Wharton
UNIVERSITY of PENNSYLVANIA

JACOBS LEVY EQUITY MANAGEMENT CENTER
FOR QUANTITATIVE FINANCIAL RESEARCH

Leverage Aversion – A Third Dimension in Portfolio Theory and Practice

Bruce I. Jacobs and Kenneth N. Levy
Principals and Co-Founders,
Jacobs Levy Equity Management

"When you add leverage on top of leverage, and then add more leverage, it usually doesn't end well."

Thomas R. Ajamie

The New York Times

"When you combine ignorance and leverage, you get some pretty interesting results."

Warren Buffett

Overview

- Unique Risks of Leverage
- Conventional Approach: Mean-Variance Model
  - Leverage constraints
- Proposed Approach: Mean-Variance-Leverage Model
  - Evaluate portfolios with investor leverage aversion
  - Optimize portfolios with investor leverage aversion
- Compare and Contrast the Two Models

Conventional Portfolio Theory and Practice

Modern Portfolio Theory (MPT)

- Investors like higher Expected Portfolio Returns (mean returns)
- Due to Risk Aversion, investors dislike higher Portfolio Volatility (variance of returns)
- MPT allows investors to trade off Expected Portfolio Returns and Portfolio Volatility
- The greater an investor's level of Risk Aversion, the greater the penalty for taking on higher levels of Portfolio Volatility
- Mean-Variance (MV) Optimization used to find the investor's Optimal Portfolio

Portfolios with Leverage

- Leverage is created either through the borrowing of cash or the borrowing of stocks to sell short
- Leverage is used to increase Expected Portfolio Returns
- Consider the following portfolios:

    Portfolio A – a long-only portfolio

        Expected Return 5%, Standard Deviation 7%

    Portfolio B – a leveraged long-short portfolio

        Expected Return 5%, Standard Deviation 7%

- MV Model is indifferent between these two portfolios

Portfolios with Leverage

- Leverage is created either through the borrowing of cash or the borrowing of stocks to sell short
- Leverage is used to increase Expected Portfolio Returns
- Consider the following portfolios:

  Portfolio A – a long-only portfolio

  Expected Return 5%, Standard Deviation 7%

  Portfolio B – a leveraged long-short portfolio

  Expected Return 5%, Standard Deviation 7%

- MV Model is indifferent between these two portfolios

But most investors would prefer Portfolio A. Why?

Components of Risks Unique to Using Leverage

- Risks and Costs of Margin Calls
  – Can force borrowers to liquidate securities at adverse prices due to illiquidity
- Risk of Losses Exceeding the Capital Invested
- Possibility of Bankruptcy

Excessive Leverage and Systemic Risk

Some Catastrophes Caused by Excessive Leverage

- Stock Market Crash triggered by Margin Calls (1929)
- Long-Term Capital Management (1998)
- Goldman Sachs Global Equity Opportunities Fund (2007)
- Leveraged Securitized Housing Debt (2007-2008)
- Bear Stearns and Lehman (2008)
- JP Morgan “Whale” (2012)

Excessive Leverage Can Give Rise To

- Systemic Risk
- Market Disruptions
- Economic Crises

Three Solutions for the Leverage Problem

- Use Traditional Mean-Variance Optimization with a Leverage Constraint
  - — But what Constraint Level is Optimal?
- Introduce a Third Dimension to Portfolio Theory: A Leverage Aversion Term
  - — Results in Mean-Variance-Leverage Optimization Model
- Build a Stochastic Margin Call Model (SMCM) to include a measure of Short-Term Portfolio Variance as a Third Dimension
  - — A formidable problem yet to be solved

For SMCM, see Markowitz (2013) and Jacobs and Levy (2013c)

Mean-Variance Utility

Maximize Utility:

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2$$

Mean-Variance Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2$$

Portfolio expected active return:

$$\alpha_P = \sum_{i=1}^{N} \alpha_i x_i$$

where α_i is the expected active return for security i, x_i denotes the active weight for security i (relative to benchmark weight b_i), and N is the number of securities.

Mean-Variance Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2$$

Variance of portfolio active return:

$$\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i \sigma_{ij} x_j$$

where σ_{ij} is the covariance between the active returns of securities i and j.

Mean-Variance Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2$$

τ_V is the investor's risk tolerance for the variance of portfolio active return.

Mean-Variance Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2$$

MV(τ_V) Investors

MV(τ_V) Portfolios

MV(τ_V) Utility

Example Using Enhanced Active Equity (EAE) Portfolio

For a 130-30 EAE Long-Short Portfolio

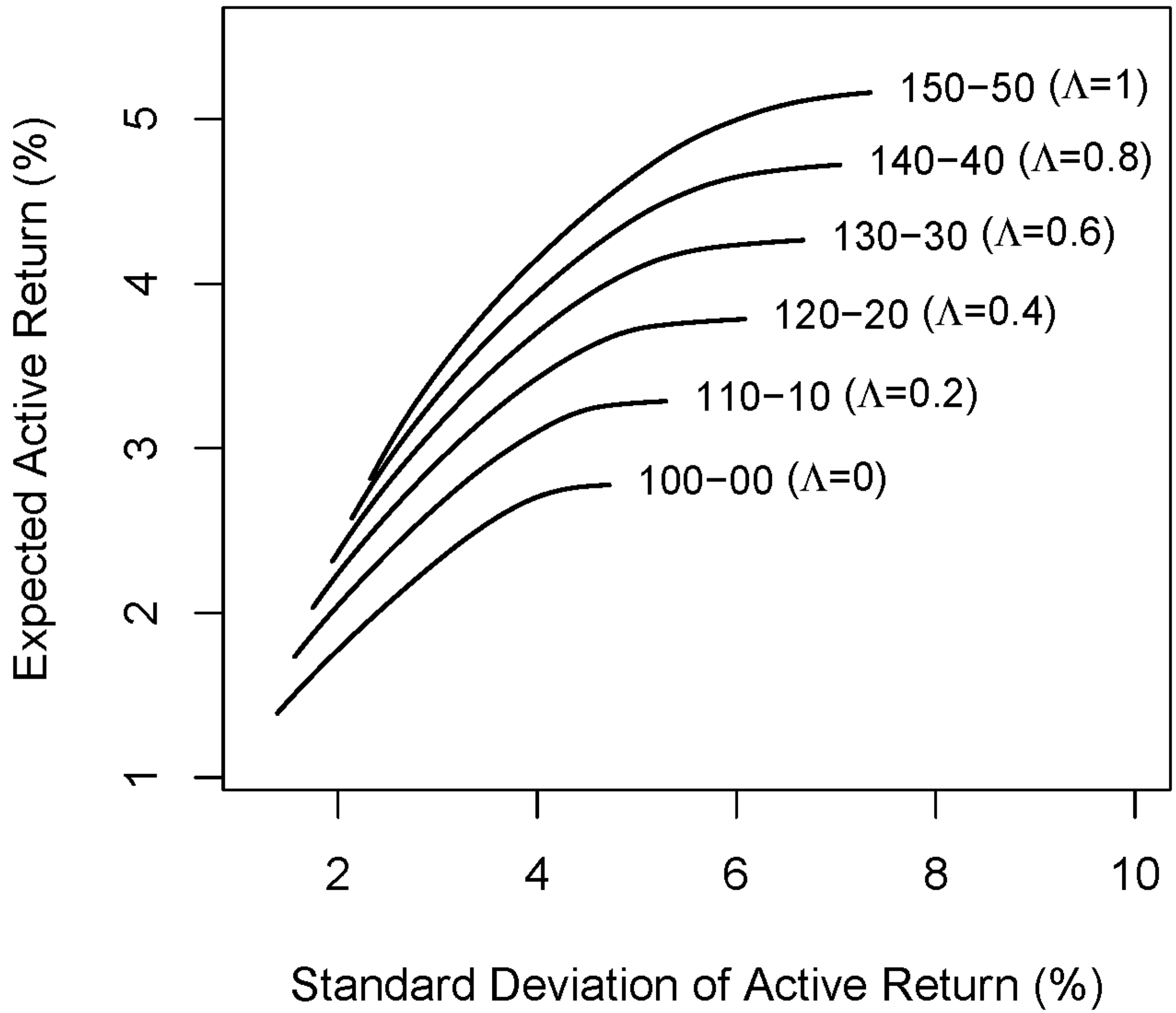
- Long Securities = 130% of Capital
- Short Securities = 30% of Capital
- Total Portfolio = 160% of Capital, or 60% in Excess of Capital
- Leverage, Λ = 0.6 (60%)
- Enhancement = $\Lambda/2$ = 0.3 (30%)
- Net Long – Short = 100% (Full Market Exposure)
- Portfolio Beta = 1

Estimation of Model Parameters

- Used Daily Return Data for Constituent Stocks in S&P 100 Index over Two Years (ending 30 September 2011)
- Estimates for Securities' Expected Active Returns: Used a skill-based transformation of daily return data given that investors have imperfect foresight
- Estimates for Variances and Covariances: Used daily return data
- Estimates for Security Betas: Used daily return data and the single index model with S&P 100 Index
- To Achieve Diversified Portfolios: Security active weight constraint at band of +/– 10 percentage points from security weight in S&P 100 Index
- Costless Self-Financing: Short proceeds finance additional long positions (in practice, there would be stock loan fees, hard-to-borrow costs, etc.)

