Precision of Ratings

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Abstract

We analyze the equilibrium precision of ratings. Our results suggest that ratings become less precise as the share of uninformed investors and the gains from trade increase. The results provide an explanation for low accuracy of ABS ratings before the financial crisis. We apply the model to evaluate the effectiveness of the recent reform proposals, including Dodd-Frank Act. We show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

JEL codes: D82, D83, G01, G18, G24, G28, L15.

Keywords: credit rating agencies, ratings accuracy, differentially informed investors, information production and selling.

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1 Introduction

Credit rating agencies (CRAs) rate securities in various asset classes. The US Securities and Exchange Commission identifies five classes of ratings, (1) financial institutions, brokers and dealers; (2) insurance companies; (3) corporate issuers; (4) issuers of asset-backed securities; and (5) issuers of government, municipal or sovereign securities. These asset classes differ substantially in terms of the extent of information asymmetries between the issuers and investors. For example, the investors’ assessment of the credit risk of Brazil’s sovereign securities can be based on publicly available sources such as IMF statistics summarizing countries macroeconomic conditions. To the contrary, investors need access to non-public information and specialized expertise to assess the credit quality of a mid-sized industrial company located in Oklahoma, US or the credit risk of an ABS portfolio. As a result, the population of investors can be differentially informed about the assets quality. The asset classes also can differ in terms of the availability of investment opportunities in a particular asset class.

The performance of CRAs during the financial crises suggests that ratings’ performance varies across asset classes. Several empirical studies report that the ratings of asset-backed securities were uninformative and inflated to highest AAA rating. At the same time, the performance of ratings in corporate bond market, utilities and insurance sectors was stable, even during the times of the financial crisis.

The purpose of the paper is to analyze what determines precision of ratings. We argue that the CRA’s incentives to produce accurate ratings depend on the market conditions measured by gains from trade, the distribution of assets in the economy and the extent of the winner’s curse problem among the heterogeneously informed investors. We build a rational model that incorporates these factors and apply it to analyze the effect of the

\[1\] Ashcraft, Goldsmith-Pinkham, and Vickery find that subprime and Alt-A mortgage backed securities (MBS) experienced a significant decline in rating standards, and 80-95% of deals were assigned a AAA rating. Stanton and Wallace (2011) document that ratings of commercial mortgage-backed securities allowed for lower subordination levels that inflated the ratings. The size of the AAA tranche of collateralized debt obligation (CDO) deals was larger than suggested by the CRA’s rating model (Griffin and Tang, 2009), with low B+ credit quality of the collateral that supported CDO issues (Benmelech and Dlugosz, 2009). In August 2011, the US Justice Department started an investigation whether one of the major CRAs, Standard and Poor’s (S&P), improperly rated mortgage backed securities in the year prior to the financial crisis.

\[2\] According to Standard and Poor’s report on corporate default rates and rating transitions, during 2008-09 only 25 companies initially rated as investment grade were in default, and the number of investment grade defaults was at most one per year during the rest of the period of 2004-2011. The rate of speculative grade companies defaults peaked to 9.5% in 2009, which is comparable to 9.7% rate following the high-tech bubble in 2001.
recent CRAs reforms proposals, in particular, the Dodd-Frank Act, on the equilibrium precision of ratings.

We model a market with issuers, investors and a monopolistic CRA. Issuers are privately informed about the value of the asset and aim to sell the issue at the highest price. Issuers need a rating to signal the asset quality to investors. The CRA designs the rating system that is composed of the information technology and a rating fee. This set up follows the information intermediation literature led by Lizzeri (1999).

We introduce two novel features to the information intermediation literature. First, we assume that issuers’ gains from trade are type dependent. More precisely, we assume that gains from trade are increasing in issuer’s asset quality. It reflects the fact that issuers with higher quality assets have better outside opportunities. Also, as we discuss below, this assumption implies that CRA can be essential to avoid market break-down, and thus it increases market efficiency.

Second, we consider a setting with differentially informed investors. Then informed investors’ ability to capture the information rent depends on the amount of information about the underlying asset contained in ratings. It implies that precision of ratings affects the extend of the winner’s curse problem in the market.

The gains from trade and the presence of differentially informed investors is what distinguishes asset classes and varies through time. Thus the model can explain the heterogeneous performance of CRAs. Also it provides a general framework that can be applied to evaluate a variety of recent policy proposals on the reforms of CRAs in the US and the EU. We discuss the policy implications of reforms on standardization of ratings across different asset classes, regulation of rating fees, introducing expert liability for overrated securities and reducing the reliance on ratings in regulation.

We obtain five main results. The first result is that the CRA’s ratings are informative but noisy. It is driven by the fact that the profit of the CRA is a product of market penetration and the fee. In the extreme case when ratings are perfectly informative about asset values, the rating fee is determined by the willingness to pay of the lowest rated issuer. The CRA can increase this issuer’s willingness to pay by assigning it high ratings with a positive probability. However, in doing so the CRA is limited by the high quality issuers decision to trade. As ratings become less informative, high quality issuers prefer to hold the asset instead of selling it at a substantial discount. This result contrasts with Lizzeri (1999) where the ratings are completely uninformative. The trade off between increasing the willingness to pay of lower quality issuers and revealing enough information to induce participation of high quality issuers determines the precision of ratings.
Second, we provide several results about the optimal information structure of the CRA. We show that the information structure is asymmetric in a sense that rating precision varies across different rating grades within the same asset class. Also, under certain conditions, the information structure must entail rating inflation. That is, lower quality issuers must be assigned higher ratings with a positive probability, but higher quality issuers are always assigned high ratings. Otherwise, higher quality issuers can refuse to trade following a low rating, which reduces CRAs profits. Thus the CRA designs a rating system under which its “mistake” is always optimistic.

The third set of results is about the precision of ratings in the market with differentially informed investors. When all investors are uninformed, gains from trade are captured by the CRA. In the presence of the winner’s curse problem, some gains from trade are captured as information rent by informed investors. The CRA can reduce the profits of informed investors by making ratings more informative. However, more informative ratings also reduce the ability of the CRA to extract the gains from trade. We show that as the pool of investors becomes less informed about asset values, the CRA reduces the precision of ratings. Also we show that the winner’s curse problem makes rating inflation feature more likely to prevail. Furthermore, when the winner’s curse problem becomes substantial, the CRA reduces the market coverage to the best quality issuers. In this case, unrated issuers do not trade and it leads to inefficiency.

