrium path, we must have \( \frac{\bar{v} + p(1,0)}{2} \leq p(1,1) \). Under this condition, if \( a = (1,1) \) then the blockholder chooses \( s = (1,1) \) for sure. When \( a = (1,0) \) we let \( \gamma \) be the probability...
Note that the blockholder chooses \( s = (1, 1) \) (whether or not \( \chi = 1 \)) and \( \eta \in [0, 1 - \gamma] \) the probability she chooses \( s = (0, 0) \) if \( \chi = 0 \). When \( a = (0, 0) \) and \( \chi = 1 \), let \( \varphi \) the probability the blockholder chooses \( s = (1, 1) \); w.p. \( 1 - \varphi \) she chooses either \( s = (1, 0) \) or \( s = (0, 1) \) with equal probability. From Bayes’ rule, we have:

\[
p(1, 1) = \overline{v} - \frac{F(\theta^*) (F(\theta^*) + (1 - F(\theta^*)) \gamma)}{F(\theta^*) (F(\theta^*) + (1 - F(\theta^*)) \gamma) + (1 - F(\theta^*)) [F(\theta^*) \delta \varphi + F(\theta^*) \gamma]} \mathbb{E}[\theta|\theta < \theta^*]
\]

\[
p(1, 0) = \overline{v} - \frac{F(\theta^*) (1 - \gamma)}{F(\theta^*) (1 - \gamma) + (1 - F(\theta^*)) \delta^2 (1 - \varphi)} \mathbb{E}[\theta|\theta < \theta^*]
\]

\[
p(0, 0) = \overline{v}.
\]

There are two cases to consider depending on whether \( \overline{v} + p(1, 0) \leq p(1, 1) \) holds with equality or strict inequality. We first consider \( \overline{v} + p(1, 0) < 2p(1, 1) \), in which case \( \gamma = \varphi = 1 \). Thus, the blockholder chooses \( s = (1, 1) \) except if \( a = (0, 0) \) and \( \chi = 0 \). Note that \( s = (1, 0) \) is off-equilibrium. We require \( \overline{v} + p(1, 0) < 2p(1, 1) \). This condition holds if and only if \( p(1, 0) < \overline{v} - \frac{2F(\theta^*)}{1 - (1 - \gamma) + \delta^2 (1 - \varphi)} \mathbb{E}[\theta|\theta < \theta^*] \). Note that the RHS is greater than \( \psi(\theta^*) \) if and only if \( \theta^* < Y(\delta) \), and so \( \theta^* < Y(\delta) \) is a necessary condition. In this equilibrium, small shock triggers \( s = (1, 1) \) for any \( a \). Therefore, similar to the case of large shocks only, manager \( i \) chooses \( a_i = 1 \) if and only if \( \psi_L(\theta^*, 1) > \theta_i \).

Since \( \psi_L(y, 1) \) is decreasing in \( y \) when \( y \leq Y(\delta) \), the equilibrium exists if and only if \( \psi_L(Y(\delta), 1) < Y(\delta) \). Second, we consider \( \overline{v} + p(1, 0) = 2p(1, 1) \). We first consider equilibria where \( p(1, 0) = \psi(\theta^*) \). The requirement that \( p(1, 0) = \psi(\theta^*) \) implies \( \varphi = 1 \) and \( p(1, 1) = \psi(\theta^*) + \eta \). The latter condition requires \( \theta^* = Y(\delta) \). Similar to the case of large shocks only, this equilibrium exists if and only if there exists \( \gamma \in [0, 1] \) such that \( \psi_L(Y(\delta), \gamma) = Y(\delta) \). Recall \( \psi_L(Y(\delta), \gamma) \) obtains its lowest value when \( \gamma = 0 \) and its highest value when \( \gamma = 1 \). Thus, this type of equilibrium exists if and only if

\[
\psi_L(Y(\delta), 0) \leq Y(\delta) \leq \psi_L(Y(\delta), 1).
\]

Next, consider an equilibrium where \( \overline{v} + p(1, 0) = 2p(1, 1) \) and \( p(1, 0) > \psi(\theta^*) \). The latter condition requires \( \varphi < 1 \). Thus, the former condition holds if and only if

\[
\varphi = 1 - \frac{(1 - F(\theta^*))^2 \delta - F(\theta^*)^2}{(1 - F(\theta^*)) \delta} (1 - \gamma).
\]

(26)

Note that \( \delta (1 - F(\theta^*))^2 - F(\theta^*)^2 \geq 0 \Leftrightarrow \theta^* < Y(\delta) \). The requirement that \( \varphi < 1 \) implies \( \gamma < \varphi \) and \( \theta^* < Y(\delta) \). In this equilibrium, manager \( i \) chooses \( a_i = 1 \) if and
Only if

\[
\bar{v} - \theta_i + \omega \left[ \frac{(F(\theta^*) + (1 - F(\theta^*)) \gamma) p(1, 1)}{1 - (1 - \delta)(1 - F(\theta^*))} \right] + \beta > \bar{v} + \omega \left[ \frac{(1 - \delta)(1 - F(\theta^*)) p(0, 0)}{1 - (1 - \delta)(1 - F(\theta^*))} \right]
\]

Note that \( p(0, 0) = p(0, 1) = \bar{v} \) and \( 2p(1, 1) = \bar{v} + p(1, 0) \). Thus, the above condition holds if and only if

\[
\beta - \omega \left[ (1 - \gamma) + (1 - \delta)(1 - F(\theta^*)) \right] \frac{\bar{v} - p(1, 0)}{2} > \theta_i.
\]

Substituting for \( p(1, 0) \) and using equation (26), manager \( i \) chooses \( a_i = 1 \) if and only if

\[
\beta - \omega \frac{(1 - \gamma) + (1 - \delta)(1 - F(\theta^*))}{1 - (1 - \delta)(1 - F(\theta^*))} F(\theta^*) \mathbb{E} [\theta \mid \theta < \theta^*] > \theta_i,
\]

which yields part (vi). Let \( \psi_{S,D}(\theta^*, \gamma) \) be the LHS of the above inequality. When \( \theta^* \leq Y(\delta) \), \( \frac{(1 - \gamma) + (1 - \delta)(1 - F(\theta^*))}{1 - (1 - \delta)(1 - F(\theta^*))} F(\theta^*) \) is increasing in \( \theta^* \) and so \( \psi_{S,D}(y, \gamma) \) decreases with \( y \). Indeed

\[
\frac{\partial}{\partial x} \left[ \frac{(1 - \gamma)x + (1 - \delta)(x - x^2)}{1 - (1 - \delta)(1 - x)^2} \right] = \frac{[(1 - \gamma) + (1 - \delta)(1 - 2x)] [1 - (1 - \delta)(1 - x)^2] - [(1 - \gamma)x + (1 - \delta)(x - x^2)] [2(1 - \delta)(1 - x)]}{[1 - (1 - \delta)(1 - x)^2]^2},
\]

which is positive if and only if

\[
(1 - \gamma) \left[ \delta + (1 - \delta)x^2 \right] + (1 - \delta) \left[ \delta(1 - x)^2 - x^2 \right] > 0.
\]

Note that

\[
\delta (1 - F(\theta^*))^2 - F(\theta^*)^2 \geq 0 \iff \theta^* \leq Y(\delta).
\]

Thus, an equilibrium exists if there is \( \gamma \in [0, 1] \) such that \( \psi_{S,D}(Y(\delta), \gamma) \leq Y(\delta) \). Since \( \psi_{S,D}(y, \gamma) \) is increasing in \( \gamma \), an equilibrium of this type exists if and only if

\[
\psi_{S,D}(Y(\delta), 0) \leq Y(\delta).
\]

Note that \( \psi_{S,D}(Y(\delta), 0) < \psi_L(Y(\delta), 0) \). Therefore, an equilibrium where \( s = (1, 1) \) is
on the equilibrium path exists if and only if $\psi_{S,D}(Y(\delta),0) \leq Y(\delta)$. Using the explicit term for $Y(\delta)$ yields the expression in the Proposition.

Proof of Lemma 5. Consider equilibria in Proposition 5. Taking a derivative of the LHS of (25) with respect to $F(\theta^*)$ yields

$$p_i(1,0) \left[-1 + \frac{\delta}{2} + \frac{1}{2}\right] + p_i(0,1) \left[\frac{\delta}{2} - \frac{1}{2}\right] + p_i(0,0)(1 - \delta)$$

$$= (1 - \delta) \left[p_i(0,0) - \left(\frac{p_i(0,1) + p_i(1,0)}{2}\right)\right].$$

This is positive because $p_i(0,0) = \pi$, the highest possible price. Thus, the LHS is increasing in $\theta^*$, and so the managers’ decisions exhibit strategic complements. Consider equilibria in Proposition 6. Based on the proof of this Proposition, if the equilibrium exhibits $\varphi = 1$ then the analysis is similar to the on with only large shocks, and hence, similar to Lemma 4, managers’ decisions exhibit strategic complements. Finally, for cases where $\varphi < 1$, we have from the proof of Proposition 6 that the manager shirks if and only if

$$\beta - \omega \left[(1 - \gamma) + (1 - \delta)(1 - F(\theta^*))\right] \frac{\pi - p(1,0)}{2} > \theta_i.$$ 

Note the LHS is increasing in $F(\theta^*)$, the probability that manager $j$ shirks, and hence, the managers’ decisions exhibit strategic complements.

Proof of Theorem 2. Note that

$$\frac{\partial}{\partial y} \psi_{S,ND}(y) = -\omega f(y) (2 - \delta) \frac{(1 - F(y)) y - \frac{1+(1-\delta)(1-F(y))^2}{1-(1-\delta)(1-F(y))^2} F(y) \mathbb{E}[\theta|\theta < y]}{1 - (1 - \delta)(1 - F(y))^2}$$

which is bounded from above. Therefore, there is $\omega' > 0$ such that if $\omega \in (0,\omega')$ then $\frac{\partial}{\partial y} \psi_{S,ND}(y) < 1$ for all $y \in (0,\pi)$. It follows, if $\omega \in (0,\pi)$ then $\psi_{S,ND}(y) = y$ has a
unique solution. Consider part (i). Note that

\[
\psi_{S,ND}(\theta^*) < \psi_B(\theta^*) \iff \frac{(2 - \delta) \left(F(\theta^*)(1 - F(\theta^*)) - \frac{\delta}{1 - \delta} \frac{1}{(1 - \delta)(1 - F(\theta^*))} \right)}{\frac{1}{(1 - \delta)(1 - F(\theta^*))}} > \frac{F(\theta^*)}{F(\theta^*) + \frac{\delta}{1 - \delta}(1 - F(\theta^*))} \iff F(\theta^*)^2 + \frac{\delta}{1 - \delta} F(\theta^*) - \frac{\delta}{1 - \delta} < 0 \iff \frac{\delta}{1 - \delta} \left(\sqrt{\frac{1}{1 - \delta} - \frac{3}{4} - \frac{1}{2}}\right) < F(\theta^*) < \frac{\delta}{1 - \delta} \left(\sqrt{\frac{1}{1 - \delta} - \frac{3}{4} - \frac{1}{2}}\right)
\]

Note that \(\frac{\delta}{1 - \delta} \left(\sqrt{\frac{1}{1 - \delta} - \frac{3}{4} - \frac{1}{2}}\right) \in (0, 1)\) for any \(\delta \in (0, 1)\). Thus, if \(\psi_{S,ND}(\theta_B^*) \leq \psi_B(\theta_B^*) = \theta_B^*\). Since \(\psi_{S,ND}(0) > 0\) then there exists \(\theta^* \in (0, \theta_B^*)\) such that \(\psi_{S,ND}(\theta^*) = \theta^*\). Therefore, there is an equilibrium with two firms that is more efficient. In contrast, if \(\psi_{S,ND}(\theta_B^*) > \psi_B(\theta_B^*) = \theta_B^*\). Since \(\psi_{S,ND}(\overline{\gamma}) = \beta < \overline{\gamma}\), there exists \(\theta^* \in (\theta_B^*, \overline{\gamma})\) such that \(\psi_{S,ND}(\theta^*) = \theta^*\).\(^{[2]}\) We first prove that the equilibrium with \(\theta^* < Y(\delta)\) and increasing in \(\gamma^*\). As in the proof of Proposition 6, such an equilibrium satisfies \(\psi_{S,D}(\theta^*, \gamma^*) = \theta^*\). Also in the proof we showed \(\psi_{S,D}(\theta^*, \gamma^*)\) is decreasing in \(\theta^*\) for \(\theta^* < Y(\delta)\) and increasing in \(\gamma^*\). Denote the equilibrium cutoff when \(\gamma^* = 0\) as \(\theta_0^*\), and the equilibrium for an alternative \(\widehat{\gamma} > 0\) as \(\theta_0^*\). Since \(\theta_0^* = \psi_{S,D}(\theta_0^*, 0) < \psi_{S,D}(\theta_0^*, \widehat{\gamma})\), it must be that \(\theta_0^* > 0\). Therefore, the equilibrium for \(\gamma^* = 0\) and \(\varphi^* < 1\) is more efficient than one in which \(\gamma^* > 0\) and \(\varphi^* < 1\). Second, in the equilibrium where \(s = (1, 1)\) is on the equilibrium path and \(\gamma^* \in [0, 1]\) and \(\varphi^* = 1\) (i.e. \(\pi + p(1,0) = 2p(1,1)\) and \(p(1,0) > q(\theta^*)\), we showed in the proof of Proposition 6 that \(\theta^* = Y(\delta)\). Therefore, trivially the equilibrium in which \(\gamma^* = 0\) and \(\varphi^* < 1\) is more efficient since we proved \(\theta_0^* < Y(\delta)\). Lastly, we compare the equilibrium where \(\gamma^* = 0\) and \(\varphi^* < 1\) to the equilibrium in which \(\gamma^* = \varphi^* = 1\). In that case, we have as a necessary condition \(\theta^* < Y(\delta)\). Denoting the cutoff in that case as \(\theta_1^*\), it must satisfy \(\psi_L(\theta_1^*, 1) = \theta_1^*\). The
equilibrium when $\gamma^* = 0$ and $\varphi^* < 1$ is more efficient iff

$$\theta_0^* = \psi_{S,D}(\theta_0^*, 0) < \psi_L(\theta_1^*, 1) = \theta_1^*$$

Since $\psi_L(\theta, 1)$ is decreasing in $\theta$ for $\theta \leq Y(\delta)$ (as shown in the proof of Proposition 2), and since $\theta_0^* < Y(\delta)$ and $\theta_1^* < Y(\delta)$ (as shown in the proof of Proposition 6), a sufficient condition for this to hold is

$$\theta_0^* = \psi_{S,D}(\theta_0^*, 0) < \psi_L(\theta_0^*, 1)$$

Given the forms of $\psi_L(\theta_0^*, 1)$ and $\psi_{S,D}(\theta_0^*, 0)$, denoting $F \equiv F(\theta_0^*)$ the above condition holds iff:

$$(F + (1-\delta)(1-F)F) \left(1 + \frac{1-F}{F} (F(1-\delta) + \delta)\right) > (1 - (1-\delta)(1-F)^2) (1 - F)(1-\delta) \iff$$

$$(F + (1-\delta)(1-F)F) \left(1 + \frac{1-F}{F} (F(1-\delta) + \delta)\right) - (1 - (1-\delta)(1-F)^2) (1 - F)(1-\delta) > 0 \iff$$

$$F + (1-\delta)(1-F)F + (1 + (1-\delta)(1-F)) (1-F) (F(1-\delta) + \delta) - (1 - (1-\delta)(1-F)^2) (1 - F)(1-\delta) > 0 \iff$$

$$(\delta - 1)F^2 + 2(1-\delta)F + \delta > 0$$

The roots of the polynomial on the LHS are given by:

$$\theta^* = 1 \pm \frac{1}{\sqrt{1-\delta}}$$

Since $\delta \in (0, 1)$, one root is larger than 1 and one is less than 0. Therefore, for all $F \in [0,1]$ (and therefore for all $\theta^* \in [0,\bar{\theta}]$), the inequality holds. This implies that $\theta_0^* < \theta_1^*$, so the equilibrium in which $\gamma^* = 0$ and $\varphi^* < 1$ is more efficient than all others in which $s = (1,1)$ is on the equilibrium path. Based on the proof of Proposition 6, if there exists an equilibrium in which $s = (1,1)$ is on the equilibrium path, then there is an equilibrium in which $s = (1,1)$ is on the equilibrium path and $\gamma = 0$, and this equilibrium is the most efficient among these equilibria. We thus focus on $\psi_{S,D}(\theta^*, 0)$. Note that $\psi_{S,D}(\theta^*, 0) < \psi_{S,N,D}(\theta^*) \forall \theta^*$. Therefore, if there is an equilibrium where $s = (1,1)$ is on the equilibrium path, then this equilibrium with $\gamma = 0$ is the most efficient among all equilibria. Recall that an equilibrium in which $s = (1,1)$ is on the equilibrium
path must satisfy $\theta^* \leq Y(\delta)$ and note that $Y(\delta) < F^{-1}\left(\frac{\delta}{1-\delta}\left(\sqrt{\frac{1}{3} - \frac{3}{4} - \frac{1}{2}}\right)\right)$. It follows that, if $\theta^* \leq Y(\delta)$, then $\psi_{S,D}(\theta^*, 0) < \psi_B(\theta^*)$. Since both $\psi_{S,D}(y, 0)$ and $\psi_B(y)$ are decreasing functions, if (16) holds then the equilibrium with two firms in which $s = (1, 1)$ is on the equilibrium path and $\gamma = 0$ exists and it is more efficient than the one-firm benchmark.