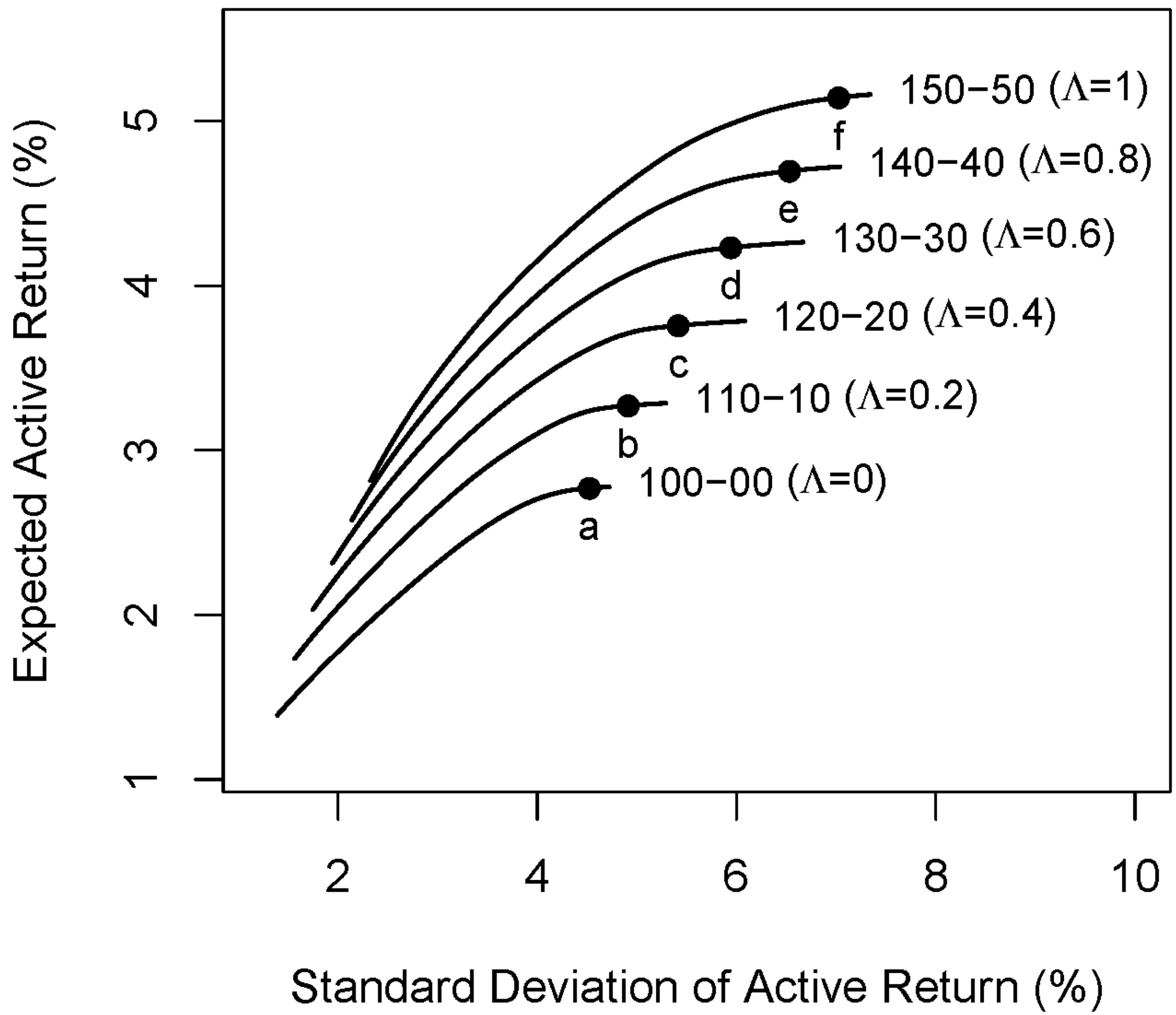
For details, see Jacobs and Levy (2012)

Efficient Frontiers for Various Leverage Constraints



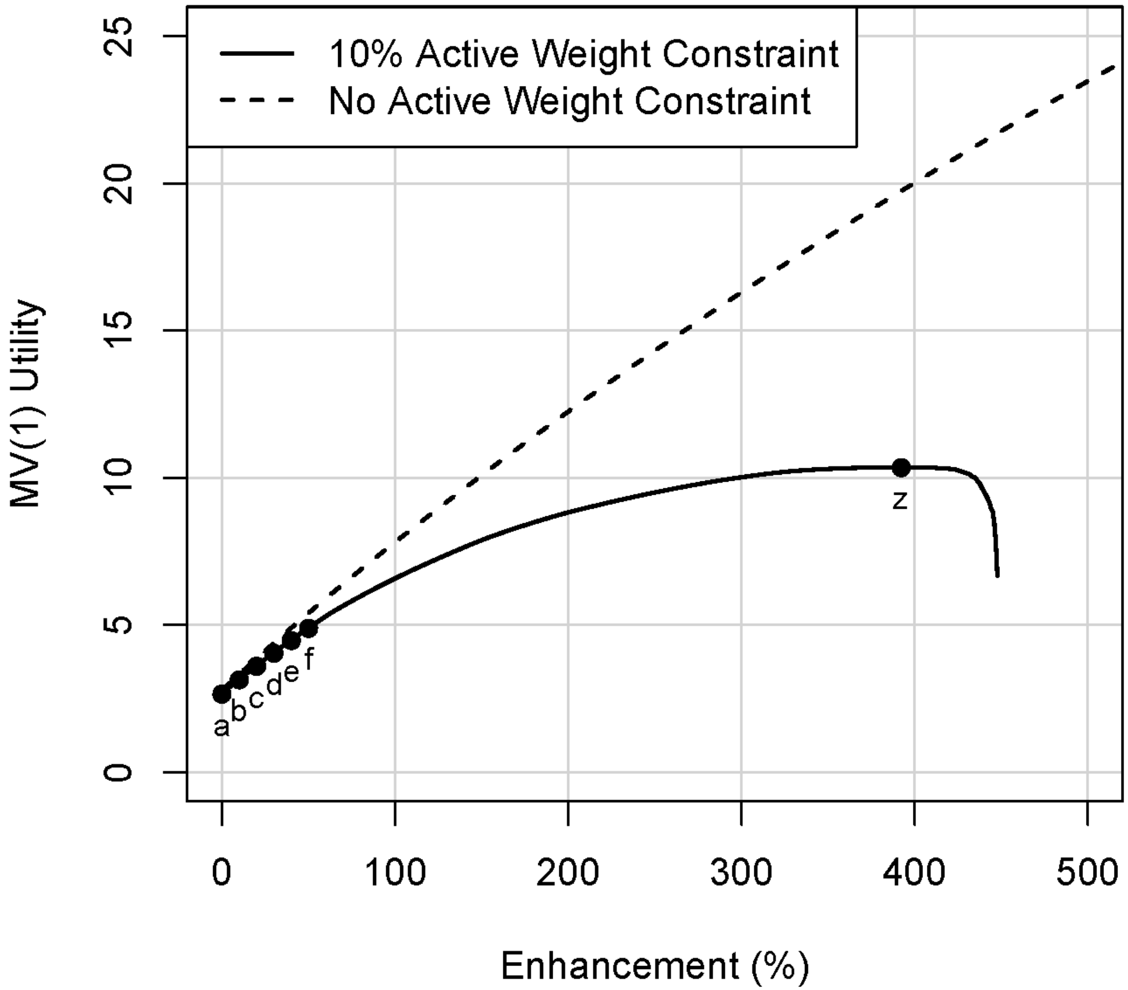
Wharton UNIVERSITY of PENNSYLVANIA | JACOBS LEVY EQUITY MANAGEMENT CENTER FOR QUANTITATIVE FINANCIAL RESEARCH

Optimal MV(1) Portfolios for Various Leverage Constraints



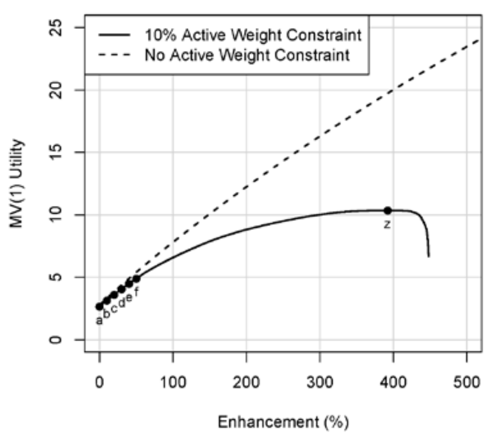
Wharton UNIVERSITY of PENNSYLVANIA | JACOBS LEVY EQUITY MANAGEMENT CENTER FOR QUANTITATIVE FINANCIAL RESEARCH

MV(1) Utility of Optimal MV(1) Portfolios as a Function of Enhancement



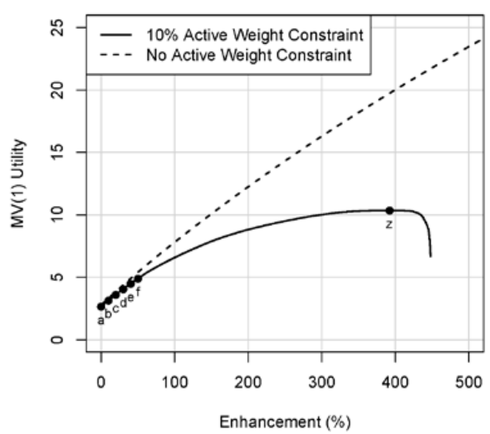
Characteristics of Optimal MV(1) Portfolios from the Perspective of an MV(1) Investor

Portfolio	EAE	Leverage	Standard Deviation of Active Return	Expected Active Return	Utility for an MV(1) Investor
a	100-0	0.00	4.52	2.77	2.67
b	110-10	0.20	4.91	3.27	3.15
c	120-20	0.40	5.42	3.76	3.61
d	130-30	0.60	5.94	4.23	4.06
e	140-40	0.80	6.53	4.70	4.48
f	150-50	1.00	7.03	5.14	4.90
z	492-392	7.84	15.43	11.55	10.36



Characteristics of Optimal MV(1) Portfolios from the Perspective of an MV(1) Investor (cont'd)

Portfolio	EAE	Leverage	Standard Deviation of Active Return	Expected Active Return	Utility for an MV(1) Investor
a	100-0	0.00	4.52	2.77	2.67
b	110-10	0.20	4.91	3.27	3.15
c	120-20	0.40	5.42	3.76	3.61
d	130-30	0.60	5.94	4.23	4.06
e	140-40	0.80	6.53	4.70	4.48
f	150-50	1.00	7.03	5.14	4.90
z	492-392	7.84	15.43	11.55	10.36



Which portfolio is optimal for an MV(1) investor who also has an aversion to leverage?