The fourth result is that precision of ratings depends on the market conditions. When an economy is in boom and issuers face profitable investment opportunities, they are willing to accept a higher discount to sell the asset. It gives the CRA a possibility to extract more surplus by making ratings less informative. Thus as the aggregate gains from trade in the economy increase, the precision of ratings is compromised.

Finally, we show that the precision of ratings depends on the distribution of asset values in the economy. As the high quality assets become more scarce, the precision of ratings decreases. The reason is that rating lower quality assets becomes a more important source of CRAs profits, and these issuers willingness to pay is increasing as ratings become less informative.

We apply the model to evaluate the recent reform proposals of the credit rating industry. Following disappointing performance of ratings of asset backed securities, regulators both in the US and in the EU developed an array of policies that aim to improve incentives of CRAs to produce accurate ratings. We discuss the Dodd-Frank proposals on ratings standardization, introducing expert liability and reliance on ratings in regulation, as well as proposals on regulation of rating fees. Our analysis suggests that some of these policies

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can be detrimental to market efficiency. In particular, we show that standardization of ratings across asset classes and introducing expert liability can reduce the precision of ratings and limit trade. To the contrary, reducing reliance on ratings in regulation and regulating rating fees can increase market efficiency.

The rest of the paper is organized as follows. The next section reviews related literature. Section 3 describes the model. Section 4 presents an example that highlights the key trade-off driving the properties of the optimal information structure. Section 5 derives several properties of the information structure when all investors are uninformed. Section 6 extends the analysis to the general case of differentially informed investors. Evaluation of policy proposals on the CRA reform is contained in Section 7, and the conclusion follows. All proofs are devoted to the Appendix.

2 Related literature

Our paper is related to Admati and Pfleiderer (1986, 1988 and 1990) who analyzed both direct and indirect information sale by a monopolistic information producer. In a direct information sale, they consider both the sale of identical information to all clients and the sale of personalized information, that is, noisy versions of the fundamental information to each trader. In an indirect sale informational monopolist trades on its private information and sells shares in its fund. Admati and Pfleiderer characterize the optimal sale under different conditions. In our paper, the CRA does not trade on its own account and thus is involved only on direct sale. However, we consider an issuer-pays system in line with the common practice. In our setting the CRA provides public information whereas Admati and Pfleiderer’s direct information sale is considered as an investor-pays system in which clients who pay for the information receive signals that are not available to other market participants.\(^3\)

We build on a framework developed in Lizzeri (1999) that delivers several important results in the information intermediation literature. Lizzeri analyzes a model with a continuum of seller types, risk neutral buyers and no restrictions on the disclosure rules that the information intermediary can offer to sellers willing to pay for certification. The type of the seller denotes its quality and is equal to the valuation of a good by the buyer.

The optimal disclosure policy is derived from the following trade-off. The profit of an intermediary is a product of the market coverage and the fee charged for the certification

\(^3\)Allen (1990) also analyzes the role of information sellers in financial markets. The focus of this paper is on the ability of the informed seller to establish that his information is reliable.
services. As buyers’ certification decision is voluntary, a certification fee is determined by the willingness to pay of the lowest rated seller. If an intermediary discloses the type of this seller perfectly, the seller is willing to pay at most the difference between its type and the expected value of uncertified sellers with lower types. However, an intermediary can increase the willingness to pay of the lowest rated type by pooling it with higher types. As higher types have no means to signal their quality other than the intermediary’s certification, they have to accept more noisy certification. Hence, the optimal disclosure of an intermediary is to pool all types. It implies that ex-post the intermediary discloses no information except that a seller is better than the lowest type. Also it is able to extract all the surplus by charging the fee equal to the expected value in the market.

One of our important modelling innovation is that we introduce winner’s curse problem to Lizzeri’s framework by assuming that investors are heterogeneously informed. Also we depart from Lizzeri’s framework in that issuers of securities have type dependent gains from trade. Both of these features are crucial in a financial market. Incorporating them in the Lizzeri’s model permits to explain the effect of changing market conditions and characteristics of an asset class on the precision of ratings. Furthermore, our model provides a unified framework that allows to analyze the consequences of an array of policy proposals on market efficiency in a single tractable model.

An important difference between Lizzeri (1999) and our model is that in our framework CRA creates surplus and helps to restore the market inefficiency. In Lizzeri’s context, information intermediation results in a pure transfer of surplus from sellers to an intermediary. The key friction in our model that makes the CRA’s services value-enhancing is the “lemon’s problem” of Akerlof (1970). As issuers prefer to hold the asset when the market price is low, their decision to trade is endogenous and depends on the rating technology of the CRA.

Our paper is closely related to recent theoretical literature that explores the incentive problems of the credit rating agencies leading to poor performance of ratings during the financial crisis. Mathis, McAndrews and Rochet (2009) and Bolton, Freixas and Shapiro (2012) show that asset complexity in the environment with naïve investors can be detrimental to CRA’s incentives. In Mathis, McAndrews and Rochet (2009), reputation is sufficient to discipline CRAs only when a large fraction of their income comes from rating simple assets. Bolton, Freixas and Shapiro (2012) build a model where a CRA may overstate the seller’s quality when there are more naïve investors. Skreta and Veldkamp (2008) study how higher complexity of rated assets affects incentives for ratings shopping and rating inflation. Their model is based on the assumption that investors cannot correct
for ratings selection bias. Then ability of sellers to obtain ratings from different CRAs and to decide which ratings to disclose leads to ratings shopping and inflation. In our framework investors can be differentially informed, but their decisions are rational, and the CRA is strategic in designing its rating technology. Thus our results are not driven by investors’ ignorance or naïveté.\footnote{In practice, sophisticated investors such as investment banks are large market makers in security markets who act on both the buy and the sell side of the market. It is implausible that these institutions are unaware about the security structure, and thus ignorance cannot fully explain the abandon evidence of rating inflation in structured finance markets documented in Benmelech and Dlugosz (2009), Coval, Jurek and Stafford (2009) and Stanton and Wallace (2010).}

The reliance on ratings in regulation and regulatory arbitrage are other factors that can potentially explain poor ratings performance. White (2010) documents that the role of ratings in prudential regulation of financial institutions has been increasing over the years. As a result, the CRAs’ compensation has been shifting from producing credit risk analysis to issuing regulatory licences. Opp, Opp and Harris (2012) develop a model where investors value highly rated bonds due to regulatory benefits. They show that rating-based regulations lead to rating inflation. But it has an ambiguous effect on CRA’s information production which depends on the distribution of firms. Analyzing a general information technology\footnote{In this respect our paper is also related to mechanism design literature on optimal information structures in auctions, Bergemann and Pesendorfer (2007).} allows us to reconsider whether rating inflation is driven only by rating-based regulation. We find that inflation can occur even in the absence of rating-based regulation, though the use of ratings in regulation makes inflation more pronounced and ratings less informative.