Proof of Proposition 7. The correlation $\rho$ has the same sign as the covariance $\sigma_{ij} = \mathbb{E}[p_i \times p_j] - \mathbb{E}[p_i] \mathbb{E}[p_j]$. In all equilibria, we have:

$$\mathbb{E}[p_i] = \mathbb{E}[p_j] = \bar{\tau} - F(\theta^*) \mathbb{E}[\theta|\theta < \theta^*]$$

We first derive $\mathbb{E}[p_i \times p_j]$. To simplify notation, we denote $F \equiv F(\theta^*)$ and $E = \mathbb{E}[\theta|\theta < \theta^*]$. We start by considering equilibria in which $s = (1, 1)$ is not on the equilibrium path. We thus have:

$$\mathbb{E}[p_i \times p_j] = (1 - \delta)(1 - F)^2 p(0, 0)^2 + \left(\delta(1 - F)^2 + 2F - F^2\right) p(1, 0) \times p(0, 1),$$

which simplifies to

$$\mathbb{E}[p_i \times p_j] = \bar{\tau}^2 - 2\bar{\tau}FE + \left(\frac{F^3 - \frac{1}{2}F^4}{F - \frac{1}{2}F^2 + \frac{1}{2}\delta(1 - F)^2}\right) E^2.$$

We thus have:

$$\sigma_{ij} = \mathbb{E}[p_i \times p_j] - \mathbb{E}[p_i] \mathbb{E}[p_j]$$

$$= \bar{\tau}^2 - 2\bar{\tau}FE + \left(\frac{F^3 - \frac{1}{2}F^4}{F - \frac{1}{2}F^2 + \frac{1}{2}\delta(1 - F)^2}\right) E^2 - (\bar{\tau} - FE)^2$$

which simplifies to

$$\sigma_{ij} = -\frac{\delta}{2} \frac{(1 - F)^2}{F - \frac{1}{2}F^2 + \frac{1}{2}\delta(1 - F)^2} F^2 E^2$$

Since $\beta \in (0, \bar{\tau})$, (15), gives us that $\theta_{S,N,D}^*(y, 0) \in (0, \bar{\tau})$, and so $\sigma_{ij}$ is strictly negative. We thus have $\rho^* < 0$. We now turn to equilibria in which $s = (1, 1)$ is on the equilibrium
path, and now use the notation

\[ A = \frac{F [F + (1 - F)\gamma^*]}{F [F + (1 - F)\gamma^*] + (1 - F) [(1 - F)\delta \varphi^* + F \gamma^*]} \]

\[ B = \frac{F(1 - \gamma^*)}{F(1 - \gamma^*) + (1 - F)\delta \frac{1}{2}(1 - \varphi^*)}. \]

We have:

\[ \mathbb{E}[p_i \times p_j] = F^2 p(1, 1)^2 + (1 - \delta)(1 - F)^2 p(0, 0)^2 + 2F(1 - F) (\gamma^* p(1, 1)^2 + (1 - \gamma^*)p(1, 0)p(0, 1)) \]

\[ + \delta(1 - F)^2 (\varphi^* p(1, 1)^2 + (1 - \varphi^*)p(1, 0)p(0, 1)) \]

\[ = \overline{v}^2 + (F^2 + 2F(1 - F)\gamma^* + (1 - F)^2\delta \varphi^*) (-2EA\overline{v} + E^2A^2) \]

\[ + (2F(1 - F)(1 - \gamma^*) + (1 - F)^2\delta(1 - \varphi^*)) (-BE\overline{v}) \]

which eventually yields:

\[ \mathbb{E}[p_i \times p_j] = \overline{v}^2 - 2FE\overline{v} + \frac{F^4 + 2F^3(1 - F)\gamma^* + F^2(1 - F)^2\gamma^*^2}{F^2 + 2F(1 - F)\gamma^* + (1 - F)^2\delta \varphi^*}E^2 \]

The covariance is thus given by:

\[ \sigma_{ij} = \overline{v}^2 - 2FE\overline{v} + \frac{F^4 + 2F^3(1 - F)\gamma^* + F^2(1 - F)^2\gamma^*^2}{F^2 + 2F(1 - F)\gamma^* + (1 - F)^2\delta \varphi^*}E^2 - (\overline{v} - FE)^2, \]

which yields:

\[ \sigma_{ij} = \left( \frac{F^2(1 - F)^2}{F^2 + 2F(1 - F)\gamma^* + (1 - F)^2\delta \varphi^*} \right) (\gamma^*^2 - \delta \varphi^*) E^2, \]

which is positive if and only if \( \gamma^*^2 > \delta \varphi^* \). Proposition 6 stated that, in an equilibrium in which \( s = (1, 1) \) is on the equilibrium path, \( \varphi^* = 1 - \frac{(1 - F)^2 - \delta - F^2}{\delta(1 - F)}(1 - \gamma^*). \) Thus, \( \sigma_{ij} > 0 \) if and only if:

\[ \gamma^*^2 - \delta \left( 1 - \frac{(1 - F)^2 - \delta - F^2}{\delta(1 - F)}(1 - \gamma^*) \right) > 0. \]

Plugging in \( \gamma^* = 0 \) shows that this equilibrium is always violated, so we have \( \rho < 0. \) \( \square \)
B Model with Large and Small Shocks

In this appendix, we consider an extension of the model in which both large and small shocks are present, i.e. \( \delta_S > 0, \delta_L > 0, \) and \( \delta_S + \delta_L < 1 \). In this setting, we will adjust notation slightly. Let \( \chi = 0 \) indicate no liquidity shock, \( \chi = 1 \) denote a small liquidity shock, and \( \chi = 2 \) denote a large liquidity shock. That is, \( \chi \) denotes the number of shares the blockholder is forced to sell due to liquidity shocks.

B.1 Single Firm Case

We start with the single firm benchmark with multiple shocks. We begin by introducing several lemmas. For simplicity, we use blockholder “type” to refer to the liquidity shock she has experienced and/or the action she has observed. For example, a “\( \chi = 1 \) and \( a = 0 \)-type” blockholder is one who has experienced a small liquidity shock and observed shirking.

**Lemma 8.** If \( \chi = 0 \) and \( a = 0 \), then \( s < 2 \).

The proof of this lemma follows the proof of Proposition 1, part (ii). Intuitively, the blockholder can earn \( 2\bar{v} \) by retaining both shares. With large liquidity shocks, there is a positive probability that \( a = 1 \) and \( s = 2 \), so \( \bar{p}(2) < \bar{v} \). Therefore, selling both shares is never optimal when \( \chi = 0 \) and \( a = 0 \).

**Lemma 9.** If \( a = 1 \), then \( s > 0 \). Furthermore, \( s = 1 \) only if \( \bar{p}(1) = \bar{v} \).

**Proof of Lemma 9.** Trivially this is true when \( \chi > 0 \). When \( \chi = 0 \) and \( a = 1 \), \( u_I(0) = 2\bar{v}(\theta^*) \). Since there are large liquidity shocks, with positive probability \( s = 2 \) and \( a = 0 \). Therefore, \( \bar{p}(2) > \bar{v}(\theta^*) \) and so \( u_I(2) = 2\bar{p}(2) > 2\bar{v}(\theta^*) = u_I(0) \).

Intuitively, the blockholder can mask the fact that she observed shirking by pretending she observed \( a = 0 \) but suffered a large liquidity shock. Therefore, it is never optimal to hold both shares when \( \chi = 0 \) and \( a = 1 \).

**Lemma 10.** If \( \chi = 1 \) and \( a = 0 \), then \( s = 1 \) for sure.

This lemma ensures that there is not an equilibrium where the blockholder never sells one share. This contrasts with the large-shock equilibrium with \( \theta^* > Y(\delta) \). In that case, \( \bar{p}(1) \) was sufficiently low that \( s = 1 \) was off-equilibrium. The intuition for this difference is that, since small shocks sometimes occur, a blockholder who observes \( a = 0 \) and \( \chi = 1 \) is sometimes forced to sell at least one share. For \( s = 1 \) to be off
the equilibrium path, it must be that \(2p(2) \geq p(1) + \overline{v}\). However, such an equilibrium cannot satisfy the intuitive criterion, since at the highest possible price \(p(1) = \overline{v}\), the \(a = 0\) and \(\chi = 1\) type would prefer to sell one share, while an \(a = 1\) type would prefer to sell both. This is not a problem in the large shocks only case, since any blockholder who receives a liquidity shock is forced to sell both shares, and we do not require a condition to prevent deviation to selling one share.

**Lemma 11.** There does not exist an equilibrium in which all blockholders observing \(a = 1\) and \(\chi < 2\) choose \(s = 1\) w.p. 1.

Lemma 11 rules out an equilibrium where the blockholder never voluntarily sells both shares. In the small shock model, it was possible for \(s = 2\) to be off the equilibrium path. In that case, \(p(2)\) could be low enough to prevent a blockholder observing \(a = 1\) from deviating to selling both shares. However, with both small and large shocks, \(s = 2\) is always on the equilibrium path, and in particular could result from \(a = 0\) and \(\chi = 2\). This imposes discipline on \(p(2)\) in that it cannot be arbitrarily low. With \(p(2)\) relatively high, a blockholder who observes \(a = 1\) and \(\chi < 2\) would prefer to deviate and sell both shares rather than retaining one share with the lowest possible value.

Thus far, we have concluded that if \(\chi = 0\) and \(a = 0\), the blockholder will sell zero or one share, the latter only if \(p(1) = \overline{v}\). If \(\chi = 1\) and \(a = 0\), then the blockholder plays \(s = 1\) w.p. 1 from Lemma 10. Trivially, \(\chi = 2\) implies \(s = 2\). Therefore, the interesting case is when \(a = 1\) and \(\chi < 2\). We know the blockholder will never play \(s = 0\) with positive probability. Therefore, the investor mixes between \(s = 1\) and \(s = 2\). By retaining one share, she can pretend that her selling is motivated not by \(a = 1\) but by a small shock; by selling both shares, she can pretend that her selling is motivated by a large shock.

Let \(\alpha\) denote the probability the blockholder chooses \(s = 1\) if \(\chi = 0\) and \(a = 1\), implying she chooses \(s = 2\) w.p. \((1 - \alpha)\). Let \(\gamma\) denote the probability the blockholder chooses \(s = 1\) if \(\chi = 1\) and \(a = 1\), implying she chooses \(s = 2\) w.p. \((1 - \gamma)\).

**Lemma 12.** In any equilibrium with both large and small shocks, prices satisfy:

\[
\begin{align*}
p(0) &= \overline{v}, \text{ if } s = 0 \text{ is on the equilibrium path} \\
p(1) &= \overline{v} - \frac{(\delta_S \gamma^* + (1 - \delta_L - \delta_S)\alpha^*)F(\theta^*)}{\delta_S(1 - F(\theta^*)) + \delta_S \gamma^* F(\theta^*) + (1 - \delta_L - \delta_S)\alpha^* F(\theta^*)} \mathbb{E} [\theta | \theta < \theta^*] \\
p(2) &= \overline{v} - \frac{(\delta_L + \delta_S(1 - \gamma^*) + (1 - \delta_L - \delta_S)(1 - \alpha^*))F(\theta^*)}{\delta_L + \delta_S(1 - \gamma^*) F(\theta^*) + (1 - \delta_L - \delta_S)(1 - \alpha^*) F(\theta^*)} \mathbb{E} [\theta | \theta < \theta^*]
\end{align*}
\]

and \(p(1) > p(2)\).
The prices above are determined by Bayes’ rule from the proposed strategies. Furthermore, if \( s = 0 \) is not on the equilibrium path, then \( p(0) \in [\psi(\theta^*), \tau] \). Furthermore, \( p(0) \) is not on the equilibrium path of play only if \( \gamma^* = \alpha^* = 0 \), because a blockholder who observes \( a = 0 \) will be willing to sell one share only if \( p(1) = \tau \). The following Lemma derives the values for \( \gamma^* \) and \( \alpha^* \) in any equilibrium. These values depend on the equilibrium cutoff rule \( \theta^* \) of the manager.

**Lemma 13.** In any equilibrium with a single firm and both large and small shocks, \( \gamma^* \) and \( \alpha^* \) must satisfy:

\[
\alpha^* = \gamma^* = 0, \text{ if } F(\theta^*) \leq \frac{\delta_L}{1 + \delta_L},
\]

and

\[
\gamma^* \delta_S + \alpha^*(1 - \delta_S - \delta_L) = \frac{\delta_S(1 + \delta_L)}{2\delta_L + \delta_S} - \frac{\delta_S \delta_L}{F(\theta^*)(2\delta_L + \delta_S)}, \text{ if } F(\theta^*) > \frac{\delta_L}{1 + \delta_L}.
\]

In the latter case there always exists \( \gamma^* \in [0, 1] \) and \( \alpha^* \in [0, 1] \) such that this equality holds.

Intuitively, a higher likelihood of shirking (a higher \( \theta^* \)) means that, holding \( \alpha^* \) fixed, \( \gamma^* \) must increase, and vice-versa. When the probability of shirking increases, \( p(2) \) falls relatively faster than \( p(1)/2 \). Therefore, to preserve the indifference between selling one or two shares, the likelihood that the blockholder sells one share upon observing \( a = 1 \) must increase, since this decreases \( p(1) \) and increases \( p(2) \). On the other hand, when \( \theta^* \) is small relative to \( \delta_L \), the blockholder strictly prefers to sell both shares and pretend that she has faced a large liquidity shock, so \( \gamma^* = \alpha^* = 0 \).

Lemma 13 shows that, given the cutoff rule of the manager, there exist \( \gamma^* \) and \( \alpha^* \) that generate prices consistent with the equilibrium strategies. However, this does not yet guarantee existence of equilibrium, since in the lemma we took \( \theta^* \) as given. In equilibrium, the manager’s optimal cutoff \( \theta^* \) will respond to changes in the mixing probabilities. Lemma 14 derives the cutoff rule of the manager, given the blockholder’s equilibrium strategy.

**Lemma 14.** Given the blockholder strategies \( (\gamma^*, \alpha^*) \) and the prices in Lemma 12, the manager shirks if and only if \( \theta^* < \theta^*_{BB} \), where \( \theta^*_{BB} \) satisfies:

\[
\theta^*_{BB} = \beta + \omega \left( (1 - \delta_S - \delta_L) \left[ \alpha^* p(1) + (1 - \alpha^*) p(2) - \tau \right] + \delta_S [\gamma^* p(1) + (1 - \gamma^*) p(2) - p(1)] \right)
\]

As is intuitive, the manager’s incentives depend on his probability of being sold and the price impact of being sold. Proposition 9 states that an equilibrium always exists,
and that the cutoff $\theta^*$ is unique given any parameterization.

**Proposition 9.** An equilibrium with $\gamma^* = \alpha^*$ always exists and is unique. Furthermore, the equilibrium cutoff $\theta^*_{BB}$ is invariant to changes in $\gamma^*$ and $\alpha^*$ so long as they satisfy the conditions in Lemma 13, given $\theta^*_{BB}$.

When the probability of shirking $F(\theta^*)$ is relatively low compared to $\delta_L$, the price impact of selling both shares is low, and so a blockholder who observes shirking and suffers a liquidity shock sells both shares, regardless of the size of her shock. This is the same intuition as in the large shock model.

However, with both small and large shocks, when $F(\theta^*)$ is relatively high this pure strategy equilibrium does not survive. This contrasts with the large shock model, where the pure strategy equilibrium always exists. With large shocks only, $s = 1$ is off the equilibrium path if $p(1)$ is sufficiently low. Indeed, the proof of the large shock model shows that $s = 1$ cannot be on the equilibrium path when $F(\theta^*) > \frac{\delta_L}{1+\delta_L}$ – the exact same cutoff for pure strategy equilibria in the multi-shock case. With the addition of small liquidity shocks (regardless of the size of $\delta_S$), $s = 1$ must be on the equilibrium path as shown in Lemma 10. This implies $p(1)$ cannot be arbitrarily low. When $F(\theta^*)$ is large, the price impact from selling both shares is also large. A blockholder who observes shirking and $\chi = 1$ would prefer to sell only one share to pretend that she has suffered a small liquidity shock. By doing so, she can sell one share at a high price, which is preferable to selling both at a low price.

This result is intuitive. With large shocks only, when $F(\theta^*)$ is low a blockholder who observes no liquidity shock and no shirking could play $s = 1$ in equilibrium. Thus, there exists a pure strategy equilibrium where the blockholder sells $s = 1$ when $a = 0$ and $\chi = 0$, and $s = 2$ otherwise. When we include small shocks with $F(\theta^*)$ low, the blockholder facing a small liquidity shock can simply mimic the strategy of the blockholder facing no shocks by trading $s = 1$ when $a = 0$ and $s = 2$ when $a = 1$. However, once $F(\theta^*)$ crosses the threshold $\frac{\delta_L}{1+\delta_L}$, mimicking is no longer possible: with large shocks only, the $\chi = 0$ blockholder will not play $s = 1$ in equilibrium, whereas a blockholder who faces a small shock must sell at least one share. Thus, $p(1)$ is too low to sustain a pure strategy equilibrium.

Even when a pure strategy does not exist, we have a mixed strategy equilibrium. When $F(\theta^*)$ is relatively high, we cannot sustain an equilibrium where a blockholder who observes shirking sells both shares w.p. 1. This is because $p(1)$ would be $\pi$ in such an equilibrium, and so the blockholder will deviate to selling one share. However, if in equilibrium a blockholder who observes shirking sells one share with positive
probability, then \( p(1) < \tau \). Furthermore, \( p(2) \) rises since \( s = 2 \) is less revealing of \( a = 1 \). At certain mixing probabilities, the above price adjustments leave the blockholder indifferent between selling one share or two shares.

Lemma 15 below states that the prices and cutoff rule from the unique equilibrium in Proposition 9 converge to those in each single shock equilibrium as either \( \delta_L \) or \( \delta_S \) approaches zero. Thus, the equilibria in the single shock case are robust to the inclusion of an arbitrarily small probability of the other shock. Even though the model with both large and small shocks involves a mixed strategy equilibrium for high values of \( \theta^* \), the mixing probabilities become arbitrarily close to zero or one as \( \delta_S \) or \( \delta_L \) goes to zero, respectively. In addition, when starting from either the large- or small-shock model, no new equilibria emerge when adding a small probability of the other shock.

**Lemma 15.** Consider the equilibrium prices and cutoff as a function of \( \delta_L \) and \( \delta_S \). The prices and cutoff of the unique equilibrium with multiple shocks converge to the prices and cutoff in the large-shock equilibrium as \( \delta_S \to 0 \), and to the prices and cutoff in the small-shock equilibrium as \( \delta_L \to 0 \).