Mean-Variance-Leverage Utility

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2 - \frac{1}{2\tau_L}\sigma_T^2\Lambda^2$$

Mean-Variance-Leverage Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2 - \frac{1}{2\tau_L}\sigma_T^2\Lambda^2$$

Variance of the leveraged portfolio's total return:

$$\sigma_T^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} h_i q_{ij} h_j$$

where q_{ij} is the covariance between the total returns of securities i and j, and h_i is the holding weight of security i.

Λ^2 is the square of the portfolio's leverage.

Intuition: Costs of leverage are higher in more volatile portfolios.

Mean-Variance-Leverage Utility (cont'd)

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2 - \frac{1}{2\tau_L}\sigma_T^2\Lambda^2$$

τ_L is the investor's tolerance for leverage risk.

Mean-Variance-Leverage Utility (cont'd)

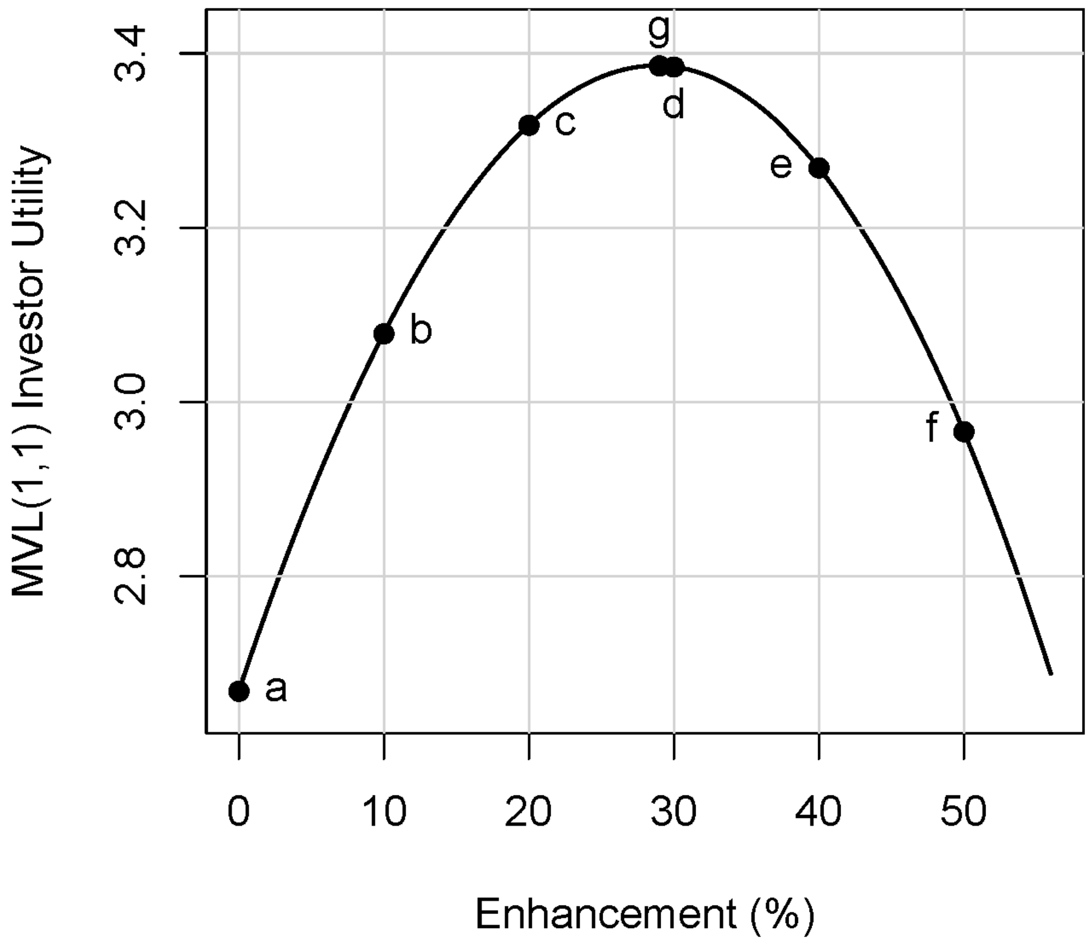
$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2 - \frac{1}{2\tau_L}\sigma_T^2\Lambda^2$$

MVL(τ_V , τ_L) Investors

MVL(τ_V , τ_L) Portfolios

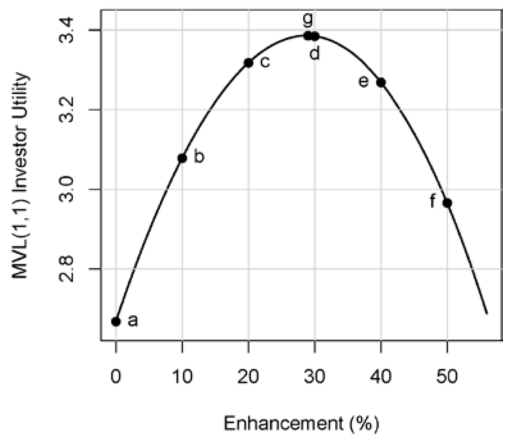
MVL(τ_V , τ_L) Utility

The Effect of Leverage Aversion: MVL(1,1) Utility of Optimal MV(1) Portfolios as a Function of Enhancement



Characteristics of Optimal MV(1) Portfolios from the Perspective of an MVL(1,1) Investor

Portfolio	EAE	Leverage	Standard Deviation of Active Return	Expected Active Return	Utility for an MVL(1,1) Investor
a	100-0	0.00	4.52	2.77	2.67
b	110-10	0.20	4.91	3.27	3.08
c	120-20	0.40	5.42	3.76	3.32
g	129-29	0.58	5.89	4.18	3.39
d	130-30	0.60	5.94	4.23	3.38
e	140-40	0.80	6.53	4.70	3.27
f	150-50	1.00	7.03	5.14	2.97



Mean-Variance Optimization as a Special Case of Mean-Variance-Leverage Optimization

$$U = \alpha_P - \frac{1}{2\tau_V}\sigma_P^2 - \frac{1}{2\tau_L}\sigma_T^2\Lambda^2$$

Special Case 1: Zero Leverage Tolerance

- MVL (τ_V, 0) reduces to MV (τ_V) with a leverage constraint of zero, resulting in a Long-Only Portfolio

Special Case 2: Infinite Leverage Tolerance

- MVL (τ_V, ∞) reduces to MV (τ_V) with no leverage constraint, resulting in a Highly Leveraged Portfolio

Mean-Variance Optimization and Mean-Variance-Leverage Optimization produce the same portfolio only in these two special cases

Solving the Mean-Variance-Leverage Optimization Problem

MV is a quadratic mathematical problem:

$$U = \sum_{i=1}^{N} \alpha_i x_i - \frac{1}{2\tau_V} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j$$

- Square of the active-weight variable x_i , including second-order cross-products
- Use quadratic solver

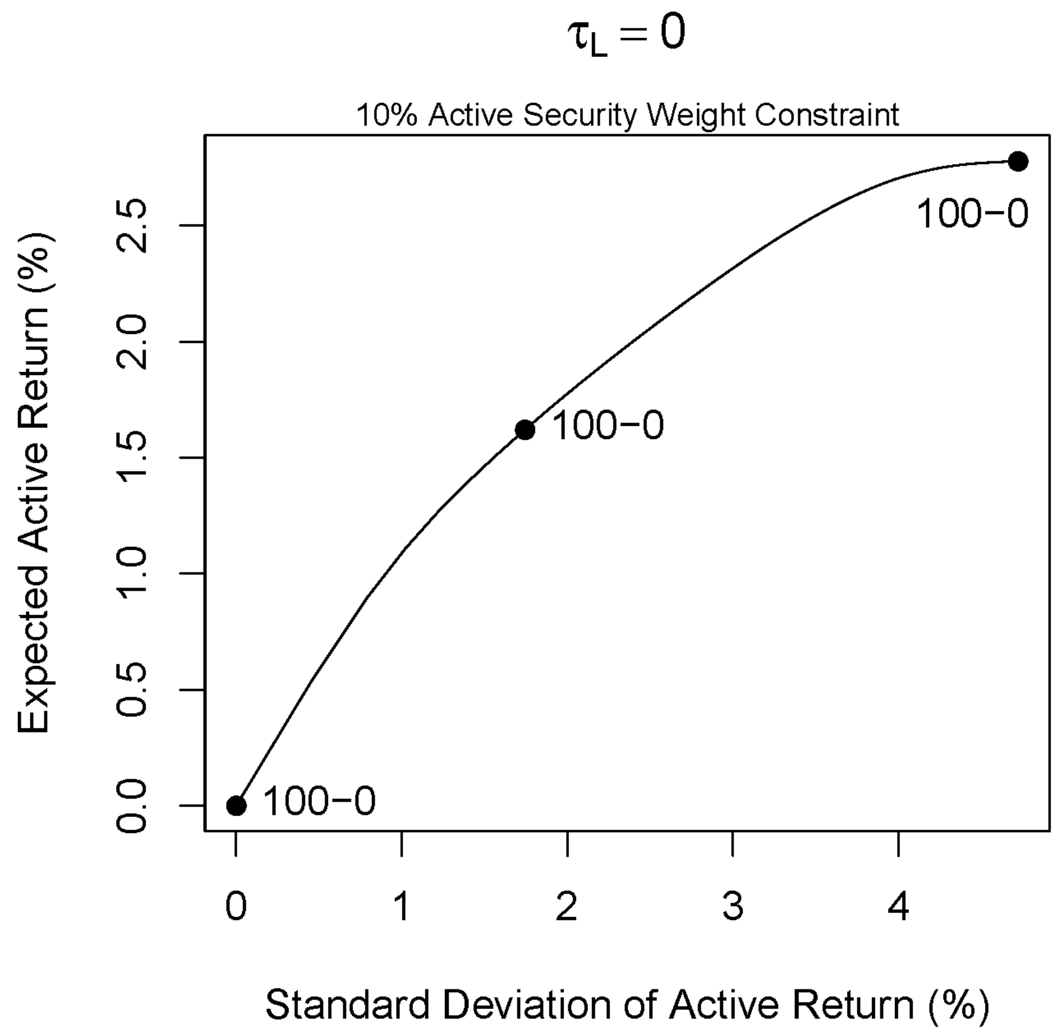
MVL is a quartic mathematical problem:

$$U = \sum_{i=1}^{N} \alpha_i x_i - \frac{1}{2\tau_V} \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \sigma_{ij} x_j - \frac{1}{2\tau_L} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} (b_i + x_i) q_{ij} (b_j + x_j) \right) \left(\sum_{i=1}^{N} |b_i + x_i| - 1 \right)^2$$