Lack of disclosure can also reduce the quality of ratings. Sangiorgi and Spatt (2012) show that lack of disclosure about the decision to solicit ratings by issuers leads to rating biases and excessive number of ratings per issuer. Pagano and Volpin (2012) analyze the effect of information transparency on the primary and secondary market liquidity. In the presence of unsophisticated investors who are unable to process disclosed information, transparency harms primary market liquidity. The reason is that transparency of information generates the winner’s curse problem between the sophisticated and unsophisticated investors. Kurlat and Veldkamp (2012) analyze the pros and cons of mandating the information disclosure in a general equilibrium framework.
3 Model

There are three groups of agents: issuers, investors and a CRA. An issuer owns an asset that is worth \( v \in V = \{v_1, v_2, v_3\} \) to investors, where \( 0 = v_1 < v_2 < v_3 \). The prior distribution of \( v \) is \( \lambda = (\lambda_1, \lambda_2, \lambda_3) \), where \( \lambda_i = \Pr(v_i) > 0 \) and \( \Sigma_i \lambda_i = 1 \). Issuers are privately informed about \( v \). The reservation value of an asset to an issuer type \( v \) is \( \delta v \), with \( \delta < 1 \). The potential gain from trade, \( v - \delta v \), can be due to several motives. It can come from the difference in discount factors between the two groups when it is more costly for the issuer to hold the asset to maturity. Then the investors’ discount factor is 1 and the issuers’ discount factor is \( \delta \) over the holding period.\(^6\) Another related motive is that an issuer has access to a positive NPV project and yet is capital constrained. Consequently, it needs to release capital to invest by selling the current assets. Then an issuer may be willing to sell at a discount in order to take advantage of the new investment opportunity. In this case, the difference of valuation, \( (1 - \delta) v \), is equivalent to the magnitude of the value creation in the new project. Third, in case of debt issuance, the gains from trade \( v - \delta v \) is the difference between the project’s expected cash flow and the capital costs.

Investors consist of two groups, informed and uninformed. Uninformed investors are purely competitive and represent a group large enough to buy the entire issue. Uninformed investors know that there is a possibility that there are some informed investors who observe the value of the asset \( v \) prior to subscribing to the issue. Demand of informed investors is not sufficient to absorb the entire issue. All investors demand a fixed amount of the issue as long as the expected value is higher than the price. Uninformed investors face a winner’s curse problem. They are more likely to obtain a larger allotment when informed investors decide not to subscribe to an issue. The rationing rule between the two groups of investors is summarized by the probability \( q \) that the uninformed investor’s demand for an underpriced security is fulfilled. Furthermore, without loss of generality we impose a normalization such that the uninformed investors’ demand is fulfilled with probability one if only uninformed investors demand. The probability \( q \) measures the severity of the winner’s curse problem. If all investors are uninformed and there is no winner’s curse, then \( q = 1 \). As \( q \) decreases, the extent of the winner’s curse problem increases. This approach builds on Rock (1986).

The CRA has an information technology to evaluate the value of the asset, but it cannot trade the asset. The signal space is denoted by \( S = \{s_1, \ldots, s_M\} \), \( M \in \mathbb{N} \). An information structure \( I \) is given by a pair \( (S, F(v, s)) \), where \( F(v, s) \) is the joint probability

\(^6\)This interpretation follows DeMarzo and Duffie (1999) and Dewatripont and Tirole (1995).
distribution over the set of asset values $V$ and the set of signals $S$. The joint probability distribution is defined in a usual way,

$$F(v, s) = \Pr(\bar{v} \leq v, \bar{s} \leq s),$$

with $f(v_j, s_i) = \Pr(v = v_j, s = s_i)$. For $F$ to be part of the information structure requires that the marginal distribution with respect to $v$ to be equal to the prior distribution of $v$, $\sum_i f(v_j, s_i) = \lambda_j$. Let $\mathcal{I}$ denote the set of information structures that satisfy this condition. For a given set of signals $S$, the precision of a signal $s_i$ on type $v_j$ is defined by

$$p_{ij} = \Pr(s_i | v_j) = \frac{f(v_j, s_i)}{\sum_i f(v_j, s_i)}.$$  \hspace{1cm} (1)

The CRA can choose any information structure. The cost of every information structure to the CRA is equal to zero. The CRA commits to reveal the signal realization to investors.\(^7\)

We consider an issuer-paid rating model in which the CRA charges a flat fee $\phi \geq 0$ to an issuer soliciting a rating. It is consistent with the information intermediation literature where the CRA has flexibility to change the quality of its “product”, that is, the signal, but is restricted from price discrimination across different types.\(^8\) Also it is the relevant setting to analyze the current credit ratings market which is dominated by issuer-paid ratings.

The profile $(I, \phi)$ defines the rating technology of the CRA. The choice of the rating technology is common knowledge among issuers and investors.

The information structure permits a very rich set of rating systems. For example, it is perfectly informative if $M \geq 3$ and $p_{ij} = 1$ if $i = j$ and $p_{ij} = 0$ otherwise. It is uninformative if for some $s_i \in S$, $p_{ij} = 1$ for all $j$, and $p_{ij} = 0$ otherwise. A rating system with rating grades can be represented with an information structure where a subset of types $V_i \subset V$ is assigned the same signal $s_i$, $p_{ij} = 1$ for all $v_j \in V_i$ and $p_{ij} = 0$ otherwise. A noisy rating system is a system where the same type can be assigned different signals, $p_{ij} < 1$ for all $i, j$.

\(^7\)As discussed in Lizzeri (1999), the properties of the equilibrium are driven by the ability of the CRA to fine-tune the information structure rather than the commitment assumption. Thus we follow the literature and assume full commitment of the CRA to the information structure.

