Leverage Aversion – A Third Dimension in Portfolio Theory and Practice

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Jacobs Levy Equity Management
“When you add leverage on top of leverage, and then add more leverage, it usually doesn’t end well.”
Thomas R. Ajamie
*The New York Times*

“When you combine ignorance and leverage, you get some pretty interesting results.”
Warren Buffett
Overview

• Unique Risks of Leverage

• Conventional Approach: Mean-Variance Model
  – Leverage constraints

• Proposed Approach: Mean-Variance-Leverage Model
  – Evaluate portfolios with investor leverage aversion
  – Optimize portfolios with investor leverage aversion

• Compare and Contrast the Two Models
Conventional Portfolio Theory and Practice

Modern Portfolio Theory (MPT)

• Investors like higher Expected Portfolio Returns (mean returns)
• Due to Risk Aversion, investors dislike higher Portfolio Volatility (variance of returns)
• MPT allows investors to trade off Expected Portfolio Returns and Portfolio Volatility
• The greater an investor’s level of Risk Aversion, the greater the penalty for taking on higher levels of Portfolio Volatility
• Mean-Variance (MV) Optimization used to find the investor’s Optimal Portfolio
Portfolios with Leverage

• Leverage is created either through the borrowing of cash or the borrowing of stocks to sell short

• Leverage is used to increase Expected Portfolio Returns

• Consider the following portfolios:
  
  - Portfolio A – a long-only portfolio
    
    Expected Return 5%, Standard Deviation 7%
  
  - Portfolio B – a leveraged long-short portfolio
    
    Expected Return 5%, Standard Deviation 7%

• MV Model is indifferent between these two portfolios
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  Expected Return 5%, Standard Deviation 7%

• MV Model is indifferent between these two portfolios

  But most investors would prefer Portfolio A. Why?
Components of Risks Unique to Using Leverage

• Risks and Costs of Margin Calls
  – Can force borrowers to liquidate securities at adverse prices due to illiquidity

• Risk of Losses Exceeding the Capital Invested

• Possibility of Bankruptcy
Excessive Leverage and Systemic Risk

Some Catastrophes Caused by Excessive Leverage

• Stock Market Crash triggered by Margin Calls (1929)
• Long-Term Capital Management (1998)
• Goldman Sachs Global Equity Opportunities Fund (2007)
• Leveraged Securitized Housing Debt (2007-2008)
• Bear Stearns and Lehman (2008)
• JP Morgan “Whale” (2012)

Excessive Leverage Can Give Rise To

• Systemic Risk
• Market Disruptions
• Economic Crises
Three Solutions for the Leverage Problem

• Use Traditional Mean-Variance Optimization with a Leverage Constraint
  — But what Constraint Level is Optimal?

• Introduce a Third Dimension to Portfolio Theory: A Leverage Aversion Term
  — Results in Mean-Variance-Leverage Optimization Model

• Build a Stochastic Margin Call Model (SMCM) to include a measure of Short-Term Portfolio Variance as a Third Dimension
  — A formidable problem yet to be solved

For SMCM, see Markowitz (2013) and Jacobs and Levy (2013c)
Mean-Variance Utility

Maximize Utility:

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_p^2 \]
Mean-Variance Utility (cont’d)

Portfolio expected active return:

\[ \alpha_P = \sum_{i=1}^{N} \alpha_i x_i \]

where \( \alpha_i \) is the expected active return for security \( i \), \( x_i \) denotes the active weight for security \( i \) (relative to benchmark weight \( b_i \)), and \( N \) is the number of securities.

\[ U = \alpha_P - \frac{1}{2\tau_V} \sigma_P^2 \]
Mean-Variance Utility (cont’d)

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_p^2 \]

Variance of portfolio active return:

\[ \sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j \]

where \( \sigma_{ij} \) is the covariance between the active returns of securities \( i \) and \( j \).
Mean-Variance Utility (cont’d)

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_p^2 \]

\( \tau_V \) is the investor’s risk tolerance for the variance of portfolio active return.
Mean-Variance Utility (cont’d)

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_p^2 \]

MV(\(\tau_V\)) Investors
MV(\(\tau_V\)) Portfolios
MV(\(\tau_V\)) Utility
Example Using Enhanced Active Equity (EAE) Portfolio