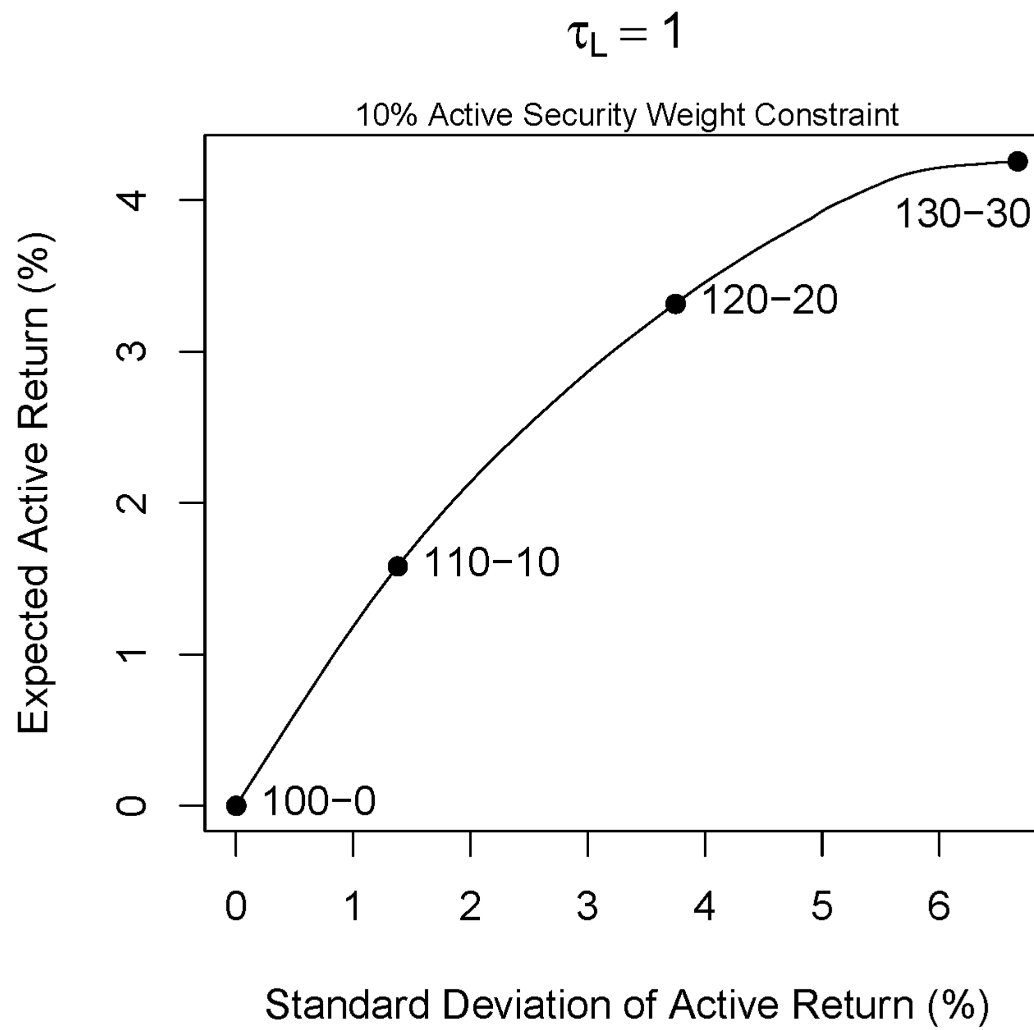
- Quartic of the active-weight variable x_i , including fourth-order cross-products
- Use Fixed-Point Iteration, which allows a quadratic solver to be applied iteratively

For details, see Jacobs and Levy (2013b,c)

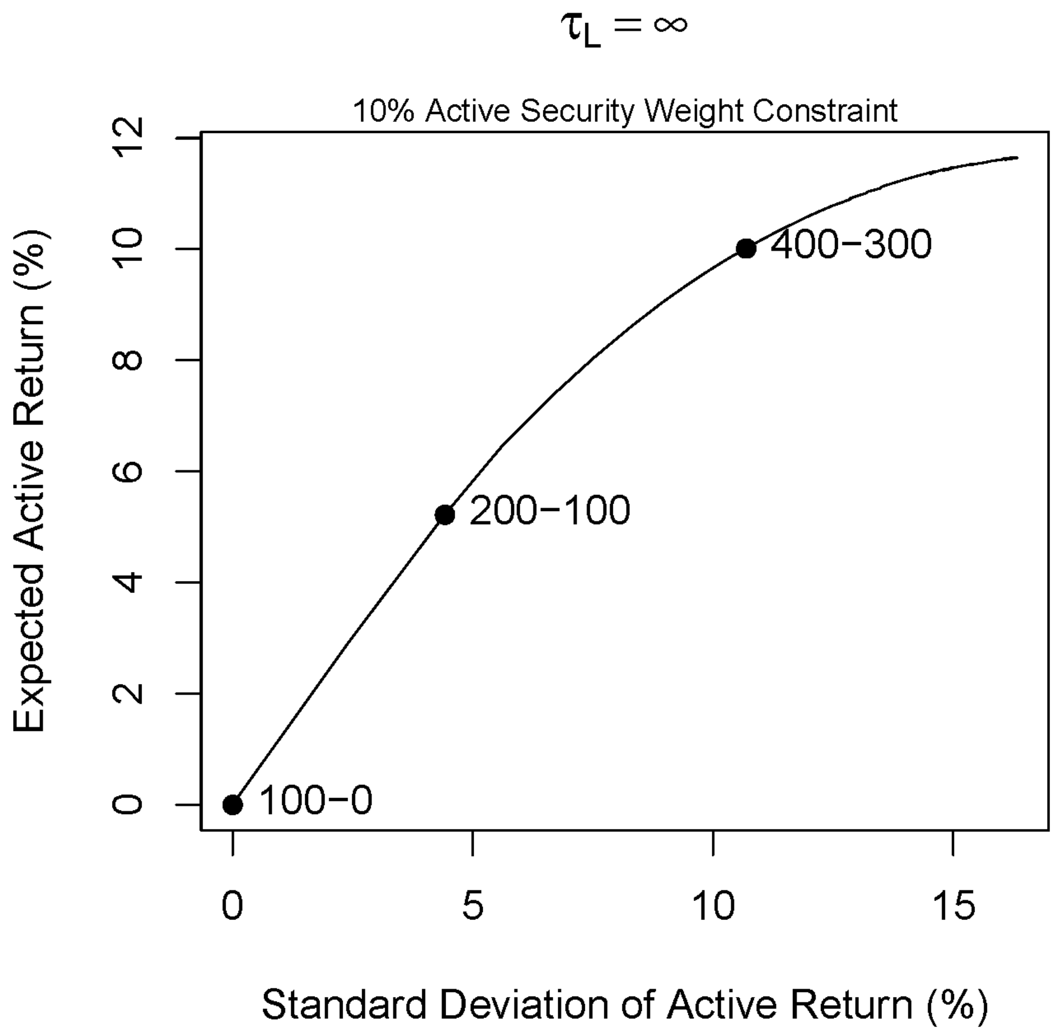
Optimal Leverage for Zero Leverage Tolerance