\(^8\)In practice, NRSRO requirements and the principles issued by the International Organization of Securities Commissions (IOSCO) prohibit ratings fees to be contingent on assigned ratings. In an alternative model where the CRA is allowed to charge different fees to different assigned signals, an equilibrium information structure is to charge each issuer the fee equal to gains of trade, and to perfectly disclose the issuer type to investors. Clearly, this outcome is uninteresting.
The structure of the game is common knowledge to issuers, informed and uninformed investors and the CRA. The timing of the game is as follows.

$t = 0$. The nature chooses the issuer’s type according to the prior distribution $\lambda$. Issuers privately learn their types $v \in V$. The rating agency commits to the rating technology $(I, \phi)$. The rating technology $(I, \phi)$ is observed by the issuers and the investors.

$t = 1$. Issuers decide whether to solicit a rating from the CRA. Issuers soliciting a rating pay a fee $\phi$. Informed investors learn the value of the asset for each issuer $v$ and the CRA learns a signal $s$ for issuers who solicited a rating. The CRA announces the ratings of rated issuers.

$t = 2$. Issuers set the price of subscription $b$.

$t = 3$. Investors who have observed whether the issuer is rated and the assigned rating at $t = 1$, decide whether to subscribe to an issue. The demand of informed and uninformed investors is fulfilled according to the rationing rule summarized above.

The strategy for the CRA is the information structure $I$ and a fee $\phi$. A behavioral strategy for the issuer is a pair of functions $d : V \times I \times \mathcal{R}_+ \rightarrow [0, 1]$ that maps the issuer’s type $v$ and the rating technology $(I, \phi)$ into the probability to solicit a rating $d$, and $b : V \times I \times \mathcal{R}_+ \times S \rightarrow \mathcal{R}_+$ that maps the issuer’s type $v$, the rating technology $(I, \phi)$ and the realization of the signal $s$ into the price of subscription $b$. A strategy of the investor is a decision to subscribe to an issue given the information available at $t = 3$. For informed investors, the subscription decision is a function $\beta_I : V \times \mathcal{R}_+ \rightarrow \{0, 1\}$ that maps the issuer’s type $v$ and the price of subscription $b$ into the decision to subscribe (1) or not (0). For uninformed investors, the subscription decision is a function $\beta_U : I \times \mathcal{R}_+ \times \{0, 1\} \times S \times \mathcal{R}_+ \rightarrow \{0, 1\}$ that maps the rating technology $(I, \phi)$, the decision of an issuer to get rated (1) or not (0), the realization of the signal $s$ and the price of the subscription $b$ into the decision to subscribe (1) or not (0). We use Perfect Bayesian equilibrium concept.

The model shares the basic framework of Lizzeri (1999). It departs from Lizzeri’s model in two important dimensions. The first difference is related to the value of the asset to issuers (sellers). In Lizzeri’s model, the issuer’s value for the asset is equal to zero for all issuer types. Assuming that the issuers’ reservation value $v$ is proportional to their types allows to capture an important feature of the financial market that issuers with highly desired assets often have better outside opportunities. In the context of Lizzeri’s model, $\delta = 0$. The issuers’ outside option also introduces a “lemon’s problem” to the market. If the market price is lower than the outside option, an issuer will hold the asset and the gains from trade will not be realized. Then the CRA’s rating technology affects
the issuers’ decision to trade and, hence, the market surplus.

The second difference is related to the information available to investors (buyers). Lizzeri assumes that all investors are uninformed which implies that competitive investors do not capture any surplus. Therefore, the total surplus captured by issuers and the CRA does not depend on the information produced by the CRA. As we will show below, in the market with differentially informed investors the CRA’s choice of the information structure affects the size of this surplus as informed investors capture informational rent. The amount of information available to investors is represented by the probability $q$ that uninformed investors’ demand for underpriced issue is fulfilled. As $q$ gets smaller, the severity of the winner’s curse problem increases. In the Lizzeri’s model all investors are uninformed which corresponds to the case $q = 1$.

There are three technical differences between our model and Lizzeri (1999) that do not affect the qualitative results but make the model tractable. Lizzeri considers a continuum of types $v$ on a bounded interval while we restrict attention to discrete finite types. The restriction simplifies the equilibrium analysis of the market with differentially informed investors. Also we model the information technology of the CRA as an information structure while Lizzeri considers a general set of disclosure policies. Given that no cost is imposed on the choice of the information structure or the disclosure policy, the two approaches are equivalent. Finally, we assume that issuers are setting the price while in Lizzeri’s model competing buyers (investors in our setting) are bidding for the asset. Again, our assumption simplifies the analysis of differentially informed investors.\footnote{Note that in Lizzeri’s model all investors are uninformed and competition between uninformed investors leads to a unique equilibrium. Lizzeri (1999) shows that uninformed investors make zero expected profit in equilibrium. If we make the same assumption in our benchmark model in which all investors are uniformed (i.e., $q = 1$), then the investors make zero expected profit in equilibrium as well. However, given that we later allow differentially informed investors, bidding by investors creates complexity that distracts the focus of our analysis. The trade-off of having the informed issuer to set the price is that it leads to additional equilibria which is common in signalling games. More specifically, uninformed investors making zero expected profit is not the unique equilibrium outcome. We discuss the equilibrium selection at the end of this section.}

For a given information structure $I$, consider the decision of investors to subscribe to an issue at time $t = 3$. Let $\gamma_{ij} = \Pr(v_j|I, s_i, q)$ denote the beliefs of uninformed investors that an issuer rated $s_i \in S$ under the rating system $I$ is type $v_j$, conditional on an issue offer. Also denote $s_0$ the event that an issuer is not rated, and $\gamma_{0j} = \Pr(v_j|I, s_0)$ the corresponding beliefs of uninformed investors conditional on the issue offer. The uninformed investors’ assessment of the asset value of an issuer rated $s_i$, $i = 1, \ldots, M$ or
not rated $s_0$, $i = 0$, under rating system $I$ is

$$U_i = \sum_j \gamma_{ij} v_j.$$  

Uninformed investors decide to subscribe to an issue rated $s_i$ if the price of subscription $b_i$ does not exceed their assessment of the asset value,

$$b_i \leq U_i.$$  

At time $t = 2$, the issuer type $v_j$ is better off selling the issue rated $s_i$ as long as the price is higher than the issuer’s asset value,

$$b_i \geq \delta v_j.$$  

It means that there are gains from trade for issuer type $v_j$ when

$$U_i \geq \delta v_j.$$  

Otherwise, the issuer holds the asset, for example, by setting the subscription price equal to $\bar{v}$, where $\bar{v} > v_3$.