For a 130-30 EAE Long-Short Portfolio

• Long Securities = 130% of Capital
• Short Securities = 30% of Capital
• Total Portfolio = 160% of Capital, or 60% in Excess of Capital
• Leverage, $\Lambda = 0.6$ (60%)
• Enhancement = $\Lambda / 2 = 0.3$ (30%)
• Net Long – Short = 100% (Full Market Exposure)
• Portfolio Beta = 1
Estimation of Model Parameters

- Used Daily Return Data for Constituent Stocks in S&P 100 Index over Two Years (ending 30 September 2011)
- Estimates for Securities’ Expected Active Returns: Used a skill-based transformation of daily return data given that investors have imperfect foresight
- Estimates for Variances and Covariances: Used daily return data
- Estimates for Security Betas: Used daily return data and the single index model with S&P 100 Index
- To Achieve Diversified Portfolios: Security active weight constraint at band of +/- 10 percentage points from security weight in S&P 100 Index
- Costless Self-Financing: Short proceeds finance additional long positions (in practice, there would be stock loan fees, hard-to-borrow costs, etc.)

For details, see Jacobs and Levy (2012)
Efficient Frontiers for Various Leverage Constraints

- 150–50 ($\Lambda=1$)
- 140–40 ($\Lambda=0.8$)
- 130–30 ($\Lambda=0.6$)
- 120–20 ($\Lambda=0.4$)
- 110–10 ($\Lambda=0.2$)
- 100–00 ($\Lambda=0$)

**Expected Active Return (%)**

**Standard Deviation of Active Return (%)**
Optimal MV(1) Portfolios for Various Leverage Constraints

![Graph showing the relationship between expected active return and standard deviation of active return for different leverage constraints.](image-url)
MV(1) Utility of Optimal MV(1) Portfolios as a Function of Enhancement

- 10% Active Weight Constraint
- No Active Weight Constraint

MV(1) Utility vs. Enhancement (%)

Points: a, b, c, d, e, f, z
Characteristics of Optimal MV(1) Portfolios from the Perspective of an MV(1) Investor

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>EAE</th>
<th>Leverage</th>
<th>Standard Deviation of Active Return</th>
<th>Expected Active Return</th>
<th>Utility for an MV(1) Investor</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100-0</td>
<td>0.00</td>
<td>4.52</td>
<td>2.77</td>
<td>2.67</td>
</tr>
<tr>
<td>b</td>
<td>110-10</td>
<td>0.20</td>
<td>4.91</td>
<td>3.27</td>
<td>3.15</td>
</tr>
<tr>
<td>c</td>
<td>120-20</td>
<td>0.40</td>
<td>5.42</td>
<td>3.76</td>
<td>3.61</td>
</tr>
<tr>
<td>d</td>
<td>130-30</td>
<td>0.60</td>
<td>5.94</td>
<td>4.23</td>
<td>4.06</td>
</tr>
<tr>
<td>e</td>
<td>140-40</td>
<td>0.80</td>
<td>6.53</td>
<td>4.70</td>
<td>4.48</td>
</tr>
<tr>
<td>f</td>
<td>150-50</td>
<td>1.00</td>
<td>7.03</td>
<td>5.14</td>
<td>4.90</td>
</tr>
<tr>
<td>z</td>
<td>492-392</td>
<td>7.84</td>
<td>15.43</td>
<td>11.55</td>
<td>10.36</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between Enhancement (%) and MV(1) Utility with and without active weight constraint.](image-url)
Characteristics of Optimal MV(1) Portfolios from the Perspective of an MV(1) Investor (cont’d)

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<thead>
<tr>
<th>Portfolio</th>
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Which portfolio is optimal for an MV(1) investor who also has an aversion to leverage?
Mean-Variance-Leverage Utility

\[ U = \alpha_p - \frac{1}{2\tau_v} \sigma_p^2 - \frac{1}{2\tau_L} \sigma_T^2 \Lambda^2 \]
Mean-Variance-Leverage Utility (cont’d)

\[
U = \alpha_p - \frac{1}{2\tau_V} \sigma_P^2 - \frac{1}{2\tau_L} \sigma_T \Lambda^2
\]

Variance of the leveraged portfolio’s total return:

\[
\sigma_T^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} h_i q_{ij} h_j
\]

where \(q_{ij}\) is the covariance between the total returns of securities \(i\) and \(j\), and \(h_i\) is the holding weight of security \(i\).

\(\Lambda^2\) is the square of the portfolio’s leverage.