### B.2 Multiple Firm Case

We now move to the multi-firm case with both large and small shocks, i.e. \( \delta_S > 0 \), \( \delta_L > 0 \), and \( \delta = \delta_L + \delta_S < 1 \).

First note that Lemmas 1 and 2 still hold in the case with multiple shocks. From Lemma 1, we know if \( \chi = 0 \) and \( a = (0, 0) \) then \( s = (0, 0) \). Furthermore, by definition of a large shock, if \( \chi = 2 \) then \( s = (1, 1) \).

**Lemma 16.** In any equilibrium, it must be that \( 2p(1, 1) \geq p(1, 0) + v(\theta^*) \). Furthermore, at least one type of blockholder with \( \chi < 2 \) will select \( s = (1, 1) \) with strictly positive probability.

With \( \delta_L > 0 \), \( s = (1, 1) \) will be played with positive probability, which disciplines the equilibrium price. When \( \delta_L = 0 \), \( s = (1, 1) \) could be off equilibrium, in which case the market maker’s beliefs upon observing this could be low enough to prevent any deviations to \( s = (1, 1) \). This is no longer the case. This lemma effectively rules out the “no double exit” equilibrium that existed with only small shocks.

For the remainder of the section, we place the following restrictions on potential equilibria. Blockholders who observe \( a = (1, 0) \) and receive either a small liquidity shock or no shock must have the same equilibrium strategy. This is not a binding restriction if \( 2p(1, 1) \neq p(1, 0) + \bar{\tau} \), since they have the same incentives. It is only
a restriction on their mixing probabilities if \(2p(1, 1) = p(1, 0) + \bar{\nu} \). Similarly, agents who observe \(a = (1, 1)\) and receive either a small liquidity shock or no shock must also have the same equilibrium strategy. This is not a binding restriction if \(2p(1, 1) \neq p(1, 0) + \bar{\nu}(\theta^*)\), again since they will have the same incentives.

**Proposition 10.** Let \(F(Y(\delta)) \equiv \frac{\sqrt{\gamma}}{1 + \sqrt{\gamma}}\). We have the following:

\[
\psi_{M,1}(Y(\delta), 0, 1) \equiv \beta - \omega \left( (1 - \delta_L - \delta_S)(1 - \frac{1}{2} F(Y(\delta))) + \frac{1}{2} \delta_S \right) (\bar{\nu} - \nu(\theta^*)) \leq Y(\delta) \tag{28}
\]

if and only if there exists an equilibrium with

\[
\xi^* = 1 - \frac{(1 - \delta_L)\delta_S(1 - F)^2 - (2\delta_L F - \delta_L + (1 - \delta_L)F^2)(1 - \delta_L)}{\delta_S(1 - F)} (1 - \gamma^*) > 0. \tag{29}
\]

that satisfies the following properties:

(i) If \(a = (1, 1)\), then \(s = (1, 1)\) for sure.

(ii) If \(a = (0, 0)\) and \(\chi = 0\), then \(s = (0, 0)\) for sure.

(iii) If \(a = (1, 0)\) and \(\chi < 2\), then \(s = (1, 1)\) w.p. \(\gamma^*\) and \(s = (1, 0)\) w.p. \(1 - \gamma^*\).

(iv) If \(a = (0, 0)\) and \(\chi = 1\), then \(s = (1, 1)\) w.p. \(\xi^*\) and \(s = (1, 0)\) or \(s = (0, 1)\) w.p. \(\frac{1}{2}\) each otherwise.

(v) Prices satisfy:

\[
p(0, 0) = p(0, 1) = \bar{\nu}
\]

\[
p(1, 0) = \bar{\nu} - \left( \frac{(1 - \delta_L)F(1 - F)(1 - \gamma^*)}{(1 - \delta_L)F(1 - F)(1 - \gamma^*) + \frac{1}{2} \delta_S(1 - F)^2(1 - \xi^*)} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*], \quad \text{if } \gamma^* \text{ or } \xi^* < 1
\]

\[
\leq 2p(1, 1) - \bar{\nu}, \quad \text{otherwise}
\]

\[
p(1, 1) = \bar{\nu} - \left( \frac{\delta_L F + (1 - \delta_L)(F^2 + F(1 - F)^2 \gamma^*)}{\delta_L + (1 - \delta_L)(F^2 + 2F(1 - F)^2 \gamma^*) + \delta_S(1 - F)^2 \xi^*} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

(vi) Manager \(i\) shirks if and only if \(\theta_i \leq \theta_{M,1}^*\), where \(\theta_{M,1}^*\) is defined by:

\[
\psi_{M,1}(\theta_{M,1}^*, \gamma^*, \xi^*) \equiv \beta + \omega ((1 - \delta_L - \delta_S)[F(p(1, 1) - \gamma^* p(1, 1) - (1 - \gamma^*) p(0, 1)] + (1 - F)(\gamma^* p(1, 1) + (1 - \gamma^*) p(1, 0) - p(0, 0)) + \delta_S[F(p(1, 1) - \gamma^* p(1, 1) - (1 - \gamma^*) p(0, 1)] + (1 - F)(\gamma^* p(1, 1) + (1 - \gamma^*) p(1, 0) - \xi^* p(1, 1) - (1 - \xi^*) \frac{1}{2} (p(1, 0) + p(0, 1)))] = \theta_{M,1}^*
\]

and prices and mixing probabilities satisfy the above conditions.

(a) An equilibrium with \(\gamma^* = \xi^* = 1\) exists if and only if \(\psi_{M,1}(Y(\delta), 1, 1) \leq Y(\delta)\).

The cutoff \(\theta^*\) associated with this equilibrium is unique.
(b) An equilibrium with $\gamma^* \in [0, 1)$, $\xi^* = 1$, and $\theta^* = Y(\delta)$ exists if and only if 
$\psi_{M,1}(Y(\delta), 1, 1) > Y(\delta) \geq \psi_{M,1}(Y(\delta), 0, 1)$, with $\gamma^*$ and the cutoff unique.

Furthermore, let

$$F(\theta^*_{LL}) \equiv \frac{-\delta_L(1 - \delta) - \delta(1 - \delta_L) + \sqrt{(\delta_L(1 - \delta) + \delta(1 - \delta_L))^2 + 4(1 - \delta_L)\delta_L(1 - \delta)^2}}{2(1 - \delta_L)(1 - \delta)}.$$ 

Then, both

$$\psi_{M,1}(Y(\delta), 0, 1) \leq Y(\delta) \quad (30)$$

and

$$\psi_{M,1}(\theta^*_{LL}, 0, 0) > \theta^*_{LL} \quad (31)$$

hold if and only if there exists an equilibrium satisfying the above conditions with $\gamma^* = 0$ and $\xi^* > 0$. The cutoff rule $\theta^* \in (\theta^*_{LL}, Y(\delta)]$ associated with this equilibrium is unique.

The equilibrium with $\gamma^* = \xi^* = 1$ is analogous to the large shock only equilibrium in which $\gamma^* = 1$, where all blockholders choose double exit unless neither manager shirks and there is no liquidity shock. It requires $F(\theta^*)$ not to be too large relative to $\delta_L + \delta_S$. As before, the intuition is straightforward: little value is destroyed for low levels of $\theta^*$. Even if a manager shirks, the value erosion is small. A relatively large $p(1, 1)$ will prevent deviations of a blockholder observing $a = (1, 0)$ to $s = (1, 0)$ or even $s = (0, 0)$, even at the lowest possible price $\underline{v}(\theta^*)$. However, for larger $\theta^*$, there is too much of a price impact to sustain this equilibrium. This equilibrium is unique up to $\delta = \delta_L + \delta_S$. This is because the small shock blockholders behave identically to the large shock blockholders so all that matters is the probability of any liquidity shock.

The equilibrium with $\gamma^* = 0$ is analogous to the equilibrium with small shocks only with double exit. In this equilibrium, $\xi^*$ is set such that $2p(1, 1) = p(1, 0) + \overline{\xi}$, which gives incentives for the $a = (0, 0)$ and $\chi = 1$ to be willing to mix between selling both and selling only one. This cannot be sustained if $\theta^*$ is too high or too low. Consider a relatively high $\theta^*$. Even at $\gamma^* = 0$ and $\xi^* = 1$, $p(1, 0) = \overline{v}(\theta^*)$ is too high relative to $p(1, 0)$ to generate indifference, so this cannot be sustained.

Now, consider a relatively low $\theta^*$. Even at $\gamma^* = 0$ and $\xi^* = 0$, $p(1, 0) + \overline{\xi}$ is too high relative to $2p(1, 1)$. To generate indifference, $\xi^*$ would necessarily be negative. It is potentially possible to allow $\gamma^*$ to rise, which increases the implicit $\xi^*$ that generates indifference.

Note that there may be a continuum of equilibria in Proposition 10, each characterized by a different $\gamma^*$. The following Lemma fully characterizes the equilibrium values of $\gamma^*$.

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Lemma 17. Suppose an equilibrium in Proposition 10 exists. Then, for each $\gamma^* \in (\gamma^*_{\min}, \gamma^*_{\max}]$ there exists an equilibrium with $\xi^* > 0$, where $\gamma^*_{\min}$ and $\gamma^*_{\max}$ are defined as follows:

(i) If $\psi_{M,1}(Y(\delta), 1, 1) \leq Y(\delta)$, $\gamma^*_{\max} = 1$. Otherwise, $\gamma^*_{\max}$ satisfies $\psi_{M,1}(Y(\delta), \gamma^*_{\max}, 1) = Y(\delta)$.

(ii) If $\psi_{M,1}(\theta^*, 0, 0) > \theta^*$, $\gamma^*_{\min} = 1$. Otherwise, let

$$1 - \tilde{\gamma}(\theta) \equiv \frac{\delta_S(1 - F(\theta))}{(1 - \delta_L)\delta_S(1 - F(\theta))^2 - (2\delta_L F(\theta) - \delta_L + (1 - \delta_L) F(\theta)^2)(1 - \delta_L)}.$$  

Then, $\gamma^*_{\min} = \tilde{\gamma}(\theta_{\min})$ where $\theta_{\min}$ satisfies:

$$\psi_{M,1}(\theta_{\min}, \tilde{\gamma}(\theta_{\min}), 0) = \theta_{\min}.$$  

Furthermore, for each possible equilibrium value of $\gamma^*$, there is a unique cutoff rule for the manager $\theta^*$.

The following lemma addresses equilibria analogous to the large shock only case in which $\eta^* = 0$, and shows that introducing a positive probability of a small liquidity shock eliminates this equilibrium unless $\theta^* = Y(\delta)$.

Lemma 18. An equilibrium in which an investor who observes $a = (1, 0)$ and $\chi = 0$ chooses $s = (0, 0)$ with positive probability exists only if $\theta^* = Y(\delta)$.

The next proposition discusses an additional type of equilibrium, in which the blockholder observing $a = (0, 0)$ and $\chi = 1$ always sells only one share, with the probability of either share being sold as $\frac{1}{2}$. That is, where $\xi^* = 1$. There is an intermediate region of $\theta^*$ such that this equilibrium is in pure strategies. For low and high values of $\theta^*$, there may only exist mixed strategy equilibria of this form.

Proposition 11. Let $\delta \equiv \delta_S + \delta_L$. An equilibrium exists with the following properties:

(i) If $\chi = 2$, $s = (1, 1)$ for sure.

(ii) If $\chi < 2$ and $a = (1, 1)$, then $s = (1, 1)$ w.p. $\alpha^* \in [0, 1]$ and either $s = (1, 0)$ or $s = (0, 1)$ w.p. $1 - \alpha^*$, each w.p. $\frac{1}{2}$.

(iii) If $\chi < 2$ and $a = (1, 0)$, then $s = (1, 1)$ w.p. $\gamma^* \in [0, 1]$ and $s = (1, 0)$ w.p. $1 - \gamma^*$.

(iv) If $\chi = 0$ and $a = (0, 0)$, then $s = (0, 0)$ for sure.

(v) If $\chi = 1$ and $a = (0, 0)$, then $s = (1, 0)$ or $s = (0, 1)$, each w.p. $\frac{1}{2}$. 
(vi) Prices satisfy:

\[ p(0, 0) = \overline{v} \]
\[ p(0, 1) = \overline{v} - \left( \frac{1}{2} (1 - \delta_L)(1 - \alpha^*)(F^2 + (1 - F)(1 - \gamma^*)) + \frac{1}{2} \delta_S(1 - F)^2 \right) \mathbb{E} [\theta_i \mid \theta_i < \theta^*] \]
\[ p(1, 0) = \overline{v} - \left( \frac{1}{2} (1 - \delta_L)(F^2(1 - \alpha^*) + F(1 - F)(1 - \gamma^*)) \right) \mathbb{E} [\theta_i \mid \theta_i < \theta^*] \]
\[ p(1, 1) = \overline{v} - \left( \frac{\delta_L F^2 + (1 - \delta_L)(F^2\alpha^* + F(1 - F)\gamma^*)}{\delta_L + (1 - \delta_L)(F^2\alpha^* + 2F(1 - F)\gamma^*)} \right) \mathbb{E} [\theta_i \mid \theta_i < \theta^*] \]

(vii) There exist \( \theta_{M,2}^* \in [0, \overline{v}] \) and mixing probabilities \( \gamma^* \) and \( \alpha^* \) satisfying

\[
\psi_{M,2}(\theta_{M,2}^*, \gamma^*, \alpha^*) \equiv \beta + \omega((1 - \delta_L - \delta_S)[F(\alpha^*p(1, 1) + (1 - \alpha^*)\frac{1}{2}(p(1, 0) + p(0, 1)) - \gamma^*p(1, 1) - (1 - \gamma^*)p(0, 1)) + (1 - F)(\gamma^*p(1, 1) + (1 - \gamma^*)p(1, 0) - p(0, 0))] + \delta_S[F(\alpha^*p(1, 1) + (1 - \alpha^*)\frac{1}{2}(p(1, 0) + p(0, 1)) - \gamma^*p(1, 1) - (1 - \gamma^*)p(0, 1)) + (1 - F)(\gamma^*p(1, 1) + (1 - \gamma^*)p(0, 1) - \frac{1}{2}(p(1, 0) + p(0, 1)))] = \theta_{M,2}^*
\]

where prices are evaluated at \( \theta_{M,2}^* \) and the mixing probabilities, such that the manager shirks if and only if \( \theta < \theta_{M,2}^* \).

Furthermore, let \( \theta_{L}^{**} \) and \( \theta_{U}^{**} \) be defined as:

\[
F(\theta_{L}^{**}) = -\delta_L(1 - \delta) - \delta(1 - \delta_L) + \sqrt{(\delta_L(1 - \delta) + \delta(1 - \delta_L))^2 + 4(1 - \delta_L)\delta_L(1 - \delta)^2}
\]
\[
F(\theta_{U}^{**}) = \min \left\{ 1, \frac{\delta_L}{\delta_S} - \frac{\delta_L}{1 - \delta_L} + \frac{1}{\delta_L}(\overline{\delta_S} + 4\delta_L(1 - \delta_L)(1 - \delta)) \right\}.
\]

which satisfy \( \theta_{L}^{**} < \theta_{U}^{**} \). Then:

(a) If \( \psi_{M,2}(\theta_{L}^{**}, 0, 1) < \theta_{L}^{**} \), then an equilibrium exists with \( \theta^* < \theta_{L}^{**}, \alpha^* = 1 \), and \( \gamma^* \in (0, 1) \) such that \( 2p(1, 1) = p(1, 0) + \overline{v} \).

(b) If \( \psi_{M,2}(\theta_{L}^{**}, 0, 1) \geq \theta_{L}^{**} \) and \( \psi_{M,2}(\theta_{U}^{**}, 0, 1) \leq \theta_{U}^{**} \), then an equilibrium exists with \( \theta^* \in [\theta_{L}^{**}, \theta_{U}^{**}], \gamma^* = 0 \) and \( \alpha^* = 1 \).

(c) If \( \psi_{M,2}(\theta_{U}^{**}, 0, 1) > \theta_{U}^{**} \), then an equilibrium exists with \( \theta^* > \theta_{U}^{**}, \gamma^* = 0 \) and \( \alpha^* \in (0, 1) \) such that \( 2p(1, 1) = p(1, 0) + \overline{v}(\theta^*) \).

The following four lemmas show that the multiple shock equilibria discussed in the above Propositions converge to single shock equilibria as either \( \delta_L \) or \( \delta_S \) approaches zero.
Lemma 19. Consider a set of parameters excluding $\delta_S$ such that $\exists \delta_S^* > 0$ such that $\forall \delta_S \leq \delta_S^*$, an equilibrium with $\gamma^* = \xi^* = 1$ in Proposition 10 exists. Then, the cutoff rule $\theta^*$ and prices associated with those strategies played in with positive probability in the associated equilibria converge to the cutoff rule and prices in the large shocks only equilibrium with $\eta^* = 0$.

Lemma 20. Consider a set of parameters excluding $\delta_L$ such that $\exists \delta_L^* > 0$ such that $\forall \delta_L \leq \delta_L^*$, an equilibrium with $\gamma^* = 0$ and $\xi^* > 0$ in Proposition 10 exists. Then, the cutoff rule $\theta^*$ and prices associated with those strategies played with positive probability in the associated equilibria with $\gamma^* = 0$ converge to the cutoff rule and prices in the small shocks only equilibrium with double exit.

Lemma 21. As $\delta_S \to 0$, the equilibrium cutoff rule and prices in the associated equilibria in Proposition 11 converge to the cutoff rule and prices in the large shocks only equilibrium with $\eta^* = 0$.

Lemma 22. As $\delta_L \to 0$, the equilibrium cutoff in Proposition 11 converges to the no double exit equilibrium with only small shocks.

These previous lemmas all demonstrate that the equilibria with one shock are generally robust to the small increases in the probability of the other shock. Next, we move to efficiency comparisons. We start by ordering the efficiency of the different types of equilibria with multiple shocks and multiple firms.

Lemma 23. Among equilibria in Proposition 10, the efficiency of equilibria decreases monotonically as $\gamma^*$ increases.

This lemma shows that equilibria with lower values of $\gamma^*$ will be more efficient among those in Proposition 10. When $\gamma^* = 0$, for instance, if manager $i$ works, he will not be sold unless the blockholder faces a liquidity shock. A larger value of $\gamma^*$ implies that he is more likely to be sold even if he does not shirk, weakening his incentives and leading to a higher cutoff for the manager.