Optimal Leverage for Leverage Tolerance of 1

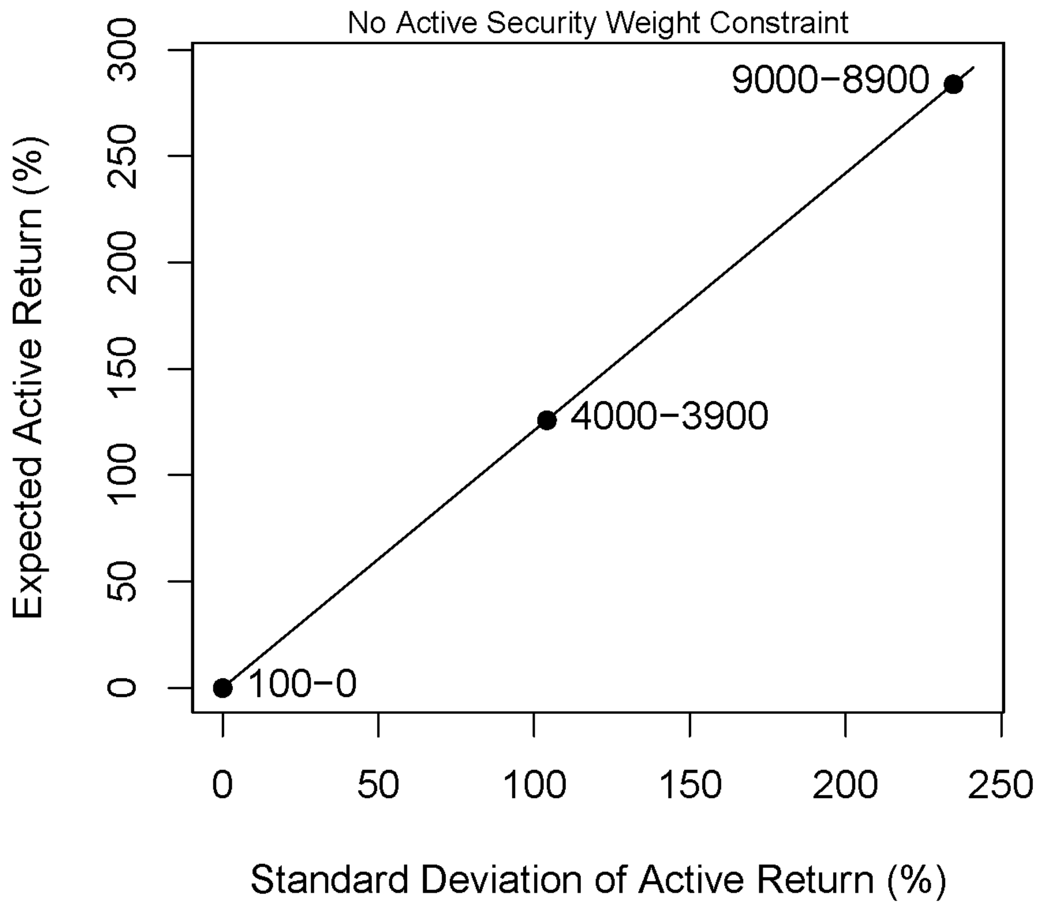


Optimal Leverage for Infinite Leverage Tolerance

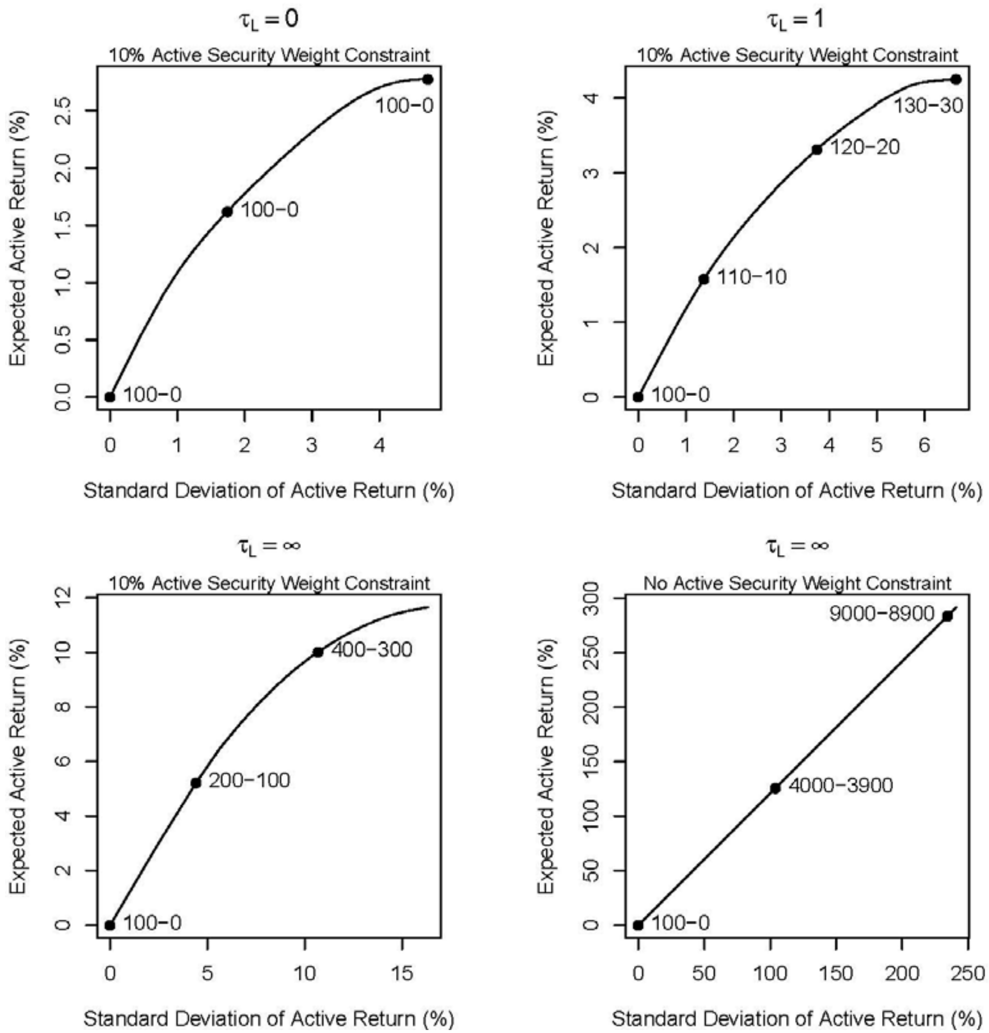


Optimal Leverage for Infinite Leverage Tolerance with No Active Security Weight Constraint

$\tau_L = \infty$

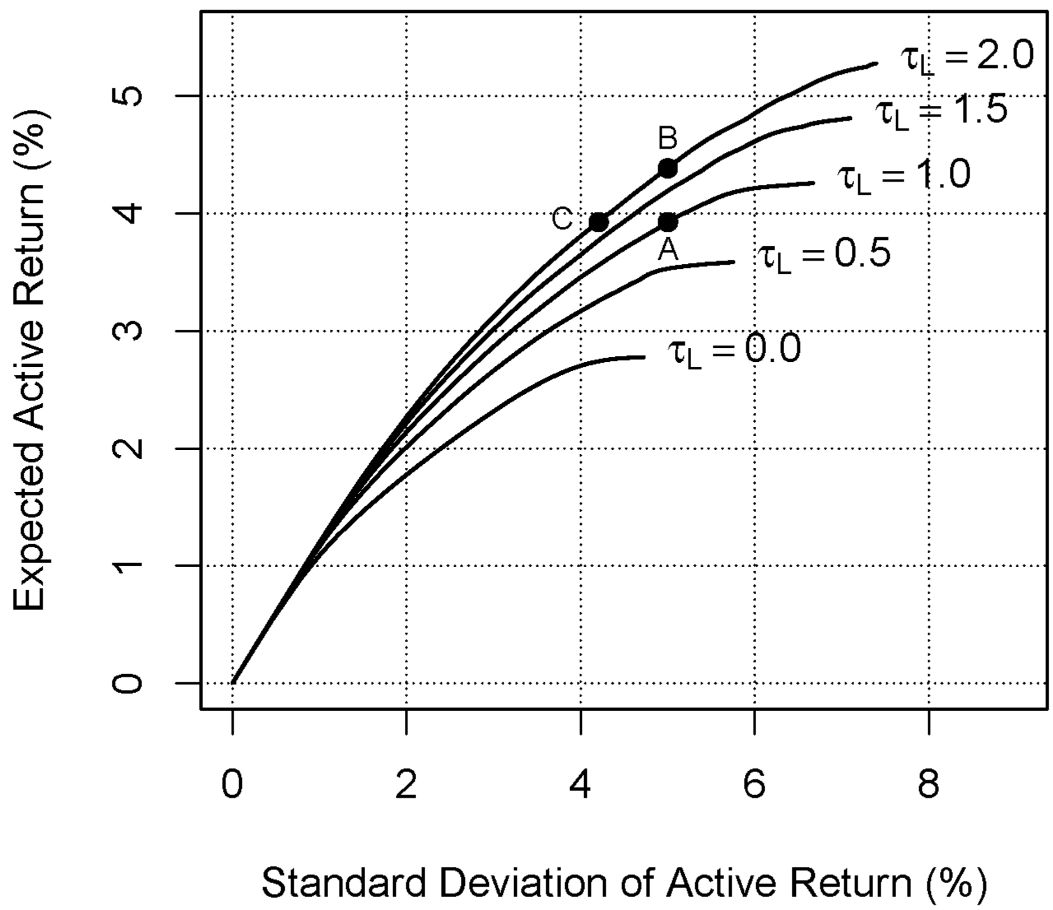


Optimal Leverage for Various Leverage-Tolerance Cases



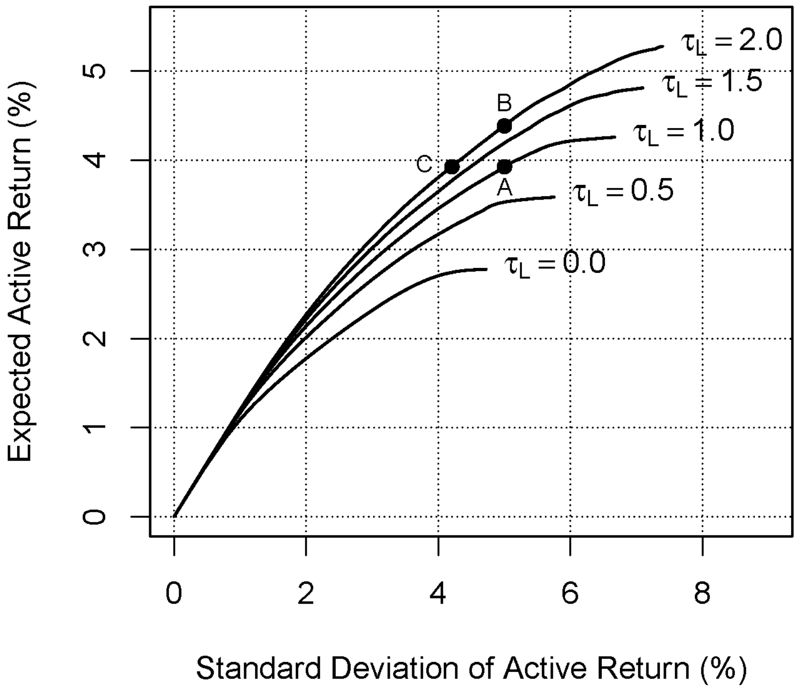
Wharton
University *of* Pennsylvania
JACOBS LEVY EQUITY MANAGEMENT CENTER
FOR QUANTITATIVE FINANCIAL RESEARCH

Mean-Variance-Leverage Efficient Frontiers for Various Leverage-Tolerance Cases (with the 10% Security Active Weight Constraint)