Intuition: Costs of leverage are higher in more volatile portfolios.
Mean-Variance-Leverage Utility (cont’d)

\[ U = \alpha_P - \frac{1}{2\tau_V} \sigma_P^2 - \frac{1}{2\tau_L} \sigma_T^2 \Lambda^2 \]

\( \tau_L \) is the investor’s tolerance for leverage risk.
Mean-Variance-Leverage Utility (cont’d)

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_p^2 - \frac{1}{2\tau_L} \sigma_T^2 \Lambda^2 \]

MVL(\(\tau_V, \tau_L\)) Investors
MVL(\(\tau_V, \tau_L\)) Portfolios
MVL(\(\tau_V, \tau_L\)) Utility
The Effect of Leverage Aversion: MVL(1,1) Utility of Optimal MV(1) Portfolios as a Function of Enhancement
Characteristics of Optimal MV(1) Portfolios from the Perspective of an MVL(1,1) Investor

<table>
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<tr>
<th>Portfolio</th>
<th>EAE</th>
<th>Leverage</th>
<th>Standard Deviation of Active Return</th>
<th>Expected Active Return</th>
<th>Utility for an MVL(1,1) Investor</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>100-0</td>
<td>0.00</td>
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<td>3.76</td>
<td>3.32</td>
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<tr>
<td>g</td>
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<td>e</td>
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<td>3.27</td>
</tr>
<tr>
<td>f</td>
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<td>1.00</td>
<td>7.03</td>
<td>5.14</td>
<td>2.97</td>
</tr>
</tbody>
</table>
Mean-Variance Optimization as a Special Case of Mean-Variance-Leverage Optimization

\[ U = \alpha_p - \frac{1}{2\tau_V} \sigma_P^2 - \frac{1}{2\tau_L} \sigma_T^2 \Lambda^2 \]

**Special Case 1: Zero Leverage Tolerance**

- MVL \((\tau_V, 0)\) reduces to MV \((\tau_V)\) with a leverage constraint of zero, resulting in a Long-Only Portfolio

**Special Case 2: Infinite Leverage Tolerance**

- MVL \((\tau_V, \infty)\) reduces to MV \((\tau_V)\) with no leverage constraint, resulting in a Highly Leveraged Portfolio

Mean-Variance Optimization and Mean-Variance-Leverage Optimization produce the same portfolio only in these two special cases
Solving the Mean-Variance-Leverage Optimization Problem

**MV is a quadratic mathematical problem:**

\[
U = \sum_{i=1}^{N} \alpha_i x_i - \frac{1}{2\tau_V} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j
\]

- Square of the active-weight variable \( x_i \), including second-order cross-products
- Use quadratic solver

**MVL is a quartic mathematical problem:**

\[
U = \sum_{i=1}^{N} \alpha_i x_i - \frac{1}{2\tau_V} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j - \frac{1}{2\tau_L} \left( \sum_{i=1}^{N} \sum_{j=1}^{N} (b_i + x_i) q_{ij} (b_j + x_j) \right) \left( \sum_{i=1}^{N} |b_i + x_i| - 1 \right)^2
\]

- Quartic of the active-weight variable \( x_i \), including fourth-order cross-products
- Use Fixed-Point Iteration, which allows a quadratic solver to be applied iteratively

For details, see Jacobs and Levy (2013b,c)
Optimal Leverage for Zero Leverage Tolerance

\[ \tau_L = 0 \]

10% Active Security Weight Constraint

- Expected Active Return (%)
- Standard Deviation of Active Return (%)

Points:
- 100-0, 0-0
Optimal Leverage for Leverage Tolerance of 1

\[ \tau_L = 1 \]

10% Active Security Weight Constraint

Expected Active Return (%) vs. Standard Deviation of Active Return (%)

- 100-0
- 110-10
- 120-20
- 130-30
Optimal Leverage for Infinite Leverage Tolerance

\[ \tau_L = \infty \]

10% Active Security Weight Constraint

Expected Active Return (%) vs. Standard Deviation of Active Return (%)

- 100-0
- 200-100
- 400-300
Optimal Leverage for Infinite Leverage Tolerance with No Active Security Weight Constraint

\[ \tau_L = \infty \]

No Active Security Weight Constraint

- 100–0
- 4000–3900
- 9000–8900
Optimal Leverage for Various Leverage-Tolerance Cases

\[ \tau_L = 0 \]