Condition (2) defines the set of issuers $T_i \subset V$ that are willing to trade after obtaining a rating $s_i$ under the rating system $I$,

$$T_i = \{ v_j | U_i \geq \delta v_j \}.$$  

If competitive uninformed investors make zero expected profit in equilibrium, issuers $T_i$ optimally set the price

$$b_i = U_i.$$  

At this price, the uninformed investors break even. If the issue is underpriced, informed investors gain a positive rent equal to the difference between the asset value and the price, $v_j - U_i$.

At time $t = 1$, if an issuer type $v_j$ solicits a rating, it is assigned a rating $s_i$ with probability $p_{ij}$. Given the rating, at stage $t = 2$ the issuer can either charge the price $U_i$ or hold the asset and realize the value $\delta v_j$. Thus issuer’s expected value of a rating is

$$R_j = \sum_i p_{ij} \max\{U_i, \delta v_j\}.$$  

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If an issuer does not solicit a rating, it can either sell the issue unrated at price $U_0$ or hold the asset. Then denote $R_0$ the payoff of unrated issuer equal to $\max\{U_0, \delta v_j\}$. Given a rating technology $(I, \phi)$, denote $d_j \in \{0, 1\}$ the decision of type $v_j$ to solicit a rating,

$$d_j = \begin{cases} 
1 & \text{if } R_j - \phi - R_0 \geq 0, \\
0 & \text{otherwise}.
\end{cases}$$

At stage $t = 0$, the CRA chooses a rating technology $(I, \phi)$ that maximizes its expected profit,

$$\Pi(I, \phi) = \sum_j \lambda_j d_j \phi.$$ 

The main focus of the analysis is to characterize how the choice of the information structure $I$ depends on the extend of the winner’s curse problem and the value of issuing a security. We restrict attention to equilibria in which uninformed investors make zero profit.\(^{10}\)

### 4 Example

We start with a motivating example to illustrate the trade-offs leading to the results presented in the subsequent sections. Consider a market with a set of issuers with $v_1 = 0$, $v_2 = 4$ and $v_3 = 8$, a prior distribution $\lambda = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, uninformed investors only, $q = 1$, and $\delta = \frac{3}{4}$. Under these assumptions, the expected value of the asset $E[v]$ is equal to 4, and the ex-ante market surplus $(1 - \delta)E[v]$ is equal to 1. Also we restrict the signal space to three signals, $S = \{s_1, s_2, s_3\}$.

In this market, the CRA is essential to realize the gains from trade. Indeed, if there is no CRA and buyers trade under the prior distribution, participation of all sellers results in market price of an asset equal to the expected value $E[v] = 4$. However, it is not sufficiently high to induce participation of issuers type $v_3$ as they obtain a higher payoff by holding an asset, $4 < \delta v_3 = 6$. Similarly, if only issuers’ types $v_2$ and $v_1$ trade, the market price of 2 is lower than the outside option of type $v_2$, $2 < \delta v_2 = 3$. As a result, in equilibrium with no CRA the gains from trade are not realized.

\(^{10}\)Note that the assumption that the issuer sets a price leads to multiple equilibrium outcomes. In particular, there are additional equilibria in which the uninformed investors make strictly positive profits. These equilibria are supported by off-the-equilibrium-path beliefs that assign a sufficiently high probability on the worse types if the issuer deviates and offer a price higher than the equilibrium price. As it is well-known, these type of equilibria are not robust to some of the stronger refinements concepts such as Perfect Sequential Equilibrium (Grossman and Perry, 1986). Proof is available from authors upon request.
Next we discuss the CRA’s choice of the information structure. We start with a benchmark of perfectly informative ratings. A perfectly revealing information structure can be represented as

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<tr>
<td>$s_3$</td>
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<td>$s_2$</td>
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where each element $p_{ij}$ is the precision of signal $s_i$ on type $v_j$, $p_{ij} = \Pr(s_i | v_j)$. It is perfectly revealing in a sense that a signal $s_i$ perfectly identifies an issuer type $v_i$. As a result, the investors’ assessment of the securities rated $s_3$, $s_2$, and $s_1$ equals to the true asset values, $U_3 = 8$, $U_2 = 4$ and $U_1 = 0$, respectively. Also issuer’s expected value of a rating is equal to its type, $R_j = v_j$. Then the issuers’ willingness to pay for a rating is $R_j - \delta v_j = v_j - \delta v_j$, where

$$R_3 - \delta v_3 = 2, \ R_2 - \delta v_2 = 1, \ R_1 - \delta v_1 = 0.$$  

Given the information structure, the CRA needs to select a rating fee. Charging a lower rating fee increases the issuers’ demand for ratings. In fact, the issuer type $v_1$ demands a rating only if its cost is zero, resulting in zero profits for CRA. Clearly, the CRA can do better by charging a positive fee and excluding participation of the lowest type $v_1$. It is easy to see that setting a fee $\phi = 1$ induces types $v_3$ and $v_2$ to solicit a rating and yields the profits of $(\frac{1}{3} + \frac{1}{3})1 = \frac{2}{3}$. Under $\phi = 1$ issuer type $v_3$ gain a positive rent, leading to expected issuer surplus of $\frac{1}{3}(2 - 1) = \frac{1}{3}$. Importantly, all gains from trade are realized and the market surplus is maximized, $\frac{2}{3} + \frac{1}{3} = 1$. Thus under the perfectly revealing information structure, the CRA can rate two types of issuers\(^{11}\) and the surplus is split between the CRA and the highest type issuers $v_3$.