B.3 Proofs for Single Firm Case

Proof of Lemma 10. First, we argue that there is no equilibrium in which $s = 1$ is off the equilibrium path. To show this, we prove by contradiction. Suppose that $s = 1$ is not played with positive probability. In this case, $\chi = 1$ implies that $s = 2$, and $\chi = 0$ and $a = 1$ implies $s = 2$. In such an equilibrium,

$$p(2) = \bar{\pi} - E [\theta | \theta < \theta^*] \left( \frac{F(\theta^*)}{\delta_L + \delta_S + (1 - \delta_S - \delta_L)F(\theta^*)} \right)$$  \hspace{1cm} (32)
For this equilibrium to be sustained, it must be that
\[ 2p(2) \geq \pi + p(1) > \psi(\theta^*) + p(1). \]

However, any equilibrium of this type will fail the intuitive criterion. If \( p(1) = \pi \), then an investor who observes \( \chi = 1 \) and \( a = 0 \) would surely deviate to \( s = 1 \). An investor who observes \( a = 1 \) would not deviate, since her payoff upon deviating would be \( \pi + \psi(\theta^*) \), and we have:
\[ 2p(2) \geq \pi + p(1) \geq \pi + \psi(\theta^*). \]

Therefore, upon observing \( s = 1 \), the market maker would assign probability one to \( a = 0 \), and therefore any \( p(1) \) that satisfies equation (32) would not satisfy the intuitive criterion. It follows then that \( s = 1 \) must be played with positive probability. We now complete the proof. Suppose there exists an equilibrium in which an investor observing \( \chi = 1 \) and \( a = 0 \) selects \( s = 2 \) with positive probability. It must be the case then that
\[ p(1) + \psi(\theta^*) < p(1) + \pi \leq 2p(2). \]
Therefore, any investor observing \( s = 1 \) with positive probability must have observed \( a = 0 \), we have \( p(1) = \pi \). However, this means \( p(1) + \pi = 2\pi > 2p(2) \), a contradiction.

**Proof of Lemma 11.** Suppose that such an equilibrium exists. Equilibrium prices must then satisfy \( p(1) + \psi(\theta^*) \geq 2p(2) \). Furthermore, an investor observing \( \chi = 1 \) and \( a = 0 \) chooses \( s = 1 \) since \( \pi + p(1) > \psi(\theta^*) + p(1) \). An investor observing \( \chi = 0 \) and \( a = 0 \) then chooses \( s = 0 \) w.p. one. Equilibrium prices would then be:
\[
\begin{align*}
p(0) &= \pi \\
p(1) &= \pi - \frac{F(\theta^*)(1 - \delta_L)}{\delta_S + (1 - \delta_S - \delta_L)F(\theta^*)} \mathbb{E}[\theta|\theta < \theta^*] \\
p(2) &= \pi - \frac{F(\theta^*)\delta_L}{\delta_L} \mathbb{E}[\theta|\theta < \theta^*] = \pi - F(\theta^*)\mathbb{E}[\theta|\theta < \theta^*].
\end{align*}
\]

The inequality \( 2p(2) \leq \psi(\theta^*) + p(1) \) then holds if and only if:
\[
2\pi - 2F(\theta^*)\mathbb{E}[\theta|\theta < \theta^*] \leq 2\pi - (1 + \frac{\delta_S}{\delta_S + (1 - \delta_S - \delta_L)F(\theta^*)}) \mathbb{E}[\theta|\theta < \theta^*] \iff \frac{\delta_S}{\delta_S + (1 - \delta_S - \delta_L)F(\theta^*)} \leq 1 - \delta_S - \delta_L \iff \delta_S F(\theta^*) + 2(1 - \delta_L)F(\theta^*) \leq 2\delta_S F(\theta^*) + 2F(\theta^*)^2(1 - \delta_S - \delta_L) \iff 0 \leq 2(1 - \delta_S - \delta_L)F(\theta^*)^2 + (\delta_S - 2(1 - \delta_L - \delta_S)) F(\theta^*) - \delta_S \iff
\]
0 ≤ (F(θ∗) − 1) (2 − δ_L − δ_S) F(θ∗) + δ_S). The roots of the RHS are F(θ∗) = 1 and 
F(θ∗) = − \frac{δ_S}{2(1 − δ_L − δ_S)} < 0. Since the leading coefficient is larger than zero, it is a convex 
parabola and for all F(θ∗) ∈ (0, 1), the inequality is violated. Therefore, this is not an 
equilibrium. \square

**Proof of Lemma 12.** The prices are determined by Bayes’ rule. The fact that p(1) > p(2) must hold is seen from the following argument. We know that in any equilibrium, 
we must have 2p(2) ≥ p(1) + v(θ∗) since from Lemma 11, some blockholder type 
observing a = 1 must choose s = 1. If 2p(2) = p(1) + v(θ∗), then since min{p(1), p(2)} > v(θ∗), we must have p(1) > p(2). If 2p(2) > p(1) + v(θ∗), then we must have γ∗ = α∗ = 0 
which means p(1) = \overline{v} > p(2). \square

**Proof of Lemma 13.** Throughout the proof we use the notation F ≡ F(θ∗). First, 
suppose in equilibrium F ≤ \frac{δ_L}{1 + δ_L}. Then, at γ∗ = α∗ = 0, we have p(1) = \overline{v} and 
p(2) = \overline{v} − \frac{δ_L}{(1 + δ_L)δ_L + (1 − δ_L)δ_L} \mathbb{E}[θ|θ < θ∗] = 
\overline{v} − \frac{1}{2} \mathbb{E}[θ|θ < θ∗] = \overline{v} + v(θ∗)

Therefore, under the restriction F ≤ \frac{δ_L}{1 + δ_L} and with γ∗ = α∗ = 0, we have 2p(2) ≥ 
p(1) + v(θ∗) = \overline{v} + v(θ∗). Furthermore, we cannot have γ∗ > 0 or α∗ > 0. If this was 
the case, we require 2p(2) = p(1) + v(θ∗). However, since p(2) is increasing and p(1) is 
decreasing in γ∗ and α∗, this implies that 2p(2) > p(1) + v(θ∗). This is inconsistent 
with any a = 1 type investor playing s = 1 with positive probability. Therefore, if 
F ≤ \frac{δ_L}{1 + δ_L} we must have γ∗ = α∗ = 0. To prove the second part of the lemma, suppose 
F > \frac{δ_L}{1 + δ_L}. From the above argument, this is inconsistent with γ∗ = α∗ = 0. For any 
equilibrium in which either α∗ > 0 or γ∗ > 0, we require 2p(2) = p(1) + v(θ∗). Given 
the prices in the previous lemma, this condition is equivalent to:

\begin{align*}
\frac{2(δ_L + δ_S(1 − γ∗) + (1 − δ_L − δ_S)(1 − α∗))F}{δ_L + δ_S(1 − γ∗)F + (1 − δ_L − δ_S)(1 − α∗)F} = 1 & \iff \frac{(δ_Sγ∗ + (1 − δ_L − δ_S)α∗)F}{δ_S(1 − F) + δ_Sγ∗F + (1 − δ_L − δ_S)α∗F} \\
\frac{2(1 − δ_Sγ∗ − (1 − δ_L − δ_S)α∗)F}{δ_L + (1 − δ_L)F − δ_Sγ∗F − (1 − δ_L − δ_S)α∗F} = & \frac{δ_S(1 − F) + 2(δ_Sγ∗ + (1 − δ_L − δ_S)α∗)F}{δ_S(1 − F) + (δ_Sγ∗ + (1 − δ_L − δ_S)α∗)F}
\end{align*}

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To ease notation, we define $X \equiv \delta_S \gamma^* + (1 - \delta_L - \delta_S) \alpha^*$. Then, the above simplifies to:

\[
\frac{2(1 - X) F}{\delta_L + (1 - \delta_L) F - X F} = \frac{\delta_S (1 - F) + 2 X F}{\delta_S (1 - F) + X F} \iff
\]

\[
2 F (1 - F) \delta_S - 2 F (1 - F) \delta_S X + 2 F^2 X - 2 F^2 X^2 =
\]

\[
(\delta_L + (1 - \delta_L) F) \delta_S (1 - F) + 2 (\delta_L + (1 - \delta_L) F) X F - F (1 - F) \delta_S X - 2 X^2 F^2 \iff
\]

\[
X (-F (1 - F) (\delta_S + 2 \delta_L)) = \delta_L \delta_S (1 - F)^2 - F (1 - F) \delta_S \iff
\]

\[
X = \frac{\delta_S (1 + \delta_L)}{\delta_S + 2 \delta_L} - \frac{\delta_S \delta_L}{F (\delta_S + 2 \delta_L)}
\]

This gives the condition from the lemma. Note that the RHS is increasing in $F$, and the LHS is increasing in both $\gamma$ and $\alpha$. Furthermore, since $\gamma \geq 0$ and $\alpha \geq 0$, we have the LHS is at least 0. At $F = \frac{\delta_L}{1 + \delta_L}$, we have RHS $= 0$. Therefore, for $F \leq \frac{\delta_L}{1 + \delta_L}$, the RHS is at most 0, and therefore it must be that $\gamma^* = \alpha^* = 0$. For $F \geq \frac{\delta_L}{1 + \delta_L}$, $\gamma^*$ and $\alpha^*$ must adjust to preserve the equality above. To complete the proof, we argue that for any $F (\theta^*) \in \left[\frac{\delta_L}{1 + \delta_L}, 1\right]$, there exist feasible mixing probabilities that satisfy the necessary equality. To see this, the RHS is maximized at $F = 1$ and attains a value of $\frac{\delta_S}{2 \delta_L + \delta_S}$. The LHS is increasing in $\gamma^*$ and $\alpha^*$. We have that the $LHS = 0$ at $\gamma^* = \alpha^* = 0$, and the $LHS = 1 - \delta_L$ at $\gamma^* = \alpha^* = 1$. Lastly, we have:

\[
1 - \delta_L > \frac{\delta_S}{2 \delta_L + \delta_S} \iff
\]

\[
(1 - \delta_L) (2 \delta_L + \delta_S) > \delta_S \iff
\]

\[
2 \delta_L - 2 \delta_L^2 + \delta_S - \delta_S \delta_L > \delta_S \iff
\]

\[
2 - 2 \delta_L - \delta_S > 0
\]

The last line holds since $2 - 2 \delta_L - \delta_S > 1 - \delta_L > 0$. Therefore, for any $F$, there exist $\gamma^*$ and $\alpha^*$ such that the equality in the lemma holds. \hfill \Box

**Proof of Lemma 14.** If the manager shirks, given the strategy of the investor, his value is:

\[
\beta + \bar{\nu} - \theta + \omega ((1 - \delta_L - \delta_S) [\alpha^* p(1) + (1 - \alpha^*) p(2)] + \delta_S [\gamma^* p(1) + (1 - \gamma^*) p(2)] + \delta_L p(2))
\]

If he does not shirk, his value is:

\[
\bar{\nu} + \omega ((1 - \delta_L - \delta_S) \bar{\nu} + \delta_S p(1) + \delta_L p(2))
\]
Therefore, the manager shirks if and only if:

\[
\beta + \varpi - \theta + \omega((1 - \delta_L - \delta_S)[\alpha^* p(1) + (1 - \alpha^*) p(2)] + \delta_S [\gamma^* p(1) + (1 - \gamma^*) p(2)] + \delta_L p(2)) > \\
\varpi + \omega((1 - \delta_L - \delta_S)\varpi + \delta_S p(1) + \delta_L p(2)) \iff \\
\beta + \omega((1 - \delta_L - \delta_S)[\alpha^* p(1) + (1 - \alpha^*) p(2) - \varpi] + \delta_S [\gamma^* p(1) + (1 - \gamma^*) p(2) - p(1)]) > \theta
\]

Since the LHS is constant given prices, and the RHS is strictly increasing in \(\theta\), the manager follows a cutoff rule. \(\square\)

**Proof of Proposition 9.** First, consider equilibria with \(\gamma^* = \alpha^*\). An equilibrium in this case is a pair \((\gamma^*, \theta^*)\) such that \(\gamma^*\) and \(\theta^*\) satisfy the conditions in Lemma 13, and

\[
\psi_{BB}(\gamma^*, \theta^*) \equiv \beta + \omega((1 - \delta_L) [\gamma^* p(1) + (1 - \gamma^*) p(2) - \varpi] + \delta_S [\gamma^* p(1) + (1 - \gamma^*) p(2) - p(1)]) = \theta^*
\]

where the prices implicitly depend on \(\gamma^*\) and \(\theta^*\). Throughout the proof we will define \(Y(\delta_L) = F^{-1}\left(\frac{\delta_S}{1 + \delta_S}\right)\). First, consider the case with

\[
\psi_{BB}(0, Y(\delta_L)) \leq Y(\delta_L)
\]

Note that \(\psi_{BB}(0, \theta)\) is decreasing in \(\theta\) since \(p(2)\) is decreasing in \(\theta\) holding fixed \(\gamma^*\). Furthermore, \(\psi_{BB}(0, 0) = \beta > 0\). Therefore, in this case there exists a unique \(\theta^* \in (0, Y(\delta_L)]\) such that \(\psi_{BB}(0, \theta^*) = \theta^*\). Since here we have \(\theta^* \leq Y(\delta_L), \gamma^* = \alpha^* = 0\) satisfies the conditions in Lemma 13, so this is an equilibrium. Now, consider the case with

\[
\psi_{BB}(0, Y(\delta_L)) > Y(\delta_L)
\]

Since \(\psi_{BB}(0, \theta^*)\) is decreasing in \(\theta^*\), there is no pure strategy equilibrium in this case. Slightly abusing notation, let \(\gamma^*(\theta^*)\) denote the mixing probability that satisfies the conditions in Lemma 13 for a given \(\theta^*\). That is, it must satisfy:

\[
\gamma^*(\theta^*) = \frac{1}{1 - \delta_L} \left(\frac{\delta_S (1 + \delta_L)}{2\delta_L + \delta_S} - \frac{\delta_S \delta_L}{F(\theta^*) (2\delta_L + \delta_S)}\right)
\]

\[
= \frac{\delta_S (1 - \delta_S - \delta_L) F(\theta^*) + \delta_S (2\delta_L + \delta_S) F(\theta^*) - \delta_S \delta_L}{(1 - \delta_L) (2\delta_L + \delta_S) F(\theta^*)}
\]

\[
= \frac{\delta_S (1 + \delta_L) F(\theta^*) - \delta_L}{1 - \delta_L (2\delta_L + \delta_S) F(\theta^*)}
\]

Note that \(\gamma^*(\theta^*)\) is strictly increasing in \(\theta^*\). At \(\theta^* = Y(\delta)\), we have \(\gamma^*(\theta^*) = 0\). At \(\theta^* = \varpi\), we have \(\gamma^*(\theta^*) = \frac{1}{1 - \delta_L} \frac{\delta_S}{2\delta_L + \delta_S} < 1\), with the final inequality holding due to the
same logic as in the proof of Lemma 13. At \( \bar{\tau} \), we have \( p(1) = p(2) = \bar{\psi}(\theta^*) \). This implies:

\[
\psi_{BB}(\gamma^*(\bar{\tau}), \bar{\tau}) = \beta - \omega(1 - \delta_L - \delta_S)(\bar{\tau} - \bar{\psi}(\theta^*)) < \beta < \bar{\tau}
\]

Therefore, there exists a \( \theta^* \in (Y(\delta_L), \bar{\tau}) \) such that \( \psi_{BB}(\gamma^*(\theta^*), \theta^*) = \theta^* \). This implies that we have existence of an equilibrium in which \( \gamma^* = \alpha^* \). Uniqueness of equilibrium follows. Suppose that, given a fixed \( \gamma^* = \alpha^* \), there exist \( \theta_1^* \neq \theta_2^* \) where each cutoff is an equilibrium. Since these are equilibria, \( \gamma^* \), \( \theta_1^* \), and \( \theta_2^* \) must satisfy:

\[
\gamma^* = \frac{1}{1 - \delta_L} \left( \frac{\delta_S(1 + \delta_L)}{2\delta_L + \delta_S} - \frac{\delta_S\delta_L}{F(\theta_1^*)(2\delta_L + \delta_S)} \right) = \frac{1}{1 - \delta_L} \left( \frac{\delta_S(1 + \delta_L)}{2\delta_L + \delta_S} - \frac{\delta_S\delta_L}{F(\theta_2^*)(2\delta_L + \delta_S)} \right).
\]

This is immediately a contradiction since \( \theta_1^* \neq \theta_2^* \). Combined with the uniqueness when we have \( \psi_{BB}(0, Y(\delta_L)) \leq Y(\delta_L) \), this implies there is a unique equilibrium. We now argue that the cutoff is invariant when we consider equilibria in which \( \gamma^* \neq \alpha^* \). For a given set of parameters, consider the equilibrium \( \theta^* \) associated with \( \gamma^* = \alpha^* \). First, if \( \theta^* \leq Y(\delta) \), \( \gamma^* = \alpha^* = 0 \) is the unique equilibrium. If \( \theta^* > Y(\delta_L) \), consider another \((\gamma^{**}, \alpha^{**}) \in [0,1] \times [0,1] \) with \( \gamma^{**} \neq \alpha^{**} \) such that:

\[
\gamma^{**}\delta_S + \alpha^{**}(1 - \delta_S - \delta_L) = \frac{\delta_S(1 + \delta_L)}{2\delta_L + \delta_S} - \frac{\delta_S\delta_L}{F(\theta^*)(2\delta_L + \delta_S)}
\]

Given these mixing probabilities, \( \theta^* \) continues to be the equilibrium cutoff if and only if it satisfies:

\[
\beta + \omega \left((1 - \delta_L - \delta_S)[\alpha^{**}p(1) + (1 - \alpha^{**})p(2) - \bar{\tau}] + \delta_S[\gamma^{**}p(1) + (1 - \gamma^{**})p(2) - p(1)]\right) = \theta^*
\]

where prices are evaluated at \( \gamma^{**}, \alpha^{**}, \) and \( \theta^* \). First note that since \( \gamma^{**}\delta_S + \alpha^{**}(1 - \delta_S - \delta_L) = \gamma\delta_S + \alpha(1 - \delta_S - \delta_L) \), the prices do not change. Furthermore, we can rewrite the LHS above as:

\[
\beta + \omega \left(((1 - \delta_L - \delta_S)[\alpha^{**} + \delta_S\gamma^{**}](p(1) - p(2)) + (1 - \delta_L - \delta_S)(p(2) - \bar{\tau}) + \delta_S(p(2) - p(1)) \right).
\]

The first term in the parentheses is the same as when evaluated at \( \alpha^* = \gamma^* \). Therefore the LHS at \( \theta^* \) is the same value as at \( \alpha^* = \gamma^* \), implying that it is still the equilibrium cutoff rule.