10% Active Security Weight Constraint

- Expected Active Return (%)
- Standard Deviation of Active Return (%)

\[ \tau_L = 1 \]

10% Active Security Weight Constraint

- Expected Active Return (%)
- Standard Deviation of Active Return (%)

\[ \tau_L = \infty \]

10% Active Security Weight Constraint

- Expected Active Return (%)
- Standard Deviation of Active Return (%)

\[ \tau_L = \infty \]

No Active Security Weight Constraint

- Expected Active Return (%)
- Standard Deviation of Active Return (%)
Mean-Variance-Leverage Efficient Frontiers for Various Leverage-Tolerance Cases (with the 10% Security Active Weight Constraint)
Characteristics of MVL ($\tau_V$, $\tau_L$) Portfolios A, B, and C

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\tau_L$</th>
<th>$\tau_V$</th>
<th>EAE</th>
<th>$\sigma_P$</th>
<th>$\alpha_P$</th>
<th>$U_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.00</td>
<td>0.24</td>
<td>125-25</td>
<td>5.00</td>
<td>3.93</td>
<td>2.93</td>
</tr>
<tr>
<td>B</td>
<td>2.00</td>
<td>0.14</td>
<td>139-39</td>
<td>5.00</td>
<td>4.39</td>
<td>2.72</td>
</tr>
<tr>
<td>C</td>
<td>2.00</td>
<td>0.09</td>
<td>135-35</td>
<td>4.21</td>
<td>3.93</td>
<td>2.68</td>
</tr>
</tbody>
</table>
Mean-Variance-Leverage Efficient Region for Various Leverage and Volatility Tolerance Cases (with No Security Active Weight Constraint)
Mean-Variance-Leverage Optimal Enhancement Surface for Various Combinations of Volatility Tolerance and Leverage Tolerance
Contour Map of Mean-Variance-Leverage Optimal Enhancement for Various Combinations of Volatility Tolerance and Leverage Tolerance
Locating the Optimal MVL(1,1) Portfolio on the Mean-Variance-Leverage Efficient Surface

Enhancement (%) vs Leverage Tolerance and Volatility Tolerance
Locating the Optimal MVL(1,1) Portfolio on a Contour Map of the Mean-Variance-Leverage Efficient Surface
Utility of Optimal MVL(1, \(\tau_L\)) Portfolios

MV(1) Utility, same as MVL(1,∞) Utility
Characteristics of Optimal MVL(1,τ_L) Portfolios
Conclusion

• Mean-Variance Optimization assumes either Zero Leverage Tolerance (long-only) or Infinite Leverage Tolerance

• Without a Leverage Constraint, Mean-Variance Optimization can result in Highly Leveraged Portfolios

• With a Leverage Constraint, Mean-Variance Optimization will lead to the Optimal Portfolio for a Leverage-Averse Investor only by Chance

• Two ways to identify the Optimal Portfolio for a Leverage-Averse Investor:
  – Consider numerous Leverage-Constrained Optimal Mean-Variance Portfolios and evaluate each one using the Investor’s Mean-Variance-Leverage Utility Function
  – Use Mean-Variance-Leverage Optimization Directly

Both Methods produce the Same Portfolio
Conclusion (cont’d)

• Both Volatility Tolerance (or Aversion) and Leverage Tolerance are Critical for Portfolio Selection

• Investors are willing to sacrifice some Expected Return in order to reduce Leverage Risk, just as they sacrifice some Expected Return in order to reduce Volatility Risk.

• Mean-Variance-Leverage Optimization balances Expected Return against Volatility Risk and Leverage Risk

• Leverage Aversion can have a large effect on an Investor’s Portfolio Choice
# Volatility and Leverage Polar Cases

## Underlying Portfolio Volatility

<table>
<thead>
<tr>
<th>Portfolio Leverage</th>
<th>Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>Long-Only Index Fund</td>
<td>Enron Employee’s Single-Stock Holding</td>
</tr>
<tr>
<td>High</td>
<td>LTCM’s Leveraged Low-Risk Arbitrage Positions</td>
<td>CEO’s Leveraged Chesapeake Energy Stock</td>
</tr>
</tbody>
</table>
Conventional portfolio theory says not to hold all your eggs in one basket.

Using excessive leverage is like piling baskets of eggs on top of one another until the pile becomes unsteady.
References