Can the CRA design an information structure that gains higher profits than the perfectly revealing information structure profits of $\frac{2}{3}$? The answer is yes. Under perfectly revealing information structure, the CRA does not extract full surplus because rated issuers types $v_2$ and $v_3$ have different willingness to pay for rating. Then the CRA is constrained to charge the fee equal to the lowest willingness to pay, in this case of issuers’ type $v_2$. If the CRA can design an information structure that leads to the same willingness to pay by rated types, charging the fee equal to the willingness to pay will allow the CRA

\(^{11}\)Note that the CRA can achieve the same profit $\frac{2}{3}$ by setting a fee $\phi = 2$. Then only type $v_3$ solicits a rating and trades. Type $v_2$ does not solicit a rating as $R_2 - \delta v_2 = 1 < \phi = 2$, and does not trade as the price of unrated asset $2$ is lower than its outside option $\delta v_2 = 3$. Thus this solution has lower market surplus of $\frac{2}{3}$ and is Pareto dominated.
to extract the market surplus. Below is an example of a noisy information structure that achieves this objective.

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The highest issuer type $v_3$ is assigned a signal $s_3$ with probability one. The medium type $v_2$ is assigned a signal $s_2$ with probability $\frac{6}{7}$ and a signal $v_3$ with probability $\frac{1}{7}$. As a result, a signal $s_2$ reveals to investors that an issuer type is $v_2$, while a signal $s_3$ can be associated with two types of issuers, $v_2$ and $v_3$.

The noisy information structure leads to the following investors’ assessment of the asset value conditional on the rating,

$$U_3 = \Pr(v_3 | s_3) \cdot 8 + \Pr(v_2 | s_3) \cdot 4 = \frac{\frac{1}{3} \cdot \frac{1}{7}}{1 + \frac{1}{3} \cdot \frac{1}{7}} \cdot 8 + \frac{\frac{1}{3} \cdot \frac{1}{7}}{1 + \frac{1}{3} \cdot \frac{1}{7}} \cdot 4 = \frac{15}{2} < v_3 = 8,$$

$$U_2 = \Pr(v_3 | s_2) \cdot 8 + \Pr(v_2 | s_2) \cdot 4 = 0 \cdot 8 + 1 \cdot 4 = 4 = v_2.$$  

As a result, the issuers’ willingness to pay for the rating,

$$R_j - \delta v_j = \Pr(s_3 | v_j)U_3 + \Pr(s_2 | v_j)U_2,$$

is equalized between the two rated types,

$$R_3 - \delta v_3 = 1 \cdot U_3 - \frac{3}{4} \cdot 8 = \frac{3}{2} \text{ and } R_2 - \delta v_2 = \frac{1}{7}U_3 + \frac{6}{7}U_2 - \frac{3}{4} \cdot 4 = \frac{3}{2}.  $$

The noisy information structure equates issuers willingness to pay by increasing the medium type $v_2$ value of the rating due to a possibility of an optimistic mistake $s_3$ and reducing the high type $v_3$ value of the rating due to making the highest rating signal $s_3$ noisy. Then the CRA rates types $v_2$ and $v_3$, charges $\phi = \frac{3}{2}$ and extracts the market surplus, $(\frac{1}{3} + \frac{1}{3})\frac{3}{2} = 1$. An interesting feature of the noisy information structure is that it can be interpreted as rating inflation. When the CRA assigns a signal $s_3$ to type $v_2$, the reported signal is always optimistic in a sense that the signal $s_3$ is also assigned to a higher type $v_3$ whereas $s_2$ is not to assigned to type $v_3$.\(^{12}\) Another interesting feature of the noisy rating system is that the precision of ratings is asymmetric for different issuer

\(^{12}\)Note that assigning $s_2$ to type $v_3$ may lead type $v_3$ to withdraw since $\delta v_3$ may be strictly greater than $U_2$.  

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types. While the highest type $v_3$ and the lowest type $v_1$ are assigned the corresponding signals $s_3$ and $s_1$ surely, the CRA chooses to be less precise in learning the medium issuers type $v_2$.

The example illustrates that the CRA can strategically choose noisy ratings, even when the precision of ratings has no costs. The analysis of a general model in the following sections builds on this economic intuition.

## 5 CRA’s choice of noisy ratings

The example of the previous section illustrates a general idea that the CRA benefits most when the information structure leads to the equal willingness to pay among the rated issuers. Otherwise, the CRA can increase the profits by fine-tuning the information structure and changing the issuers’ expected values of a rating $R_i$ and $R_j$. Then a necessary condition of an optimal information structure can be written as

$$R_i - \delta v_i = R_j - \delta v_j = \phi \text{ for all } i, j.$$  

In this section we develop properties of the CRA’s choice of noisy ratings driven by its incentives to equalize willingness to pay among the rated issuers. In order to isolate this effect from the winner’s curse problem that arises when investors are heterogeneously informed, this section focuses on the case of uninformed investors, $q = 1$. The following section extends the analysis to a general case of differentially informed investors, $q < 1$.

We first characterize an equilibrium information structure. Then we discuss its distinctive properties, and analyze whether these properties persist in every equilibrium information structure.

Consider an information structure with three signals, $S_3 = \{s_1, s_2, s_3\}$. Given $S_3$, the CRA chooses the probability distribution on the set of signals and types $S_3 \times V$ which can be represented in terms of $p_{ij} = \Pr(s_i | v_j)$,

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with $0 \leq p_{ij} \leq 1$ and $\Sigma_i p_{ij} = 1$. Then there exists an equilibrium with the following properties.
Proposition 1 There exists an equilibrium in which CRA extracts all gains from trade. The CRA designs an information structure with three signals $S_3$ and the probability distribution

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where $p_{22} = \frac{\delta(\lambda_3+\lambda_2)}{\lambda_3+\delta\lambda_2} < 1$, and charges a positive rating fee $\phi = (1-\delta)\frac{\lambda_3v_3+\lambda_2v_2}{\lambda_3+\lambda_2}$. At $t = 1$, issuers types $v_3$ and $v_2$ solicit a rating, while issuers type $v_1$ do not solicit a rating. At $t = 2$, conditional on rating $s_3$, issuers types $v_3$ and $v_2$ set the same price $b_3 = U_3 = \frac{\lambda_3v_3+\lambda_2v_2+\delta\lambda_2(v_3-v_2)}{\lambda_3+\lambda_2} > \delta v_3$; conditional on rating $s_2$, issuers type $v_2$ set the price $b_2 = U_2 = v_2$. At $t = 3$, all rated issuers' offers are fully subscribed and traded.