**Proof of Lemma 15.** To show this, we first consider as \( \delta_S \to 0 \). We will argue convergence by showing that the equilibrium cutoff \( \theta^*(\delta_S) \) is continuous in \( \delta_S \) at \( \delta_S = 0 \).
For a given $\delta_S$, the equilibrium cutoff is defined by the $\theta^*$ satisfying:

$$
\beta + \omega ((1 - \delta_L - \delta_S)[\alpha^* p(1) + (1 - \alpha^*) p(2) - \bar{\pi}] + \delta_S[\gamma^* p(1) + (1 - \gamma^*) p(2) - p(1)]) - \theta^* = 0
$$

(33)

Consider first a parameterization (besides $\delta_S$) such that there exists $\varepsilon > 0$ such that for all $0 < \delta_S < \varepsilon$, $\theta^*(\delta_S) \leq \frac{\delta_S}{1+\delta_L}$. In this case, for $\delta_S$ small we have $\gamma^* = \alpha^* = 0$. In this case, equation (33) simplifies to:

$$
\beta - \omega (1 - \delta_L) \frac{F(\theta^*)}{\delta_S + (1 - \delta_L) F(\theta^*)} \mathbb{E} [\theta | \theta < \theta^*] - \theta^* = 0
$$

Note that this is independent of $\delta_S$, so $\theta^*(\delta_S) \rightarrow \theta^*(0)$, where $\theta^*(0)$ is the equilibrium in the large shocks only case. Strategies (for those players who still exist in the limit) and prices (for those on the equilibrium path of play) then trivially converge. Now, consider a case where there exists $\varepsilon > 0$ such that for all $0 < \delta_S < \varepsilon$, $\theta^*(\delta_S) > \frac{\delta_S}{1+\delta_L}$. In this case, we have mixed strategies in every equilibrium as $\delta_S$ converges. We consider the equilibria in which $\gamma^* = \alpha^*$ since the cutoff is invariant to changes in the mixing probabilities. In such a case, equation (33) becomes:

$$
\beta - \omega \left( (1 - \delta_L - \delta_S) \left( \frac{\gamma^*(1 - \delta_L) \gamma^* F(\theta^*)}{\delta_S (1 - F(\theta^*)) + (1 - \delta_L) \gamma^* F(\theta^*)} + \frac{(1 - \gamma^*) (\delta_L + (1 - \delta_L)(1 - \gamma^*) F(\theta^*))}{\delta_S (1 - F(\theta^*)) + (1 - \delta_L)(1 - \gamma^*) F(\theta^*)} \right) \right) + \mathbb{E} [\theta | \theta < \theta^*] - \theta^* = 0
$$

Now, note that $\gamma^*$ is continuous and differentiable in $\delta_S$ at $\delta_S = 0$. Therefore, by the implicit function theorem, $\theta^*(\delta_S)$ is continuous and differentiable at $\delta_S = 0$, so $\theta^*(\delta_S) \rightarrow \theta^*(0)$. Furthermore, since $\gamma^* \rightarrow 0$ as $\delta_S \rightarrow 0$, $\theta^*(0)$ is defined by the $\theta^*$ that satisfies:

$$
\beta - \omega (1 - \delta_L) \frac{F(\theta^*)}{\delta_S + (1 - \delta_L) F(\theta^*)} \mathbb{E} [\theta | \theta < \theta^*] - \theta^* = 0
$$

This again yields the same equilibrium cutoff as in the benchmark large shocks only case. Furthermore, strategies and prices also converge. To complete the proof for $\delta_S \rightarrow 0$, note that there cannot be a case in which, as $\delta_S \rightarrow 0$, $\theta^*$ jumps between being smaller than $\frac{\delta_S}{1+\delta_L}$ and larger than $\frac{\delta_S}{1+\delta_L}$. This is because, if there exists any $\delta_S > 0$ such that $\theta^*(\delta_S) \leq \frac{\delta_S}{1+\delta_L}$, $\theta^*(\delta_S)$ for all $\delta_S \leq \delta_S$. Now, to show convergence as $\delta_L \rightarrow 0$,
first note that for any fixed parameters besides $\delta_L$, we have $Y(\delta) \to 0$ as $\delta_L \to 0$. We show this implies that for any parameters, there exists $\varepsilon > 0$ such that, for all $0 < \delta_L < \varepsilon$, $\theta^*(\delta_L) > \frac{\delta_L}{1+\delta_L}$. Consider that for any $\delta_L > 0$, the equilibrium cutoff is defined by:

$$\theta^* = \beta + \omega \left( (1 - \delta_L - \delta_S)\alpha^*p(1) + (1 - \alpha^*)p(2) - \overline{v} \right) + \delta_S [\gamma^*p(1) + (1 - \gamma^*)p(2) - p(1)]$$

At $\theta^* = 0$ we have $LHS = 0$, and $RHS = \beta$ since $p(1) = p(2) = \overline{v}$ at $\theta^* = 0$. Since the RHS and the LHS are both continuous in $\theta^*$, we must have, for $\delta_L$ small enough, the $\theta^*$ satisfying the previous equation larger than $\frac{\delta_L}{1+\delta_L}$. Therefore, we consider the case as $\delta_L$ is small enough such that the equilibrium cutoff satisfies $\theta^*(\delta_L) \geq \frac{\delta_L}{1+\delta_L}$. To show convergence of the cutoff we employ the same strategy as in the previous part of the proof. Again, the cutoff $\theta^*$ must satisfy:

$$\begin{align*}
\beta - \omega \left( (1 - \delta_L - \delta_S) \left( \frac{\gamma^*(1 - \delta_L)\gamma^*F(\theta^*)}{\delta_S(1 - F(\theta^*)) + (1 - \delta_L)\gamma^*F(\theta^*)} + \frac{(1 - \gamma^*)(\delta_L + (1 - \delta_L)(1 - \gamma^*))F(\theta^*)}{\delta_L + (1 - \delta_L)(1 - \gamma^*)F(\theta^*)} \right) \mathbb{E}[\theta|\theta < \theta^*] + \\
\delta_S \left( \frac{(1 - \gamma^*)(\delta_L + (1 - \delta_L)(1 - \gamma^*))F(\theta^*)}{\delta_L + (1 - \delta_L)(1 - \gamma^*)F(\theta^*)} - \frac{(1 - \delta_L)\gamma^*F(\theta^*)}{\delta_S(1 - F(\theta^*)) + (1 - \delta_L)\gamma^*F(\theta^*)} \right) \mathbb{E}[\theta|\theta < \theta^*] \right) - \theta^* = 0
\end{align*}$$

The LHS is continuous and differentiable in $\delta_L$ at $\delta_L = 0$, since again $\gamma^*$ is continuous and differentiable at $\delta_L = 0$. Therefore, by the implicit function theorem, $\theta^*(\delta_L)$ is continuous and differentiable at $\delta_L = 0$, so $\theta^*(\delta_L) \to \theta^*(0)$. Since in this case $\gamma^* \to 1$, we have $\theta^*(0)$ defined by the $\theta^*$ satisfying:

$$\begin{align*}
\beta - \omega \frac{F(\theta^*)}{\delta_S(1 - F(\theta^*)) + F(\theta^*)} \mathbb{E}[\theta|\theta < \theta^*] - \theta^* = 0
\end{align*}$$

This is the same cutoff as in the small shocks only equilibrium. Therefore, the cutoff converges. Furthermore, prices and strategies also converge to the small shocks only equilibrium since $\gamma^* \to 1$. Thus, we have shown convergence.

\[ \square \]

**B.4 Proofs for Multiple Firm Case**

**Proof of Lemma 16.** Suppose instead that there exists an equilibrium in which $2p(1, 1) < p(1, 0) + \overline{v}(\theta^*)$. Then, no investor who observes $\chi < 2$ will select $s = (1, 1)$. Further-
more, note that an investor observing $\chi = 1$ and $a = (0, 0)$ will choose $s = (1, 0)$ or $s = (0, 1)$, each w.p. $\frac{1}{2}$. This implies that $p(1, 0) > \varrho(\theta^*)$. Other investor types have the following strategies. An investor observing $a = (1, 0)$ will choose $s = (1, 0)$, while an investor observing $a = (0, 1)$ will choose $s = (0, 1)$. An investor observing $s = (1, 1)$ will choose $s = (1, 0)$ or $s = (0, 1)$, each w.p. $\frac{1}{2}$. Given these equilibrium strategies, prices in must be:

\[ p(1, 1) = \pi - F \mathbb{E}[\theta | \theta < \theta^*] \]
\[ p(1, 0) = \pi - \left( \frac{(1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))}{\delta_S(1 - F)^2 + (1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))} \right) \mathbb{E}[\theta | \theta < \theta^*] \]

where $F \equiv F(\theta^*)$. This equilibrium holds if and only if such prices satisfy $2p(1, 1) < p(1, 0) + \varrho(\theta^*)$:

\[ 2p(1, 1) < p(1, 0) + \varrho(\theta^*) \iff \]
\[ 2\pi - 2F \mathbb{E}[\theta | \theta < \theta^*] < 2\pi - \left( 1 + \frac{(1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))}{\delta_S(1 - F)^2 + (1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))} \right) \mathbb{E}[\theta | \theta < \theta^*] \iff \]
\[ \frac{\delta_S(1 - F)^2 + 2(1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))}{\delta_S(1 - F)^2 + (1 - \delta_L)(\frac{1}{2}F^2 + F(1 - F))} < 2F \iff \]
\[ 0 < -F^3(1 - \delta_L - \delta_S) + F^2(2(1 - \delta_L - \delta_S) + (1 - \delta_L) - \frac{1}{2}\delta_S) - F(2(1 - \delta_L - \delta_S)) - \frac{1}{2} \delta_S \]

Denote the right-hand side of the above inequality as $RHS(F)$. Notice that $RHS(0) = -\frac{1}{2}\delta_S < 0$ and $RHS(1) = 0$. Also note that:

\[ \frac{dRHS}{dF}(0) = -2(1 - \delta_L - \delta_S) < 0 \]

and

\[ \frac{dRHS}{dF}(1) = (1 - \delta_L) > 0 \]

Since a cubic can have at most two local extrema, we therefore cannot have $RHS(F) > 0$ for any $F \in [0, 1]$, since this would require three local extrema. Therefore, this inequality is never satisfied so there does not exist an equilibrium with $2p(1, 1) < p(1, 0) + \varrho(\theta^*)$. The second part of the lemma follows trivially since we cannot have $2p(1, 1) = p(1, 0) + \varrho(\theta^*)$ for any $F(\theta^*) \in (0, 1)$ under the prices given above, which are the equilibrium prices when no investor who observes $\chi < 2$ chooses double exit.

\[ \square \]

**Proof of Proposition 10.** We start by proving existence and uniqueness of the pure strategy equilibrium with $\xi^* = \gamma^* = 1$. In such an equilibrium, we require $2p(1, 1) \geq
Given the equilibrium prices derived from Bayes’ rule, and setting \( p(1, 0) = \Xi(\theta^*) \) this inequality holds (with \( \delta \equiv \delta_L + \delta_S \) iff:

\[
2\overline{v} - \frac{2F}{\delta_L + \delta_S + (1 - \delta_L - \delta_S)(1 - (1 - F)^2)} \mathbb{E}[\theta_i|\theta_i < \theta^*] \geq 2\overline{v} - \mathbb{E}[\theta_i|\theta_i < \theta^*] \iff 1 \geq \frac{2F}{\delta_L + \delta_S + (1 - \delta_L - \delta_S)(1 - (1 - F)^2)} \iff \\
\delta_L + \delta_S + (1 - \delta_L - \delta_S)(1 - (1 - F)^2) \geq 2F \iff -(1 - \delta)F^2 - \delta F + \delta \geq 0
\]

The LHS has one positive root and one negative root. The positive root is:

\[
-\frac{2\delta - \sqrt{4\delta}}{2(1 - \delta)} = \frac{\sqrt{\delta}}{1 + \sqrt{\delta}}
\]

Since it is a concave parabola, this inequality holds for \( F(\theta^*) \leq \frac{\sqrt{\delta}}{1 + \sqrt{\delta}} \). Notice also that the intuitive criterion here holds, since an investor observing \( \chi = 1 \) and \( a = (0, 0) \) has the same incentives to deviate as \( a = (1, 0) \). We move now to the manager’s incentives. Given the manager \( j \) has a cutoff \( \theta^* \), if manager \( i \) shirks upon observing \( \theta_i \), he receives:

\[
\overline{v} - \theta_i + \beta + \omega [\delta_L p(1, 1) + \delta_S p(1, 1) + (1 - \delta_L - \delta_S)p(1, 1)]
\]

If he does not shirk he receives:

\[
\overline{v} + \omega [\delta_L p(1, 1) + \delta_S p(1, 1) + (1 - \delta_L - \delta_S)(F(\theta^*)p(1, 1) + (1 - F(\theta^*))\overline{v})]
\]

Therefore, rearranging we get that the manager will shirk if and only if:

\[
\psi_{M,1}(\theta^*, 1, 1) \equiv \beta + \omega [(1 - \delta_L - \delta_S)(1 - F(\theta^*))p(1, 1) - \overline{v}] \geq \theta_i
\]

Note that after plugging in for \( p(1, 1) \) and \( \gamma^* = \xi^* = 1 \), we have \( \psi_{M,1}(\theta^*, 1, 1) = \psi_L(\theta^*, 1, 0) \) from the large shocks only case (equation (19)), except with \( \delta \equiv \delta_S + \delta_L \). As shown previously, \( \psi_L(\theta, 1, 0) \) is decreasing in \( \theta \) for \( \theta \leq Y(\delta) \equiv F^{-1}\left(\frac{\sqrt{\delta}}{1 + \sqrt{\delta}}\right) \). Therefore, the restriction for existence and uniqueness of cutoff is

\[
\psi_{M,1}(Y(\delta), 1, 1) \leq Y(\delta)
\]

The intermediate value theorem implies this is a necessary condition for this type of equilibrium to exist. Furthermore, the fact that \( \psi_{M,1}(\theta, 1, 1) \) is decreasing in \( \theta \leq Y(\delta) \)
implies that it is a sufficient condition, and that the associated cutoff rule $\theta^*$ is unique. Lastly, $\gamma^* = \xi^*$ trivially satisfies equation (29). We now move to analysis of the equilibria in which $\xi^* > 0$ and $\gamma^* < 1$. The prices in the statement of the Proposition are again simply derived from Bayes’ rule, and the equation for the manager’s cutoff rule is derived as usual. In any equilibrium of this form, prices must satisfy $2p(1, 1) = p(1, 0) + \bar{v}$ to ensure that blockholders who mix are indifferent between selling both and selling only one. Therefore, given a manager’s cutoff rule $\theta^*$ and letting $F = F(\theta^*)$, $\gamma^*$ and $\xi^*$ must satisfy:

$$(1 - \delta_L)F(1 - F)(1 - \gamma^*) \quad \frac{2\delta_L F + 2(1 - \delta_L)(F^2 + F(1 - F)\gamma^*)}{(1 - \delta_L)F(1 - F)(1 - \gamma^*) + \frac{1}{2}\delta_S(1 - F)^2(1 - \xi^*)} = \frac{2\delta_L F + 2(1 - \delta_L)(F^2 + F(1 - F)\gamma^*)}{\delta_L + (1 - \delta_L)(F^2 + 2F(1 - F)\gamma^*) + \delta_S(1 - F)^2\xi^*}$$

This holds if and only if:

$$\xi^* = 1 - \frac{(1 - \delta_L)\delta_S(1 - F)^2 - (2\delta_L F - \delta_L + (1 - \delta_L)F^2)(1 - \delta_L)}{\delta_S(1 - F)}(1 - \gamma^*)$$

This is simply equation (29) in the Proposition. First, note that if $\gamma^* = 1$, then we must have $\xi^* = 1$, which is the case we first considered. Furthermore, there exist $(\gamma^*, \xi^*) \in [0, 1) \times [0, 1]$ that satisfy the previous equation if and only if:

$$\frac{(1 - \delta_L)\delta_S(1 - F)^2 - (2\delta_L F - \delta_L + (1 - \delta_L)F^2)(1 - \delta_L)}{\delta_S(1 - F)} = \frac{-(1 - \delta_L)(1 - \delta_L - \delta_S)F^2 - 2(1 - \delta_L)(\delta_L + \delta_S)F + (1 - \delta_L)(\delta_L + \delta_S)}{\delta_S(1 - F)} \geq 0$$

The numerator has one negative root and one positive root, with the positive root given by:

$$F(Y(\delta)) = \frac{\sqrt{\delta_L + \delta_S}}{1 + \sqrt{\delta_L + \delta_S}}$$

Therefore, since the numerator is a concave parabola in $F$, and since the denominator is positive for all $F < 1$, for all $F \leq F(Y(\delta))$ we have the coefficient is larger than zero. For all $F > F(Y(\delta))$, we cannot have such an equilibrium. Thus, $F \leq F(Y(\delta))$ is a necessary condition for this equilibrium type. Furthermore, at $Y(\delta)$, equation (29) implies that $\xi^* = 1$ and any $\gamma^*$ will satisfy $2p(1, 1) = p(1, 0) + \bar{v}$. The manager’s cutoff
rule is defined by:

$$
\psi_{M,1}(\theta^*, \gamma^*, \xi^*) \equiv \beta + \omega((1 - \delta_L - \delta_S)[F(p(1, 1) - \gamma^*p(1, 1) - (1 - \gamma^*)p(0, 1)] + (1 - F)(\gamma^*p(1, 1) + (1 - \gamma^*)p(1, 0) - p(0, 0)) + \delta_S[F(p(1, 1) - \gamma^*p(1, 1) - (1 - \gamma^*)p(0, 1)] + (1 - F)(\gamma^*p(1, 1) + (1 - \gamma^*)p(1, 0) - \xi^*p(1, 1) - (1 - \xi^*)\frac{1}{2}(p(1, 0) + p(0, 1)))) = \theta^*_M
$$