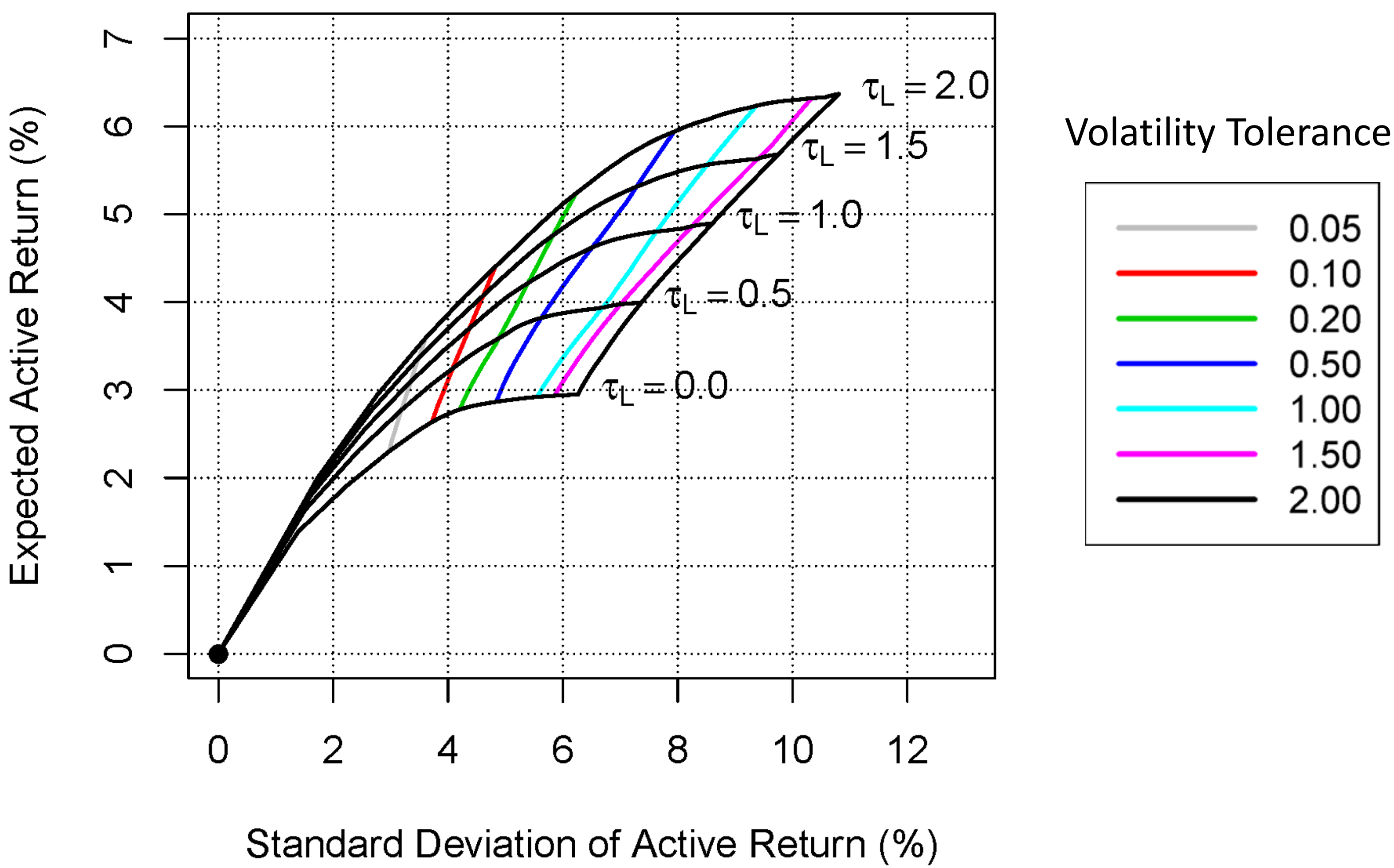


Characteristics of MVL (τ_V , τ_L) Portfolios A, B, and C

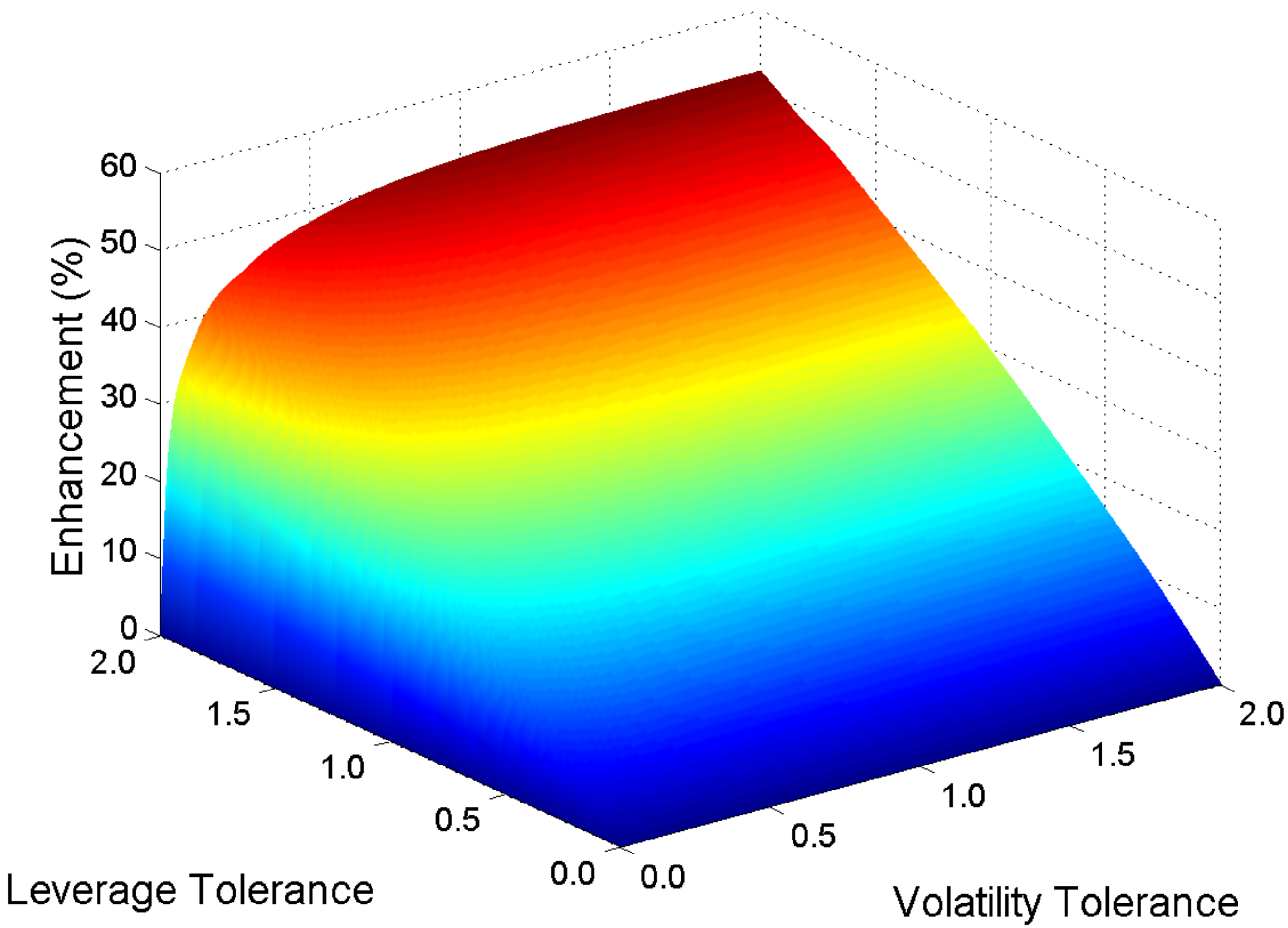
Portfolio	τ_L	τ_V	EAE	σ_P	α_P	U_A
A	1.00	0.24	125-25	5.00	3.93	2.93
B	2.00	0.14	139-39	5.00	4.39	2.72
C	2.00	0.09	135-35	4.21	3.93	2.68



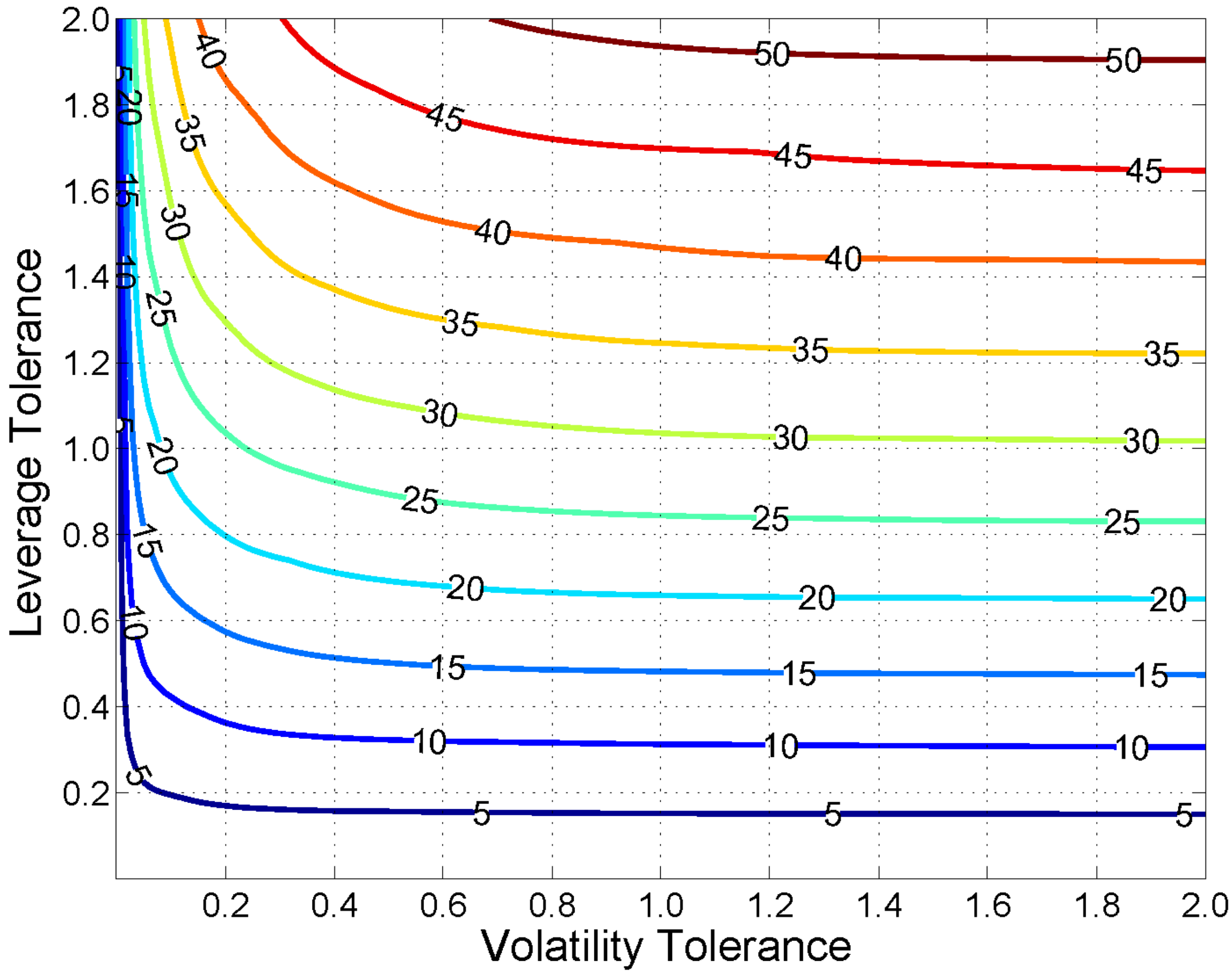
Mean-Variance-Leverage Efficient Region for Various Leverage and Volatility Tolerance Cases (with No Security Active Weight Constraint)