The equilibrium has several interesting features. First, as discussed earlier, the information structure induces equal willingness to pay for ratings among rated issuers,

$$R_3 - \delta v_3 = U_3 - \delta v_3 = \phi,$$

$$R_2 - \delta v_2 = p_{22}U_2 + (1-p_{22})U_3 - \delta v_2 = \phi.$$

As a result, the CRA extracts the market surplus by charging the fee equal to the issuers' willingness to pay, $(\lambda_2 + \lambda_3)\phi = (1-\delta)E[v]$. In particular, the ability to extract the market surplus implies that restricting the set of signals to three signals $S_3$ is without loss of generality.

Second, ratings are informative but noisy. While rating $s_2$ reveals the issuer type $v_2$, rating $s_3$ can be assigned to two types, $v_3$ and $v_2$, and leaves investors uncertain about the type. From the perspective of issuers, while type $v_3$ is certain to be rated $s_3$, type $v_2$ can be rated $s_2$ and $s_3$ with probabilities $p_{22}$ and $1-p_{22}$, respectively. Then the probability $p_{22}$ can be interpreted as rating precision, as higher values of $p_{22}$ make rating $s_3$ a more precise signal about type $v_3$.

Noisy ratings imply that the equilibrium security prices divert from assets' fundamental values. Issuers type $v_3$ are certain to sell the issue at price $U_3$. However, these issuers sell their security at a discount, $U_3 < v_3$. The reason is that investors rationally anticipate that an asset rated $s_3$ can have value $v_2$,

$$U_3 = \Pr(v_3|s_3)v_3 + \Pr(v_2|s_3)v_2 = \frac{\lambda_3v_3 + (1-p_{22})\lambda_2v_2}{\lambda_3 + (1-p_{22})\lambda_2}.$$ 

At the same time, issuers type $v_2$ rated $s_3$ sell the issue at a price above the fundamental value, $U_3 > v_2$. Rating $s_2$ perfectly reveals the issuer type $v_2$ and leads to price $U_2 = v_2$. 

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The third distinctive feature of the equilibrium information structure is that it entails rating inflation. Indeed, the intermediate type \( v_2 \) can be rated either \( s_2 \) or \( s_3 \), effectively pooling with type \( v_3 \) with probability \( 1 - p_{22} \). Thus the CRA intentionally makes an optimistic mistake in rating issuers \( v_2 \). The benefit of rating inflation is that it adjusts issuers’ willingness to pay compared to the situation of perfectly informative ratings. Pooling of types \( v_2 \) and \( v_3 \) increases the willingness to pay of the intermediate type \( v_2 \) and decreases the willingness to pay of the highest type \( v_3 \).

Is rating inflation necessary for the CRA to extract all surplus? The answer depends on the value of issuer’s outside option. In equilibrium, an information structure needs to satisfy both ex-ante and interim participation constraints for the set of rated issuers. The ex-ante constraint at \( t = 1 \) states that the expected value of a rating is at least as high as the value of an outside option. The interim constraint at time \( t = 2 \) requires that rated issuers are willing to trade after the rating has been assigned. Under rating inflation structure of Proposition 1, trade is an optimal continuation strategy for both types of issuers. Type \( v_3 \) is certain to be rated \( s_3 \) and sell a security at price \( U_3 > \delta v_3 \). Type \( v_2 \) is rated \( s_2 \) or \( s_3 \) and sells at prices \( U_2 \) or \( U_3 \) which are higher than no trade alternative payoff of \( \delta v_2 \). Given the equilibrium security prices, the interim participation constraints are satisfied for any value of an outside option \( \delta \).

Intuitively, if the outside option of the issuers \( \delta \) is low, the interim constraint can be satisfied under other information structures that permit CRA to gain the market surplus. Consider an example of an information structure with rating deflation where type \( v_3 \) receives a noisy signal,

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<td>( s_2 )</td>
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with \( z = \frac{\delta (\lambda_3 + \lambda_2)}{\delta \lambda_3 + \lambda_2} \). It exhibits rating deflation in a sense that the CRA makes a pessimistic mistake as it assigns a rating \( s_2 \) to type \( v_3 \) with a probability \( 1 - z \). Under this information structure, the equilibrium security prices are

\[
\hat{U}_3 = v_3 \text{ and } \hat{U}_2 = \frac{(1 - \delta)\lambda_3}{\lambda_3 + \lambda_2} v_3 + \frac{\delta \lambda_3 + \lambda_2}{\lambda_3 + \lambda_2} v_2.
\]

If both types of issuers choose to trade following the rating assignment, rating deflation induces equal willingness to pay among the rated issuers and yields market surplus to the CRA. However, as the outside option of issuers \( \delta \) increases, type \( v_3 \) assigned a lower rating \( s_2 \) can refuse to trade at price \( \hat{U}_2 \). It occurs when the price is not sufficiently high to
make trade attractive, \( \hat{U}_2 < \delta v_3 \). From the perspective of the CRA, interim withdrawal of issuers type \( v_3 \) from the market is inefficient as it reduces all issuers willingness to pay for a rating. At the same time, trade is always interim optimal under rating inflation. The next proposition provides general conditions under which rating inflation is a necessary property of an equilibrium rating system.

**Proposition 2** Restrict attention to the set of equilibria in which only types \( v_2 \) and \( v_3 \) solicit a rating. Then for \( \delta > \bar{\delta} = \frac{\lambda_2 v_2 + \lambda_3 v_3}{\lambda_2 v_3 + \lambda_3 (2v_3 - v_2)} \) the equilibrium information structure must entail rating inflation.

The fourth feature of the equilibrium information structure is the effect of the issuers’ outside option \( \delta \) on ratings precision. Lower value of outside option \( \delta \) means that the issuer is more eager to sell the asset. As we discussed earlier, one interpretation of a lower value of outside option corresponds to the situation when an economy is in a boom. Then the relationship between the rating precision and the value of the outside option explains how the quality of CRA’s information depends on the economic cycle. Another interpretation of \( \delta \) is the value of investment opportunities in an asset class. Then the effect of outside option on ratings precision explains the heterogeneity of information precision between asset-backed securities and more traditional asset classes like corporate bonds.

**Proposition 3** The precision of ratings declines as the opportunity costs of holding an asset increase, \( \frac{dp_{22}}{d\delta} > 0 \).