At $Y(\delta)$, $\psi_{M,1}(Y(\delta), \gamma^*, 1)$ is increasing in $\gamma^*$. To see this, first note that both $p(1, 1)$ and $p(1, 0)$ are invariant to $\gamma^*$ at $\xi^* = 1$ and $Y(\delta)$, since $2p(1, 1) = p(1, 0) + \overline{v} = v(\theta^*) + \overline{v}$ for any $\gamma^*$ at those values. Then, we have:

$$
\frac{\partial \psi_{M,1}(Y(\delta), \gamma^*, 1)}{\partial \gamma^*} = \omega(1 - \delta_L)\frac{1}{2}(\overline{v} - v(\theta^*)) > 0.
$$

Therefore, if $\psi_{M,1}(Y(\delta), 1, 1) \geq Y(\delta) \geq \psi_{M,1}(Y(\delta), 0, 1)$, then by the intermediate value theorem, there exists a unique $\gamma^*$ such that $\psi_{M,1}(Y(\delta), \gamma^*, 1) = Y(\delta)$. That is, we have an equilibrium with cutoff rule $Y(\delta)$ and a unique $\gamma^* \in [0, 1]$ with $\xi^* = 1$. That $\psi_{M,1}(Y(\delta), \gamma^*, 1)$ is increasing in $\gamma^*$ also implies that this condition is sufficient. Now, consider the case where $Y(\delta) > \psi_{M,1}(Y(\delta), 1, 1)$. To argue that there is no equilibrium with $\xi^* > 0$, we first argue that for any fixed $\theta \leq Y(\delta)$, $\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*))$ is increasing in $\gamma^*$, where $\xi^*(\theta, \gamma^*)$ is the $\xi^*$ that satisfies equation (29) given $\theta$ and $\gamma^*$. To see this, note that since

$$
1 - \xi^*(\theta, \gamma^*) = A(\theta)(1 - \gamma^*)
$$

with $A(\theta) \equiv \frac{(1 - \delta_L)\delta_S(1 - F)^2 - (2\delta_L - \delta_S)(1 - \delta_L)F(1 - F)}{\delta_S(1 - F)}$, we have for any $\theta$:

$$
p(1, 0) = \overline{v} - \left(\frac{(1 - \delta_L)F(1 - F)(1 - \gamma^*)}{(1 - \delta_L)F(1 - F)(1 - \gamma^*) + \frac{1}{2}\delta_S(1 - F)^2(1 - \xi^*)}\right) \mathbb{E}[\theta_i | \theta_i < \theta] = \overline{v} - \left(\frac{(1 - \delta_L)F(1 - F)}{(1 - \delta_L)F(1 - F) + \frac{1}{2}\delta_S(1 - F)^2A(\theta)}\right) \mathbb{E}[\theta_i | \theta_i < \theta].
$$

That is, fixing $\theta$, we have $p(1, 0)$ invariant to changes in $\gamma^*$ allowing $\xi^*$ to also change. Furthermore, since for these values of $\gamma^*$ and $\xi^*(\theta, \gamma^*)$ we have $2p(1, 1) = p(1, 0) + \overline{v}$, we necessarily have $p(1, 1)$ also invariant to changes in $\gamma^*$. Then, we have:

$$
\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*)) \equiv \beta + \omega((1 - \delta_L - \delta_S)[F(1 - \gamma^*) \frac{1}{2}(p(1, 0) - \overline{v}) + (1 - F)(1 - \frac{1}{2}\gamma^*)p(1, 0) - \overline{v}) + \delta_S(1 - \gamma^*) \frac{1}{2}(p(1, 0) - \overline{v})]
$$

Clearly then, since $p(1, 0) < \overline{v}$ when $\xi^* < 1$, we must have $\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*))$ increasing.
ing in $\gamma^\ast$ given $\theta^\ast$. Furthermore, we show that, for any fixed $\gamma^\ast$, $\psi_{M,1}(\theta, \gamma^\ast, \xi^\ast(\theta, \gamma^\ast))$ is decreasing in $\theta$. We have:

$$\psi_{M,1}(\theta, \gamma^\ast, \xi^\ast(\theta, \gamma^\ast)) \equiv \beta + \omega((1 - \delta_L - \delta_S)[F(p(1, 1) - \gamma^\ast p(1, 1) - (1 - \gamma^\ast)p(0, 1))] + (1 - F)(\gamma^\ast p(1, 1) + (1 - \gamma^\ast)p(1, 0) - p(0, 0)) + \delta_S[F(p(1, 1) - \gamma^\ast p(1, 1) - (1 - \gamma^\ast)p(0, 1)) + (1 - F)(\gamma^\ast p(1, 1) + (1 - \gamma^\ast)p(1, 0) - \xi^\ast p(1, 1) - (1 - \xi^\ast)\frac{1}{2}(p(1, 0) + p(0, 1)))]

= \beta - \omega(\pi - p(1, 0))((1 - \delta_L - \delta_S)(1 - \frac{1}{2}F - \frac{1}{2}\gamma^\ast) + \frac{1}{2}\delta_S(1 - \gamma^\ast))

= \beta - \omega \left(-\frac{1}{2}(1 - \delta_L - \delta_S)F^2 + (1 - \delta_L - \delta_S)F + \frac{1}{2}(\delta_L + \delta_S) \right) \mathbb{E}[\theta_i | \theta_i \leq \theta]

This is decreasing in $\theta$ for $\theta \leq Y(\delta)$ if the term in parentheses is increasing. Since the denominator of the derivative of that term with respect to $F$ is positive (i.e. it is $(-\frac{1}{2}(1 - \delta_L - \delta_S)F^2 + (1 - \delta_L - \delta_S)F + \frac{1}{2}(\delta_L + \delta_S))^2$), we can determine the sign of the derivative as the sign of its numerator, which is given by:

$$\left((1 - \delta_L - \delta_S)(1 - F) + \frac{1}{2}\delta_S - \frac{1}{2}\gamma^\ast(1 - \delta_L)\right) \left(-\frac{1}{2}(1 - \delta_L - \delta_S)F^2 + (1 - \delta_L - \delta_S)F + \frac{1}{2}(\delta_L + \delta_S) \right)$$

$$= \left((1 - \delta_L - \delta_S)(1 - F) + \frac{1}{2}\delta_S - \frac{1}{2}\gamma^\ast(1 - \delta_L)\right) \left(-\frac{1}{2}(1 - \delta_L - \delta_S)F^2 + (1 - \delta_L - \delta_S)F + \frac{1}{2}(\delta_L + \delta_S) \right)$$

This can be simplified to:

$$\frac{1}{4}(1 - \delta_L - \delta_S)(\delta_S - \gamma^\ast(1 - \delta_L))F^2 - \frac{1}{2}(1 - \delta_L - \delta_S)(\delta_L + \delta_S)F + \frac{1}{2}(1 - \delta_L - \delta_S)(\delta_L + \delta_S) + \frac{1}{2}(1 - \delta_L - \delta_S)(\delta_L + \delta_S)$$

Note that, for any $F$ this is decreasing in $\gamma^\ast$. We show that at $\gamma^\ast = 1$, the term in (34) is positive for all $F \leq F(Y(\delta))$. This will then imply that, for all $\gamma^\ast$, $\psi_{M,1}(\theta, \gamma^\ast, \xi^\ast(\theta, \gamma^\ast))$ is decreasing in $\theta$ for $\theta \leq Y(\delta)$. When $\gamma^\ast = 1$, the parabola in (34) becomes:

$$-\frac{1}{4}(1 - \delta_L - \delta_S)^2F^2 - \frac{1}{2}(1 - \delta_L - \delta_S)(\delta_L + \delta_S)F + \frac{1}{4}(1 - \delta_L - \delta_S)(\delta_L + \delta_S)$$

which is a concave parabola in $F$. This has one positive root and one negative root, with the positive root being $F(Y(\delta)) = \frac{\sqrt{\delta_S + \delta_L} - \delta_S}{1 + \sqrt{\delta_S + \delta_L}}$. Therefore, for all $F \in [0, F(Y(\delta))]$, (34) is positive. This proves that for all $\gamma^\ast$, $\psi_{M,1}(\theta, \gamma^\ast, \xi^\ast(\theta, \gamma^\ast))$ is decreasing in $\theta$ for $\theta \leq Y(\delta)$. Thus, we have concluded: (i) for any fixed $\theta \leq Y(\delta)$, $\psi_{M,1}(\theta, \gamma^\ast, \xi^\ast(\theta, \gamma^\ast))$ is
increasing in $\gamma^*$; and (ii) for any fixed $\gamma^*$, $\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*))$ is decreasing in $\theta \leq Y(\delta)$.

Finally, to see that $\psi_{M,1}(Y(\delta), 0, 1) > Y(\delta)$ implies that an equilibrium of this form does not exist, we have for any $\theta < Y(\delta)$ and $\gamma^* \in [0, 1]$:

$$
\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*)) \geq \psi_{M,1}(\theta, 0, \xi^*(\theta, 0)) \\
\geq \psi_{M,1}(Y(\delta), 0, \xi^*(Y(\delta), 0)) \\
= \psi_{M,1}(Y(\delta), 0, 1) \\
> Y(\delta) \\
\geq \theta.
$$

Therefore, we cannot have any $\theta \leq Y(\delta)$ and $\gamma^*$ such that $\psi_{M,1}(\theta, \gamma^*, \xi^*(\theta, \gamma^*)) = \theta$, and therefore no equilibrium of this type. Last, to show when we can guarantee existence of the equilibrium in which $\gamma^* = 0$ and $\xi^* > 0$, note that the coefficient on $(1 - \gamma^*)$ is less than one if and only if:

$$
-(1 - \delta_L)(1 - \delta_L - \delta_S)F^2 - 2(1 - \delta_L)(\delta_L + \delta_S)F + (1 - \delta_L)(\delta_L + \delta_S) \leq \delta_S(1 - F) \\\iff 0 \leq (1 - \delta_L)(1 - \delta_L - \delta_S)F^2 + 2(1 - \delta_L)(\delta_L + \delta_S) - \delta_SF - (1 - \delta_L)(\delta_L + \delta_S) + \delta_S
$$

which has one negative root and one positive root, which is $F(\theta^*_L)$ as defined in the Proposition. Therefore, this inequality holds only for $F \geq F(\theta^*_L)$. Thus, $\xi^* > 0$ when $\gamma^* = 0$ if and only if $\theta^* \in (\theta^*_L, Y(\delta)]$. Suppose that both conditions (30) and (31) hold. Then, by intermediate value theorem we have there exists a $\theta^* \in (\theta^*_L, Y(\delta)]$ and $\xi^* > 0$, with $\gamma^* = 0$ satisfying equation (29) such that $\psi_{M,3}(\theta^*, 0, \xi^*(\theta^*, 0)) = \theta^*$. This constitutes an equilibrium. Note also that since $\theta^* \in (\theta^*_L, Y(\delta)]$, $\xi^*(\theta^*, 0) > 0$. Since $\psi_{M,3}(\theta^*, 0, \xi^*(\theta^*, 0))$ is decreasing in $\theta^* < Y(\delta)$, we can also conclude that the cutoff is unique for the $\gamma^* = 0$ equilibrium if it exists, and that conditions (30) and (31) are sufficient for existence of a $\gamma^* = 0$ equilibrium.

**Proof of Lemma 17.** Suppose as is stated in the Lemma that an equilibrium with $\xi^*$ exists. First, if $\psi_{M,1}(Y(\delta), 1, 1) \leq Y(\delta)$, then trivially as is stated in Proposition 10 an equilibrium with $\gamma^* = \xi^* = 1$ exists. Obviously, since $\gamma^*$ cannot take a larger value, we have $\gamma^*_{\max} = 1$. If instead $\psi_{M,1}(Y(\delta), 1, 1) \geq Y(\delta) > \psi_{M,1}(Y(\delta), 0, 1)$, as stated in the Proposition there is a unique $\gamma^{**}$ satisfying $\psi_{M,1}(Y(\delta), \gamma^{**}, 1) = Y(\delta)$ such that an equilibrium exists. To see that this is the equilibrium with the largest possible $\gamma^*$, note that we show in the proof of Proposition 10 that: (i) for any fixed $\theta \leq Y(\delta)$, $\psi_{M,1}(\theta, \gamma, \xi^*(\theta, \gamma))$ is increasing in $\gamma$; and (ii) $\psi_{M,1}(\theta, \gamma, \xi^*(\theta, \gamma))$ is decreasing
in $\theta \leq Y(\delta)$. For any $\tilde{\gamma} > \gamma^{**}$ and $\tilde{\theta} < Y(\delta)$, we have:

$$Y(\delta) = \psi_{M,1}(Y(\delta), \gamma^{**}, 1) < \psi_{M,1}(Y(\delta), \tilde{\gamma}, 1) < \psi_{M,1}(\tilde{\theta}, \tilde{\gamma}, \xi^*(\tilde{\theta}, \tilde{\gamma}))$$

Therefore, $\gamma^{**}$ is the largest possible equilibrium mixing probability in such an equilibrium. This yields the $\gamma^{*}_{\max}$ described in the proposition. For $\gamma^{*}_{\min}$, we have from Proposition 10 that if $\psi_{M,1}(\theta_{L}^{**}, 0, 0) > \theta_{L}^{**}$, an equilibrium with $\gamma^* = 0$ and $\xi^* > 0$ exists. Therefore, in this case $\gamma^{*}_{\min} = 1$. Suppose instead that $\psi_{M,1}(\theta_{L}^{**}, 0, 0) \leq \theta_{L}^{**}$. Then recall that from equation (29) we have:

$$\xi^* = 1 - \frac{(1 - \delta_L)\delta_S(1 - F(\theta))^2 - (2\delta_L F(\theta) - \delta_L + (1 - \delta_L)F(\theta)^2)(1 - \delta_L)}{\delta_S(1 - F(\theta))}(1 - \gamma^*) > 0.$$ 

Let the coefficient on $(1 - \gamma^*)$ as a function of $\theta$ be denoted $A(\theta)$. We show in the proof of Proposition 10 that for $\theta \leq \theta_{L}^{**}$, we have $A(\theta) \geq 1$. Therefore, rearranging we have $\xi^* > 0$ if and only if $(1 - \gamma^*) < \frac{1}{A(\theta)}$, with $A'(\theta) < 0$. Thus, for a given $\theta^*$, the minimum possible mixing probability such that $\xi^*$ is well-defined is $\gamma^* = \frac{1}{A(\theta^*)}$, which is $\tilde{\gamma}(\theta^*)$ defined in equation (17). Furthermore, $\xi^*(\theta^*, \tilde{\gamma}(\theta^*)) = 0$. Then, the lowest possible equilibrium $\gamma^*$ is the $\tilde{\gamma}(\theta^{**})$ satisfying:

$$\psi_{M,1}(\theta^{**}, \tilde{\gamma}(\theta^{**}), 0) = \theta^{**}$$

To see that this is the smallest equilibrium mixing probability, first note that for any $\theta < \theta^{**}$, $\tilde{\gamma}(\theta^{**}) < \tilde{\gamma}(\theta)$. For any $\theta > \theta^{**}$, we prove by contradiction. Suppose that there exists a $\tilde{\gamma} < \tilde{\gamma}(\theta^{**})$ such that it is an equilibrium mixing probability with $\theta$ as the equilibrium cutoff, i.e.

$$\psi_{M,1}(\theta, \tilde{\gamma}, \xi^*(\theta, \tilde{\gamma})) = \theta.$$

We then have:

$$\theta = \psi_{M,1}(\theta, \tilde{\gamma}, \xi^*(\theta, \tilde{\gamma})) < \psi_{M,1}(\theta, \tilde{\gamma}(\theta^{**}), \xi^*(\theta, \tilde{\gamma}(\theta^{**}))) < \psi_{M,1}(\theta^{**}, \tilde{\gamma}(\theta^{**}), \xi^*(\theta^{**}, \tilde{\gamma}(\theta^{**}))) = \theta^{**}$$

This gives us our desired contradiction. Therefore, in this case $\gamma^{*}_{\min}$ is as described in the Lemma. Uniqueness of the cutoff rule for each $\gamma^*$ trivially comes from the fact that $\psi_{M,1}(\theta, \gamma, \xi^*(\theta, \gamma))$ is decreasing in $\theta$, given $\gamma$. 

\[73\]
Proof of Lemma 18. In any equilibrium in which an investor who observes \(a = (1, 0)\) and \(\chi = 0\) chooses \(s = (0, 0)\) with positive probability must satisfy \(p(1, 0) = \underline{v}(\theta^*)\). We denote throughout the proof this probability to be \(\eta^*\). This is because the payoff of the blockholder from selling only one share is \(p(1, 0) + \overline{v}\), whereas the payoff from holding both is \(\underline{v}(\theta^*) + \overline{v}\). Therefore, unless \(p(1, 0) = \underline{v}(\theta^*)\), selling one share will strictly dominate. Furthermore, we must have \(2p(1, 1) \leq \overline{v} + \underline{v}(\theta^*)\), otherwise selling both shares will strictly dominate holding both. This also then implies that an investor who observes \(a = (0, 0)\) and \(\chi = 1\) must also choose \(s = (1, 1)\) w.p. one in such an equilibrium, that is \(\xi^* = 1\). If she plays \(a = (1, 0)\) with positive probability, then this contradicts \(p(1, 0) = \underline{v}(\theta^*)\). As such, this also requires \(2p(1, 1) \geq \underline{v}(\theta^*) + \overline{v}\), which together with the previous restriction implies \(2p(1, 1) = \underline{v}(\theta^*) + \overline{v}\). Let \(\gamma^*\) denote as before the probability that an \(a = (1, 0)\) and \(\chi = 0\) or \(1\) blockholder sells both shares. Then, in such an equilibrium we have as before:

\[
p(1, 1) = \overline{v} - \left(\frac{\delta_L F + (1 - \delta_L)(F^2 + F(1 - F)\gamma^*)}{\delta_L + (1 - \delta_L)(F^2 + 2F(1 - F)\gamma^*) + \delta_S(1 - F)^2}\right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