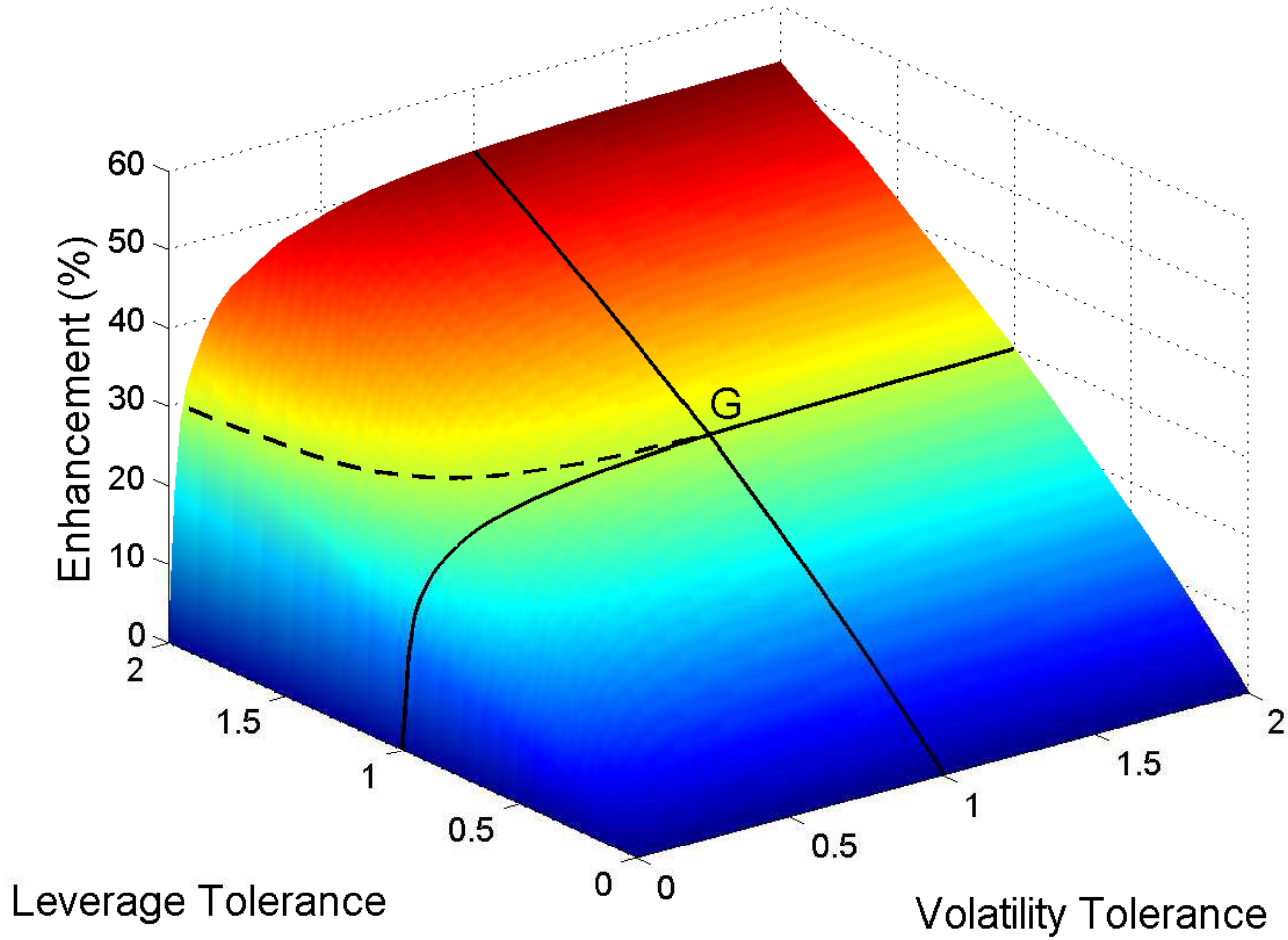
Mean-Variance-Leverage Optimal Enhancement Surface for Various Combinations of Volatility Tolerance and Leverage Tolerance



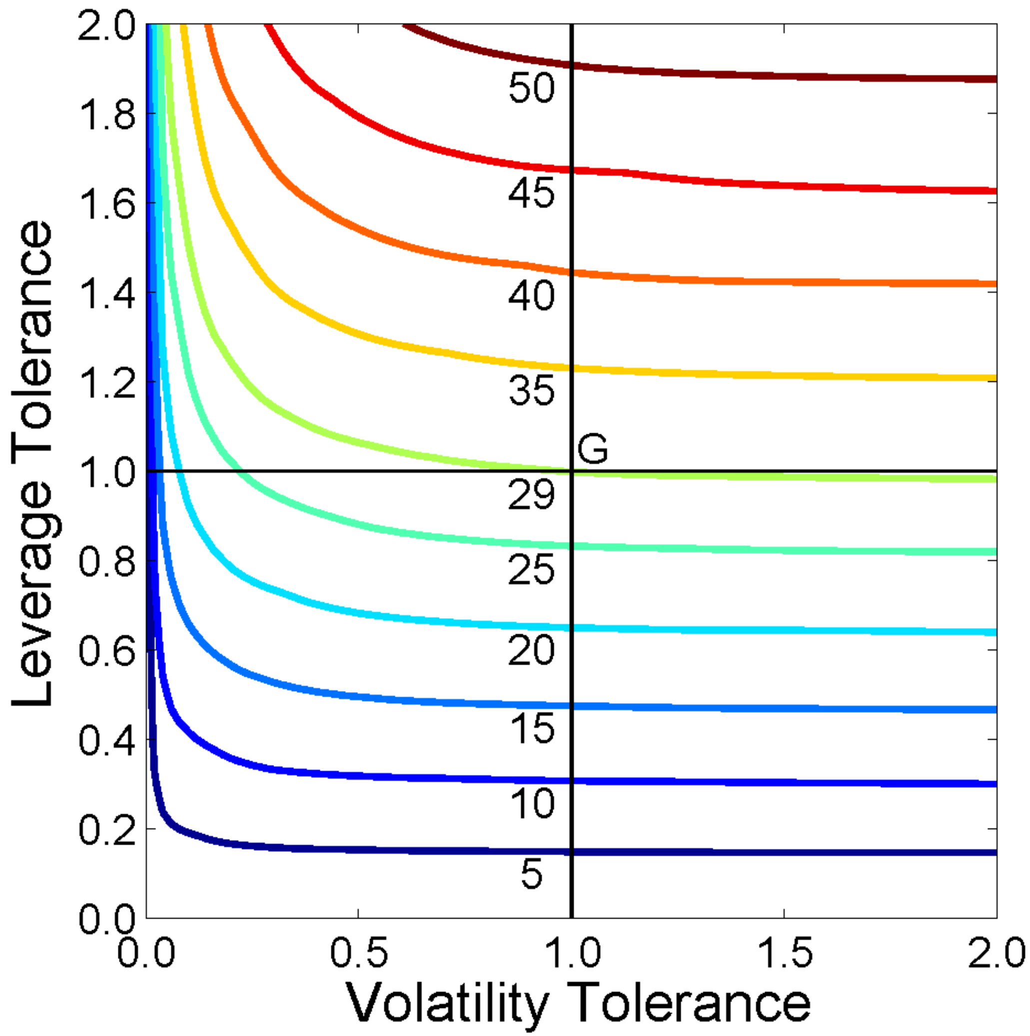
Contour Map of Mean-Variance-Leverage Optimal Enhancement for Various Combinations of Volatility Tolerance and Leverage Tolerance