Then, for a given \(\theta^*\), price is consistent with \(2p(1, 1) = \overline{v} + \underline{v}(\theta^*)\) if and only if:

\[
2 \left(\frac{\delta_L F + (1 - \delta_L)(F^2 + F(1 - F)\gamma^*)}{\delta_L + \delta_S(1 - F)^2 + 2(1 - \delta_L)(F^2 + 1 - F\gamma^*) + (1 - \delta_L)F^2}\right) = 1 \iff 2\delta_L F + 2(1 - \delta_L)(F^2 + F(1 - F)\gamma^*) + 2(1 - \delta_L)F^2 = \delta_L + \delta_S(1 - F)^2 + 2(1 - \delta_L)(F^2 + F(1 - F)\gamma^*) + (1 - \delta_L)F^2 \iff F^2(1 - \delta_L - \delta_S) + 2F(\delta_L + \delta_S) - (\delta_L + \delta_S) = 0
\]

which has one root in between zero and one, given by \(F = \frac{\sqrt{\delta_L + \delta_S}}{1 + \sqrt{\delta_L + \delta_S}}\), which does not depend on \(\gamma^*\). Therefore, we can only have an equilibrium \(\eta^* > 0\) if \(\theta^* = Y(\delta)\). \(\Box\)

Proof of Proposition 11. The equilibrium prices in the statement of the proposition are determined by Bayes’ rule. Furthermore, the manager’s function \(\psi_{M,2}(\theta, \gamma^*, \alpha^*)\) is determined as usual. Consider first the existence of the pure strategy equilibrium in which \(\gamma^* = 0\) and \(\alpha^* = 1\) (part (b) of the Proposition). This equilibrium can exist only if

\[
p(1, 0) + \overline{v} \geq 2p(1, 1) \geq p(1, 0) + \underline{v}(\theta^*)
\]
Under the conjectured equilibrium strategies, prices would be:

\[
p(1, 1) = \overline{v} - \left( \frac{F^2 + \delta_L F(1 - F)}{\delta_L + (1 - \delta_L)F^2} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

\[
p(1, 0) = \overline{v} - \left( \frac{(1 - \delta_L)F}{\delta_S \frac{1}{2}(1 - F) + (1 - \delta_L)F} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

\[
p(0, 1) = \overline{v}
\]

These prices are consistent with the inequality \( p(1, 0) + \overline{v} \geq 2p(1, 1) \) iff, at the equilibrium \( F \equiv F(\theta^*) \):

\[
F^2(1 - \delta_L)(1 - \delta_L - \delta_S) + F(\delta_S(1 - \delta_L) + 2\delta_L(1 - \delta_L - \delta_L\delta_S) - \delta_L(1 - \delta_L - \delta_S) \geq 0
\]

There is one positive root and one negative root of the quadratic equation on the LHS of the inequality. Furthermore, the positive root is in between 0 and 1, since at \( F = 0 \) we have \( LHS = -\delta_L(1 - \delta_L - \delta_S) < 0 \), and at \( F = 1 \) we have \( LHS = 1 - \delta_L > 0 \). Thus, the positive root of the above quadratic, denoted by \( F(\theta^*_{U}) \), defines the cutoff such that, for all \( F \geq F(\theta^*_{U}) \), the inequality \( p(1, 0) + \overline{v} \geq 2p(1, 1) \) holds. We have:

\[
F(\theta^*_{U}) = \frac{-\delta_L(1 - \delta) - \delta(1 - \delta_L) + \sqrt{(\delta_L(1 - \delta) + \delta(1 - \delta_L))^2 + 4(1 - \delta_L)\delta_L(1 - \delta)^2}}{2(1 - \delta_L)(1 - \delta)}
\]

This gives the lower bound on the managers’ cutoff rule \( \theta^* \) such that this pure strategy equilibrium exists. The other necessary condition for this pure strategy equilibrium is \( p(1, 0) + \overline{v}(\theta^*) \leq 2p(1, 1) \). Given equilibrium prices, this condition simplifies to:

\[
F^3(1 - \delta_L)\delta_S - F^2(\delta_L(1 - \delta_L) + 3\delta_L(1 - \delta_L - \delta_S) + \delta_S) + F(\delta_L(1 - \delta_L) + 3\delta_L(1 - \delta_L - \delta_S) + \delta_L\delta_S \geq 0
\]

Let the left-hand side of the inequality as a function of \( F \) be denoted \( LHS(F) \). We have \( LHS(0) = \delta_L\delta_S > 0 \), and \( LHS(1) = 0 \). Furthermore, since \( LHS'(0) > 0 \), and since it is a third-order polynomial and can therefore have at most two local extrema, there is a unique \( F(\theta^*_{U}) \in (0, 1) \) such that the inequality holds for all \( F \in [0, F(\theta^*_{U})] \).

We can factor the cubic equation to obtain:

\[
(F - 1)((1 - \delta_L)\delta_S F^2 + (2\delta_S\delta_L - 4\delta_L(1 - \delta_L))F - \delta_L\delta_S) \geq 0
\]

The second term (call it \( LHS_2(F) \)) is a quadratic function of \( F \) and is a convex parabola with \( LHS_2(0) = -\delta_L\delta_S < 0 \), and therefore has one negative root and one positive root.
The positive root of $LHS_2(F)$, denoted by $F^*$ is given by:

$$F^* = 2\frac{\delta_L}{\delta_S} - \frac{\delta_L}{1 - \delta_L} + \frac{\sqrt{\delta_L(\delta_S^2 + 4\delta_L(1 - \delta_L)(1 - \delta_L - \delta_S))}}{(1 - \delta_L)\delta_S}$$

We now show that $F^* \geq 1$ (and therefore $LHS(F) \geq 0$ for all $F \in [0, 1]$) if and only if $\delta_L \geq \frac{1}{2}(1 - \sqrt{1 - \delta_S})$. To derive this, first note that $F^* \geq 1$ if and only if $LHS_2(1) \leq 0$. This is because $LHS_2(0) < 0$ and $LHS_2(F)$ is a convex parabola. This condition then holds if:

$$0 \geq LHS_2(1) = (1 - \delta_L)\delta_S + \delta_S\delta_L - 4\delta_L(1 - \delta_L) = \delta_S - 4\delta_L(1 - \delta_L) \iff 0 \geq 4\delta_S^2 - 4\delta_L + \delta_S$$

The roots of this quadratic in $\delta_L$ are:

$$\left\{ \frac{1}{2} \left( 1 \pm \sqrt{1 - \delta_S} \right) \right\}$$

Since we cannot have $\delta_L \geq \frac{1}{2}(1 + \sqrt{1 - \delta_S})$, this inequality holds if and only if $\delta_L \geq \frac{1}{2}(1 - \sqrt{1 - \delta_S})$. Otherwise, $F^* < 1$. We then can in general denote

$$F(\theta_{U^*}^*) = \min \{1, F^*\}$$

Note that trivially it must be that $F(\theta_{L^*}^*) < F(\theta_{U^*}^*)$. The expression for $\psi_{M,2}(\theta, \gamma^*, \alpha^*)$ under these prices and strategies simplifies to:

$$\psi_{M,2}(\theta^*, 0, 1) = \beta + \omega \left( (1 - \delta_L - \delta_S)(F(\theta^*)p(1, 1) + (1 - F(\theta^*))p(1, 0) - \overline{v}) + \delta_S(F(\theta^*)p(1, 1) - \overline{v}) + (1 - F(\theta^*))\frac{1}{2}(p(1, 0) - \overline{v}) \right)$$

An equilibrium of this form then exists if there is a $\theta^* \in [\theta_{L^*}^*, \theta_{U^*}^*]$ such that $\psi_{M,2}(\theta^*, 0, 1) = \theta^*$. Existence then follows from the supposition that $\psi_{M,2}(\theta_{L^*}^*, 0, 1) \geq \theta_{L^*}^*$ and $\psi_{M,2}(\theta_{U^*}^*, 0, 1) \leq \theta_{U^*}^*$. The intermediate value theorem implies there must exist a $\theta_{M,2}^* \in [\theta_{L^*}^*, \theta_{U^*}^*]$ such that $\theta_{M,2}^* = \psi_{M,2}(\theta_{M,2}^*, 0, 1)$. Next, consider the equilibrium of the form $\alpha^* = 1$, and $\gamma^* \in (0, 1)$ (part (a) of the Proposition). In this case, investors who observe $\chi < 2$ and $a = (1, 0)$ must be indifferent between selling both shares and selling only the share of the shirking manager. This implies that any equilibrium of this type must satisfy
In a mixed strategy equilibrium of this type, prices are:

\[
p(1, 0) = \overline{v} - \left( \frac{(1 - \delta_L) F(1 - F)(1 - \gamma^*)}{\delta_S (1 - F^2) + (1 - \delta_L) F(1 - F)(1 - \gamma^*)} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*] \]

\[
p(1, 1) = \overline{v} - \left( \frac{\delta_L F + (1 - \delta_L)(F(1 - F)\gamma^* + F^2)}{\delta_L + (1 - \delta_L)(2F(1 - F)\gamma^* + F^2)} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*] \]

Therefore, for agents to be willing to play a mixed strategy, \(\gamma^*\) must satisfy:

\[
\frac{(1 - \delta_L) F(1 - F)(1 - \gamma^*)}{\delta_S (1 - F^2) + (1 - \delta_L) F(1 - F)(1 - \gamma^*)} = \frac{2\delta_L F + 2(1 - \delta_L)(F(1 - F)\gamma^* + F^2)}{\delta_L + (1 - \delta_L)(2F(1 - F)\gamma^* + F^2)} \iff \\
\frac{(1 - \delta_L)(1 - \gamma^*)}{\delta_S (1 - F^2) + (1 - \delta_L) F(1 - F)(1 - \gamma^*)} = \frac{2\delta_L + 2(1 - \delta_L)((1 - F)\gamma^* + F)}{\delta_L + (1 - \delta_L)(2F(1 - F)\gamma^* + F^2)} \iff \\
\gamma^* = \frac{-(1 - \delta_L)(1 - \delta_L - \delta_S) F^2 - (2\delta_L(1 - \delta_L - \delta_S) + \delta_S) F + \delta_L (1 - \delta_L - \delta_S)}{-(1 - \delta_L)(1 - \delta_L - \delta_S) F^2 - 2(1 - \delta_L)(\delta_S + \delta_S)(1 - \delta_L)}
\]

At \(F = 0\), we have \(\gamma^* = \frac{\delta_S (1 - \delta_L - \delta_S)}{(\delta_L + \delta_S)(1 - \delta_L)} \in (0, 1)\). Note also that the numerator is the same quadratic as in the previous part of the proof with one root of \(F(\theta_L^*)\) (and the other root is negative). Therefore, for \(F < F(\theta_L^*)\) the numerator is positive. Intuitively, at \(F = F(\theta_L^*)\) we have \(\gamma^* = 0\), as this coincides with the pure strategy equilibrium above.

The denominator has one negative root and is larger than the numerator for all \(F < 1\). Therefore, for all \(F \in [0, F(\theta_L^*)]\), we will have \(\gamma^* \in [0, 1]\), and for all \(F > F(\theta_L^*)\), \(\gamma^* \notin [0, 1]\). That is, we have a mixing probability that is well-defined and consistent with equilibrium strategies if and only if the equilibrium cutoff \(\theta^* \leq \theta_L^*\). Denote the mapping of the equilibrium cutoff rule \(\theta^*\) to the mixing probability above as \(\gamma^*(\theta^*)\). A cutoff rule in the mixed strategy equilibrium above must satisfy:

\[
\psi_{M, 2}(\theta^*, \gamma^*(\theta^*), 1) \equiv \beta + \omega[(1 - \delta_L)(F(1 - \gamma^*(\theta^*))(p(1, 1) - \overline{v}) + (1 - F)(1 - \gamma^*(\theta^*))(p(1, 0) - p(1, 1))) + (1 - \delta_L - \delta_S)(p(1, 1) - \overline{v})] = \theta^*
\]

where prices are functions of \(\theta^*\), and where we use the fact that \(\gamma^*\) ensures \(p(1, 1) = \frac{1}{2}(p(1, 0) + \overline{v})\). At \(\theta^* = 0\), since all prices are \(\overline{v}\), we have \(\psi_{M, 2}(\theta^*, \gamma^*(\theta^*), 1) = \beta > 0\).

At \(\theta = \theta_L^*\) we have \(\gamma^* = 0\). Therefore, if \(\psi_{M, 2}(\theta_L^*, 0, 1) < \theta_L^*\), we have existence of a mixed strategy equilibrium by the intermediate value theorem. Last, consider the equilibrium in which \(\gamma^* = 0\) and \(\alpha^* \in (0, 1)\) (type (c) in the Proposition). In this case, since blockholders who observe \(a = (1, 1)\) are mixing between selling both shares and selling only one share, we must have \(2p(1, 1) = p(1, 0) + 2\theta(\theta^*)\), which also ensures that \(\xi^* = \gamma^* = 0\) is optimal for the other blockholders. Prices in this equilibrium simplify
At the equilibrium cutoff \( \theta^* \), \( \alpha^* \) must be such that:

\[
2p(1, 1) = p(1, 0) = v(\theta^*) \iff \frac{2 \delta_L F + 2(1 - \delta_L) F^2 \alpha^*}{\delta_L + (1 - \delta_L) F^2 \alpha^*} = \frac{\delta_S (1-F)^2}{\delta_S + (1 - \delta_L) (F(1-F) + \frac{1}{2}(1-\alpha^*)^2)}
\]

We first show that, for \( F \in [0, 1] \), \( \alpha^* \in [0, 1] \) if and only if \( \theta^* \in [\theta_U^*, \pi] \). First, note that the denominator is positive for all \( F \in [0, 1] \). Second, we can rewrite the numerator as

\[
-(1 - \delta_L)(1 - \delta_L - \delta_S) F^2 + (2\delta_L(1 - \delta_L - \delta_S) + \frac{1}{2}\delta L \delta S) F + \frac{1}{2}\delta L \delta S
\]

This is positive at \( F = 0 \) and at \( F = 1 \). Since it is a concave parabola it must be positive for all \( F \in [0, 1] \). Therefore, \( \alpha^* > 0 \) for all \( F \in [0, 1] \). Now, to show that
\( \alpha^* \leq 1 \) if and only if \( F \in [F(\theta^*_U^*), 1] \), we argue as follows. First, \( \alpha^* \leq 1 \) if and only if:

\[
\frac{1}{2} \delta_L \delta_S (1 - F)(1 - 2F) + \delta_L (1 - \delta_L) F (2 - F) \leq \frac{1}{2} (1 - \delta_L) \delta_S F^2 (1 - F) + \delta_L (1 - \delta_L) F^2 \iff \\
-(1 - \delta_L) \delta_S F^3 + ((1 - \delta_L) \delta_S + 4 \delta_L (1 - \delta_L) - 2 \delta_L \delta_S) F^2 - (4 \delta_L (1 - \delta_L) - 3 \delta_L \delta_S) F - \delta_L \delta_S \geq 0 \iff \\
-(1 - \delta_L) \delta_S F^3 + (3 \delta_L (1 - \delta_L - \delta_S) + \delta_L (1 - \delta_L) + \delta_S) F^2 \\
-3 \delta_L (1 - \delta_L - \delta_S) + \delta_L (1 - \delta_L)) F - \delta_L \delta_S \geq 0
\]

Notice that the LHS is the negative of the cubic equation from which we derived \( \theta^*_U^* \).

Therefore, it has the same roots as that equation. Recall that one root is \( F = 1 \), and a second is

\[
F^* \equiv 2 \frac{\delta_L}{\delta_S} - \frac{\delta_L}{1 - \delta_L} + \frac{\sqrt{\delta_L (\delta_S^2 + 4 \delta_L (1 - \delta_L) (1 - \delta_L - \delta_S))}}{(1 - \delta_L) \delta_S}
\]

which is \( F(\theta^*_U^*) \) when \( F(\theta^*_U^*) \leq 1 \). The third root must be less than 0, since at \( F = 0 \) the LHS < 0, but as \( F \to -\infty \), LHS \to \infty \). Denote this third root as \( F^*_N \). Since this is a cubic equation with a negative leading coefficient, we must have LHS \leq 0

for all \( F \in [F^*_N, \min\{F^*, 1\}] \). Therefore, if \( F^* \geq 1 \), there is no mixing equilibrium of this type since the above inequality is violated for all possible \( \theta^* \). If \( F^* < 1 \), then we potentially have a mixed strategy equilibrium of this form. Let \( \alpha^*(\theta^*) \) denote the \( \alpha^* \) that satisfies the above equation as a function of \( \theta^* \). Note that in this case, \( F^* = F(\theta^*_U^*) \) and \( \alpha^*(\theta^*_U^*) = \alpha^*(\bar{\nu}) = 1 \). Suppose \( \psi_{M,2}(\theta^*_U^*, 0, 1) > \theta^*_U^* \). As \( \theta^* \to \bar{\nu} \), we have \( \alpha^* \to 1 \) and \( F \to 1 \). Furthermore:

\[
\lim_{\theta^* \to \bar{\nu}} \psi_{M,2}(\theta^*, 0, \alpha^*(\theta^*)) = \beta + \omega \left[ (1 - \delta_L) \lim_{\theta^* \to \bar{\nu}} (p(1, 1) - p(0, 1)) \right]
\]

We also have:

\[
p(0, 1) = \bar{\nu} - \left( \frac{1}{2} \frac{(1 - \delta_L)(1 - \alpha^*) F^2}{\delta_S (1 - F)^2 + (1 - \delta_L) (F (1 - F) + \frac{1}{2} (1 - \alpha^*) F^2)} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

\[
\to \bar{\nu} - \left( \frac{\partial \alpha^*(1)}{\partial F} \right) \frac{\partial F}{2 + \frac{\partial \alpha^*(1)}{\partial F}} \mathbb{E}[\theta] \geq \psi(\theta^*)
\]

\[
p(1, 1) = \bar{\nu} - \left( \frac{\delta_L F + (1 - \delta_L) F^2 \alpha^*}{\delta_L + (1 - \delta_L) F^2 \alpha^*} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*]
\]

\[
\to \bar{\nu} - \left( \frac{\delta_L + (1 - \delta_L)}{\delta_L + (1 - \delta_L)} \right) \mathbb{E}[\theta] = \psi(\theta^*)
\]
where $\mathbb{E}[\theta] = \lim_{\theta^{*} \to \pi} \mathbb{E}[\theta | \theta < \theta^{*}]$. Thus, $\lim_{\theta^{*} \to \pi} p(1, 1) - p(0, 1) < 0$, so:

$$\lim_{\theta^{*} \to \pi} \psi_{M, 2}(\theta^{*}, 0, \alpha^{*}(\theta^{*})) < \beta < \pi$$

Finally, by the intermediate value theorem we have existence of this mixed strategy equilibrium.

**Proof of Lemma 19.** Consider parameters as described in the Lemma. Then, for $\delta_{S} \leq \delta^{*}_{S}$, the equilibrium $\theta^{*}(\delta_{S})$ is defined by the $\theta^{*}$ that solves:

$$\beta - \omega(1 - \delta_{L} - \delta_{S})(1 - F(\theta^{*})) \left( \frac{F(\theta^{*})}{1 - (1 - \delta_{L} - \delta_{S})(1 - F(\theta^{*}))^2} \right) \mathbb{E}[\theta_{i} | \theta_{i} < \theta^{*}] - \theta^{*} = 0$$

Note that the LHS is continuous and differentiable in $\delta_{S}$ at $\delta_{S} = 0$. Therefore, by the implicit function theorem, $\theta^{*}(\delta_{S})$ is continuous and differentiable. This implies $\theta^{*}(\delta_{S}) \to \theta^{*}(0)$, where $\theta^{*}(0)$ is the $\theta^{*}$ that satisfies:

$$\beta - \omega(1 - \delta_{L})(1 - F(\theta^{*})) \left( \frac{F(\theta^{*})}{1 - (1 - \delta_{L})(1 - F(\theta^{*}))^2} \right) \mathbb{E}[\theta_{i} | \theta_{i} < \theta^{*}] - \theta^{*} = 0$$

Note that $\theta^{*}(0)$ solves the same equation as in the large shocks only case when $\gamma^{*} = 0$, as seen in equation (14). Furthermore, since for $\delta_{S}$ small we must have $\theta^{*}(\delta_{S}) \leq 1 + \frac{\sqrt{\delta_{S}^{2} + \delta_{L}^{2}}}{1 + \sqrt{\delta_{L} + \delta_{S}}}$, we must have in the limit $\theta^{*}(0) \leq 1$, which is the cutoff for $\gamma^{*} = 1$ in the large shocks only case. Strategies for large shock or no shock investors trivially converge since they are pure for all $\delta_{S} < \delta^{*}_{S}$ and the same as in the large shocks only case. Denote prices for strategies $(s_{1}, s_{2})$ as a function of $\theta^{*}$ and $\delta_{S}$ as $p_{s_{1}, s_{2}}(\theta^{*}, \delta_{S})$. Since for all $\delta_{S} < \delta^{*}_{S}$ we have $p_{s_{1}, s_{2}}(\theta^{*}(\delta_{S}), \delta_{S}) = \pi$, trivially it converges to $p_{s_{1}, s_{2}}(\theta^{*}(0), \delta_{S}) = \pi$ – the same price as in the large shocks only case. To argue convergence of the price upon observing $s = (1, 1)$, first note that $p_{s_{1}, s_{2}}(\theta^{*}, \delta_{S})$ is continuous in both its arguments. Furthermore, we already showed that $\theta^{*}(\delta_{S})$ is continuous at $\delta_{S} = 0$. We therefore have that $p_{s_{1}, s_{2}}(\theta^{*}(\delta_{S}), \delta_{S})$ is continuous in $\delta_{S}$ at $\delta_{S} = 0$, the price upon observing $s = (1, 1)$ converges to that in the large shocks only case.

**Proof of Lemma 20.** An equilibrium cutoff in Proposition 10 with $\gamma^{*} = 0$ given a $\delta_{L} < \delta^{*}_{L}$, denoted $\theta^{*}(\delta_{L})$, is given by the $\theta^{*}$ satisfying:

$$\beta + \omega((1 - \delta_{L} - \delta_{S})(F(p(1, 1) - p(0, 1)) + (1 - F)(p(1, 0) - p(0, 0))) + \delta_{S}[F(p(1, 1) - p(0, 1)) + (1 - F)(p(1, 0) - \xi^{*}p(1, 1) - (1 - \xi^{*})\frac{1}{2}(p(1, 0) + p(0, 1)))]) - \theta^{*} = 0$$
where prices and mixing probability $\xi^*$ are defined as in the proposition. Note that $\xi^*$ continuously differentiable in $\delta_L$ at $\delta_L = 0$, as are all prices. Therefore, by the same logic as in the previous lemmas, $\theta^*(\delta_L)$ is continuous and differentiable at $\delta_L = 0$ so $\theta^*(\delta_L) \to \theta^*(0)$. Also, we have as $\delta_L \to 0$:

$$
\xi^*(\delta_L) = 1 - \frac{(1 - \delta_L)\delta_S(1 - F)^2 - (2\delta_L F - \delta_L + (1 - \delta_L)F^2)(1 - \delta_L)}{\delta_S(1 - F)}
$$

$$
\to 1 - \frac{\delta_S(1 - F)^2 - F^2}{\delta_S(1 - F)} = \xi^*(0)
$$

where $\xi^*(0)$ is the same mixing probability as in Proposition 6. Furthermore, after plugging prices evaluated at $\delta_L = 0$ into the cutoff rule equation, and utilizing the fact that in this equilibrium $2p(1, 1) = p(1, 0) + \overline{\upsilon}$, $\theta^*(0)$ is the value of $\theta^*$ satisfying:

$$
\beta - \omega \frac{1 + (1 - \delta_S)(1 - F(\theta^*))}{1 - (1 - \delta_S)(1 - F(\theta^*))^2} F(\theta^*) \mathbb{E}[\theta_i | \theta_i < \theta^*] = \theta^*
$$

which is the same condition as equation (27) in the proof of Proposition 6, which defines the cutoff rule in the small shocks only case. Therefore, the equilibrium cutoff converges to that in the small shocks only case with double exit and $\gamma^* = 0$. To confirm that this is in fact an equilibrium in the limit, we note that for all $\delta_L < \delta^*_L$, we must have:

$$
\psi_{M, 3}(Y(\delta), 0, 0) = \beta + \omega \left(1 - \delta_L\right) \left(\frac{(1 - \delta_L)F(Y(\delta))(1 - F(Y(\delta)))^2 + \delta_L F(Y(\delta))(1 - F(Y(\delta)))}{(1 - \delta_L)F(Y(\delta)(1 - F(Y(\delta))) + \delta_L} + \frac{(1 - \delta_L)F(Y(\delta))^3}{(1 - \delta_L)F(Y(\delta))^2 + \delta_S(1 - F(Y(\delta)))^2} - \delta_S \frac{1 - \delta_L F(Y(\delta)) (1 - F(Y(\delta)))}{(1 - \delta_L)F(Y(\delta))^2 + \delta_S(1 - F(Y(\delta)))^2} \right) - \delta_S(1 - \delta_L)F(Y(\delta))^2 + \delta_S(1 - F(Y(\delta)))^2 \right) \mathbb{E}[\theta_i | \theta_i < Y(\delta)] \leq Y(\delta)
$$

where $F(Y(\delta)) = \frac{\sqrt{\delta_L + \delta_S}}{1 + \sqrt{\delta_S}}$, as noted in the Proposition. Note that $F(Y(\delta)) \to \frac{\sqrt{\delta_S}}{1 + \sqrt{\delta_S}}$ as $\delta_L \to 0$. Furthermore, both sides of the condition are continuously differentiable in $\delta_L$, and at $\delta_L = 0$ it simplifies to

$$
\beta + \omega \left(1 - \frac{\sqrt{\delta_S}}{2} \right) \mathbb{E}[\theta_i | \theta_i < Y(\delta)] \leq \frac{\sqrt{\delta_S}}{1 + \sqrt{\delta_S}}
$$

which is identical to equation (16) in Proposition 10. Therefore, $\theta^*(0)$ will satisfy the conditions necessary for it to be an equilibrium cutoff. Prices clearly also converge, as do strategies since the mixing probability in the multiple shock case is identical to that in the small shock only case. \qed
Proof of Lemma 21. First, note that in the equilibrium with both small and large shocks,

\[ F(\theta_u^*) = \min \left\{ 1, 2 \frac{\delta_L}{\delta_S} - \frac{\delta_L}{1 - \delta_L} + \frac{\sqrt{\delta_L(\delta_S^2 + 4\delta_L(1 - \delta_L)(1 - \delta))}}{(1 - \delta_L)\delta_S} \right\}. \]

Note that there exists a \( \delta_S^* \) such that, for all \( \delta_S \leq \delta_S^* \), \( F(\theta_u^*) = 1 \). Therefore, we do not need to consider equilibria of type (c) as \( \delta_S \to 0 \). Furthermore, as \( \delta_S \to 0 \), we have \( F(\theta_u^*) \to \sqrt{\frac{\delta_L}{1 + \sqrt{\delta_L}}} \). First, consider the case where there exists \( \delta_S^* > 0 \) such that for all \( \delta_S \leq \delta_S^* \), the equilibrium in Proposition 11 is of type (b). In this equilibrium, \( \alpha^* = 1 \) and \( \gamma^* = 0 \). In this case, \( \theta^*(\delta_S) \) is defined by the \( \theta^* \) that solves:

\[
\beta + \omega((1 - \delta_L - \delta_S)[F(p(1, 1) - p(0, 1)) + (1 - F)(p(1, 0) - \tau)] +
\delta_S[F(p(1, 1) - p(0, 1)) + (1 - F)(p(1, 0) - \frac{1}{2}(p(1, 0) + p(0, 1))) - \theta^* = 0
\]

where prices are defined as in Proposition 11. Prices are clearly differentiable in \( \delta_S \) at \( \delta_S = 0 \), so the LHS is continuous and differentiable at \( \delta_S = 0 \). Therefore, by the implicit function theorem, \( \theta^*(\delta_S) \) is continuous and differentiable at \( \delta_S = 0 \), again implying \( \theta^*(\delta_S) \to \theta^*(0) \) where \( \theta^*(0) \) is the \( \theta^* \) that satisfies:

\[
\beta - \omega(1 - \delta_L) \left( F \left( \frac{\delta_L F + (1 - \delta_L)F^2}{\delta_L + (1 - \delta_L)F^2} \right) + (1 - F) \right) \mathbb{E}[\theta_i|\theta_i < \theta^*] - \theta^* = 0
\]

Note that this is the same implicit definition of \( \theta^* \) as in equation (14) when \( \gamma^* = 0 \), so \( \theta^*(0) \) is the same cutoff as in the large shocks only equilibrium. Furthermore, note that since \( F(\theta^*(\delta_S)) \geq \frac{\sqrt{\delta_L + \delta_S}}{1 + \sqrt{\delta_L + \delta_S}} \) for all \( \delta_S \leq \delta_S^* \), we must have \( F(\theta^*(0)) \geq \frac{\sqrt{\delta_L}}{1 + \sqrt{\delta_L}} \), so we also have the cutoff in the appropriate region. The same argument as in the previous lemma shows that equilibrium prices also converge to those in the large shocks only case. Now, consider the case where there exists \( \theta_S^* > 0 \) such that for all \( \theta_S \leq \theta_S^* \), the equilibrium in Proposition 11 is of type (a). This is a mixed strategy equilibrium. Following the same logic as in the previous part of the proof, an equilibrium cutoff is
defined by $\theta^*$ satisfying:

$$
\beta + \omega((1 - \delta_L - \delta_S)[F(p(1, 1) - \gamma^*p(1, 1) - (1 - \gamma^*)p(0, 1)) + \\
(1 - F)(\gamma^* p(1, 1) + (1 - \gamma^*) p(1, 0) - p(0, 0))]) + \\
\delta_S[F(p(1, 1) - \gamma^* p(1, 1) - (1 - \gamma^*) p(0, 1)) + \\
(1 - F)(\gamma^* p(1, 1) + (1 - \gamma^*) p(1, 0) - \frac{1}{2} (p(1, 0) + p(0, 1)))) - \theta^* = 0
$$

where prices are defined as in the Proposition. Note again that mixing probabilities and prices are continuous and differentiable in $\delta_S$ at $\delta_S = 0$, so $\theta^*(\delta_S)$ is continuous and differentiable at $\delta_S = 0$. Therefore, $\theta^*(\delta_S) \to \theta^*(0)$. Note that $\gamma^*(\delta_S) \to 1$ as $\delta_S \to 0$. We then have $\theta^*(0)$ is given by the $\theta^*$ that satisfies:

$$
\beta - \omega(1 - \delta_L)(1 - F) \left( \frac{F}{1 - (1 - \delta_L)(1 - F)^2} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*] - \theta^* = 0
$$

which is the same condition as in the large shocks only case with $\gamma^* = 1$. As usual, note that $\theta^*(0)$ must also satisfy $F(\theta^*(0)) \leq \frac{\sqrt{\beta}}{1 + \sqrt{\delta_L}}$, which is consistent with $\gamma^* = 1$. By the same logic as before, prices also converge. 

\textbf{Proof of Lemma 22.} First note that, in the Proposition 11 equilibrium as $\delta_L \to 0$, we have

$$
F(\theta^*_L) \to 0 \\
F(\theta^*_U) \to 0
$$

Note also that we have existence of a type (c) equilibrium if $\psi_{M,2}(\theta^*_L, 0, 1) > \theta^*_U$. Since $\psi_{M,2}$ is continuous in $\theta$ and since $F(\theta^*_U) \to 0$ as $\delta_L \to 0$, this condition will necessarily hold for $\delta_L$ small since $\psi_{M,2}(0, 0, 1) = \beta > 0$. Therefore, we need only to analyze equilibria of type (c) – those with $\gamma^* = 0$ and $\alpha^* \in (0, 1)$ with $2p(1, 1) = p(1, 0) + q(\theta^*)$. Now, an equilibrium here is defined by the $\theta^*$ satisfying:

$$
\beta + \omega((1 - \delta_L - \delta_S)[F(\alpha^* p(1, 1) + (1 - \alpha^*) \frac{1}{2} p(1, 0) + p(0, 1)) - p(0, 1)) + \\
(1 - F)(p(1, 0) - p(0, 0)))] + \\
\delta_S[F(\alpha^* p(1, 1) + (1 - \alpha^*) \frac{1}{2} p(1, 0) + p(0, 1)) - p(0, 1)) + \\
(1 - F)(p(1, 0) - \frac{1}{2} (p(1, 0) + p(0, 1)))) - \theta^* = 0
$$

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where prices are as defined in the Proposition 11. First, note that we have for any \( \delta_L > 0 \):

\[
\alpha^* = \frac{\frac{1}{2} \delta_L \delta_S (1 - F)(1 - 2 F) + \delta_L (1 - \delta_L) F (2 - F)}{\frac{1}{2} (1 - \delta_L) \delta_S F^2 (1 - F) + \delta_L (1 - \delta_L) F^2}
\]

We then have \( \alpha^* \) continuously differentiable at \( \delta_L = 0 \) and \( \alpha^* \to 0 \) as \( \delta_L \to 0 \). Furthermore, since prices are differentiable and continuous in \( \delta_L \) at \( \delta_L = 0 \), we have the \( \theta^*(0) \) as the \( \theta^* \) that satisfies:

\[
\beta + \omega ((1 - \delta_S)[F \frac{1}{2}(p(1, 0) - p(0, 1)) + (1 - F)(p(1, 0) - \overline{v})] + \\
\delta_S[F \frac{1}{2}(p(1, 0) - p(0, 1)) + (1 - F)\frac{1}{2}(p(1, 0) - p(0, 1))]) - \theta^* = 0
\]

which simplifies, after plugging in for prices:

\[
\beta + \omega(2 - \delta_S) \left( \frac{F(1 - F)}{1 - (1 - \delta_S)(1 - F)^2} \right) \mathbb{E}[\theta_i | \theta_i < \theta^*] = 0
\]

This is the same condition as in the small shocks only equilibrium with no double exit. Therefore, we have the equilibrium cutoff converging. Strategies also converge, as do prices. \( \square \)

**Proof of Lemma 23.** Consider a \( \gamma^* \) such that there exists a \( \theta^* \) with \( \psi_{M,1}(\theta^*, \gamma^*, \xi^*(\theta^*, \gamma^*)) = \theta^* \). As shown in the proof of Proposition 10: (i) \( \psi_{M,1}(\theta, \tilde{\gamma}, \xi^*(\theta, \tilde{\gamma})) \) is increasing in \( \tilde{\gamma} \) for each \( \theta \), and (ii) decreasing in \( \theta \leq Y(\delta) \) for each \( \tilde{\gamma} \). Take any \( \tilde{\gamma} > \gamma^* \) such that there exists a \( \tilde{\theta} \leq \theta^* \) that is an equilibrium cutoff, i.e.

\[
\psi_{M,1}(\tilde{\theta}, \tilde{\gamma}, \xi^*(\tilde{\theta}, \tilde{\gamma})) = \tilde{\theta}
\]

With the previous two points we have:

\[
\theta^* = \psi_{M,1}(\theta^*, \gamma^*, \xi^*(\theta^*, \gamma^*)) < \psi_{M,1}(\theta^*, \tilde{\gamma}, \xi^*(\theta^*, \tilde{\gamma})) \leq \psi_{M,1}(\tilde{\theta}, \tilde{\gamma}, \xi^*(\tilde{\theta}, \tilde{\gamma})) = \tilde{\theta}
\]

where the last inequality comes from the supposition that \( \tilde{\theta} \leq \theta^* \). This yields a contradiction. Therefore, the cutoff rule is increasing in \( \gamma^* \), which implies that efficiency decreases as \( \gamma^* \) increases. \( \square \)