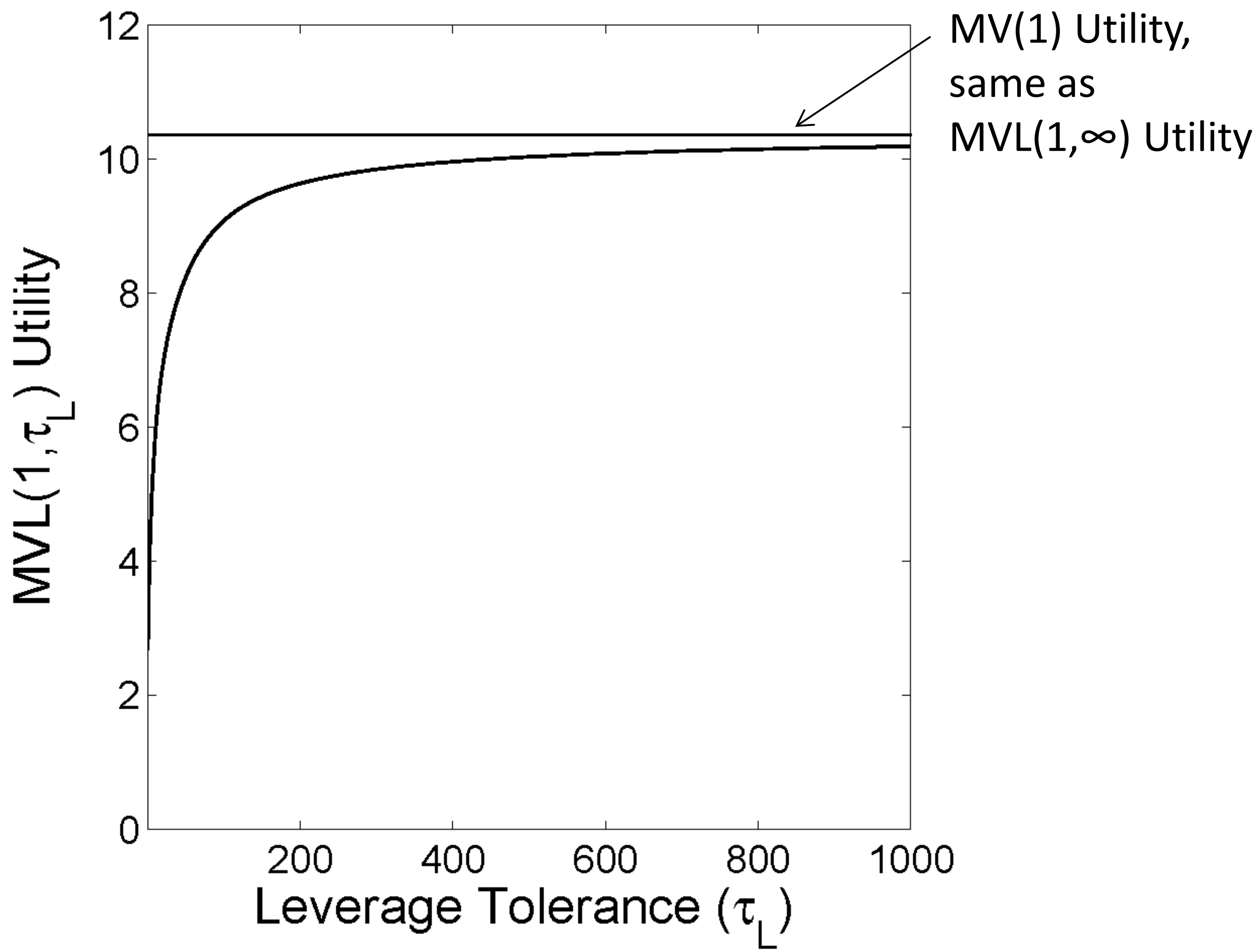
Locating the Optimal MVL(1,1) Portfolio on the Mean-Variance-Leverage Efficient Surface



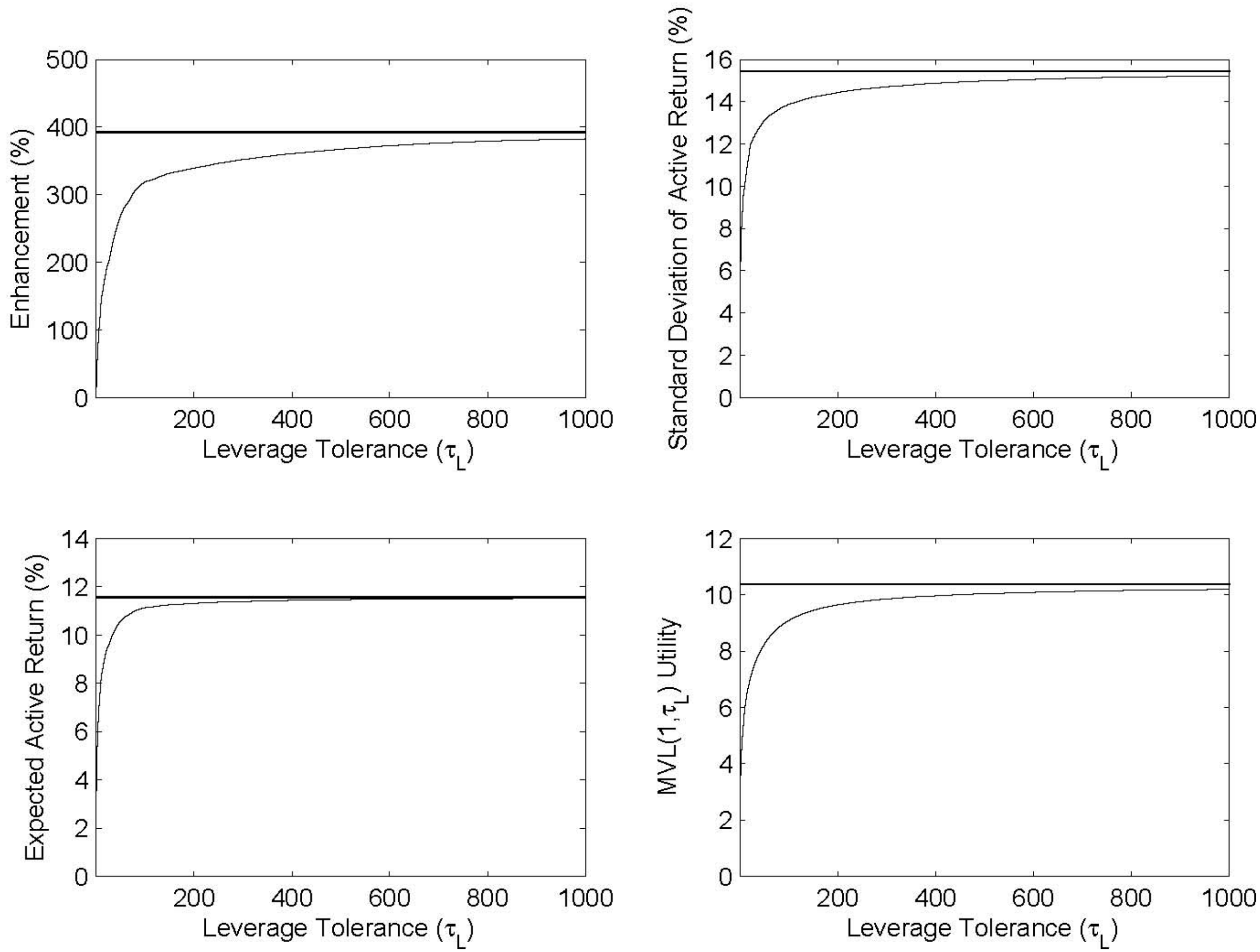
Locating the Optimal MVL(1,1) Portfolio on a Contour Map of the Mean-Variance-Leverage Efficient Surface



Utility of Optimal MVL(1, τ_L) Portfolios



Characteristics of Optimal MVL(1,τ_L) Portfolios



Conclusion

- Mean-Variance Optimization assumes either Zero Leverage Tolerance (long-only) or Infinite Leverage Tolerance

- Without a Leverage Constraint, Mean-Variance Optimization can result in Highly Leveraged Portfolios

- With a Leverage Constraint, Mean-Variance Optimization will lead to the Optimal Portfolio for a Leverage-Averse Investor only by Chance

- Two ways to identify the Optimal Portfolio for a Leverage-Averse Investor:
  - Consider numerous Leverage-Constrained Optimal Mean-Variance Portfolios and evaluate each one using the Investor's Mean-Variance-Leverage Utility Function
  - Use Mean-Variance-Leverage Optimization Directly

  Both Methods produce the Same Portfolio

Conclusion (cont'd)

- Both Volatility Tolerance (or Aversion) and Leverage Tolerance are Critical for Portfolio Selection
- Investors are willing to sacrifice some Expected Return in order to reduce Leverage Risk, just as they sacrifice some Expected Return in order to reduce Volatility Risk.
- Mean-Variance-Leverage Optimization balances Expected Return against Volatility Risk *and* Leverage Risk
- Leverage Aversion can have a large effect on an Investor's Portfolio Choice

Volatility and Leverage Polar Cases

Underlying Portfolio Volatility

Portfolio Leverage	Low	High
Low	Long-Only Index Fund	Enron Employee's Single-Stock Holding
High	LTCM's Leveraged Low-Risk Arbitrage Positions	CEO's Leveraged Chesapeake Energy Stock

Conventional portfolio theory says
not to hold all your eggs in one basket.

Using excessive leverage is like piling baskets of eggs on top
of one another until the pile becomes unsteady.

References

Burr, B. "Pair Sees MPT Flaw Over Risks of Leverage." *Pensions & Investments*, May 13, 2013.

Jacobs, B. and K. Levy. "Leverage Aversion and Portfolio Optimality." *Financial Analysts Journal*, Vol. 68, No. 5 (2012), pp. 89-94.

___________. "Introducing Leverage Aversion into Portfolio Theory and Practice." *The Journal of Portfolio Management*, Vol. 39, No. 2 (2013a), pp. 1-2.

___________. "Leverage Aversion, Efficient Frontiers, and the Efficient Region." *The Journal of Portfolio Management*, Vol. 39, No. 3 (2013b), pp. 54-64.

___________. "A Comparison of the Mean-Variance-Leverage Optimization Model and the Markowitz General Mean-Variance Portfolio Selection Model." forthcoming, *The Journal of Portfolio Management*, Vol. 39, No. 1 (2013c), pp. 1-4.

___________. "Traditional Optimization is Not Optimal for Leverage-Averse Investors." forthcoming, *The Journal of Portfolio Management*, Vol. 40, No. 2 (2014).

Markowitz H.M. "How to Represent Mark-to-Market Possibility with the General Portfolio Selection Model." *The Journal of Portfolio Management*, Vol. 39, No. 4 (2013), pp. 1-3.

Zweig, J. "Borrowing Against Yourself." *Wall Street Journal*, September 22, 2012.