The outside option affects the informativeness of ratings due to the issuers endogenous participation decision. If ratings are uninformative, the investors’ posterior assessment of issuer’s type is close to the ex-ante average asset value. It implies that high value issuers type \( v_3 \) must sell an asset at a substantial discount. If the opportunity costs of holding an asset are very high, \( \delta \to 0 \), the issuers are willing to accept a discount. In this case ratings are uninformative, \( p_{22} \to 0 \) and both types of issuers \( v_3 \) and \( v_2 \) are assigned the same rating. However, as the opportunity costs of holding an asset decline and the outside option \( \delta v \) increases, the CRA has to increase rating precision. Otherwise, high quality issuers are better off holding the asset and realizing the value \( \delta v \) instead of accepting to sell at a discount. In the limit case \( \delta = 1 \), the CRA discloses all information.

The effect of issuers’ outside option on rating precision is closely related to Lizzeri’s (1999) striking result of uninformative ratings. In Lizzeri’s model, all issuers pay a positive fee to obtain uninformative ratings in order to distinguish themselves from the worse
Proposition 3 reveals that ratings are uninformative only when the issuers have zero value of an asset, \( \delta = 0 \). In a general case \( \delta > 0 \), ratings are informative but noisy.

If different values of \( \delta \) correspond to different stages of the business cycle, the result of Proposition 3 also shows that the precision of ratings must be procyclical. When the economy is in a boom, the opportunity costs of holding the assets is high due to attractive investment opportunities and \( \delta \) is low. Then issuers are eager to sell the issue, yielding high profits to the CRA. When the economy is bust and \( \delta \) is high, there are fewer investment opportunities, and the profits of the CRA are low.\(^{14}\)

Finally, the last feature of the equilibrium information structure of Proposition 1 regards the market coverage. The CRA rates two issuers’ types \( v_3 \) and \( v_2 \), while the lowest type \( v_1 \) is not rated. Clearly, reducing the market coverage to rating either issuers type \( v_3 \) or \( v_2 \) is not profitable. By following this strategy, the CRA forgoes the surplus that can be created by issuers type \( v_{-j} \), and achieve a profit of \((1 - \delta)\lambda_j v_j\) which is always inferior to the market surplus \((1 - \delta)E[v]\).

Are there equilibria in which the CRA rates all issuers? In general, the CRA cannot extract any surplus from the lowest type \( v_1 \) as it can signal its type at no cost by not soliciting a rating. It implies that participation of the lowest type must be driven by the ability to sell a rated asset at a price above the fundamental value, \( R_1 - \delta v_1 \geq v_1 - \delta v_1 \). Then extending the market coverage from two to three types must increase the willingness to pay of the lowest type \( v_1 \) at the same time as decreasing the willingness to pay of types \( v_2 \) and \( v_3 \). Note that the size of market surplus is fixed. Consequently, as long as trade remains an optimal continuation strategy of rated types, changing types’ willingness to pay does not change CRA’s profits. The following proposition explains how the CRA’s choice of the market coverage affects the expected value of a rating for different types of issuers. Let \( R_j^k \) denote type \( v_j \)’s expected value when \( k \) highest types are rated. Similarly, \( \Pi^k \) stands for CRA’s profits when \( k \) highest types are rated.

**Proposition 4** There exists an equilibrium in which all three types are rated. Issuers’ expected value of a rating

\[
R_j^3 = (1 - \delta)E[v] + \delta v_j, \quad j = 1, 2, 3,
\]

\(^{13}\)In Lizzeti (1999), ratings are uninformative in a sense that the posterior distribution of seller types conditional on the intermediary’s disclosure policy and the disclosed signal is identical to the prior distribution.

\(^{14}\)Bar-Isaac and Shapiro (2011) and Bar-Isaac and Shapiro (forthcoming) analyze a model where pericyclicity of ratings and CRA profits are driven by labor supply of analysts.
Increasing as the CRA’s rating system excludes participation of lower types.

When investors are uninformed, \( q = 1 \), the CRA is indifferent between rating all issuer types or rating the two highest types \( v_2 \) and \( v_3 \):

\[
\Pi^2 = \sum_{j=2,3} \lambda_j (R_j^2 - \delta v_j) = \sum_{j=1,2,3} \lambda_j (R_j^3 - \delta v_j) = \Pi^3
\]

\[
= (1 - \delta)E[v].
\]

Rating only one type, \( v_3 \) or \( v_2 \), is less profitable than rating all issuers or the two highest types, \( \Pi^1 < \Pi^k \) for \( k = 2, 3 \).

6 Noisy ratings and differentially informed investors

In the market with heterogeneously informed investors, precision of ratings affects the size of the surplus of the issuers, uninformed investors and the CRA. The reason is that noisy ratings allow informed investors to earn information rent. It implies that issuers must provide a winner’s curse discount to uninformed investors who realize that they can be offered an overpriced issue that is not demanded by informed investors. As a result, the winner’s curse problem decreases the surplus between the issuers, the uninformed investors and the CRA. The winner’s curse problem can be reduced by making ratings more informative. In fact, the CRA can eliminate the winner’s curse completely by designing perfectly informative ratings. However, as the analysis of the previous section shows, perfectly informative ratings cause differences in issuers’ willingness to pay, and

\[\text{See Fulghieri, Strobl and Xia (2011) for further discussion on unsolicited ratings.}\]
reduces the ability of the CRA to extract the surplus. In this section we analyze the trade-off between the two countervailing incentives of the CRA. We build on the results of the previous section and explore the effect of the winner’s curse problem on the optimal rating system of the CRA. We show that it may lead to no trade in some market segments and thus produce inefficiencies.

The start with the effect of winner’s curse problem on the market coverage.

**Proposition 5** In the market with differentially informed investors, any information structure that induces participation of the two highest types $v_2$ and $v_3$ is more profitable than the one that induces participation of all types. Consequently, there exists no equilibria in which uninformed investors break even and CRA rates all types.

The result is in contrast to the outcome in the absence of winners curse. It implies that in the market with differentially informed investors the CRA provides partial market coverage. The economic rational for the result is that including the lowest issuer type $v_1$ hardens the underpricing problem without increasing the market surplus that the CRA can obtain from issuers. It reduces the price the investors are willing to pay for a rated asset, and consequently, the issuers’ willingness to pay for a rating. Unlike in the case of uninformed investors $q = 1$, redistribution of surplus from higher types $v_3$ and $v_2$ to the lowest type $v_1$ is reducing the market surplus between the issuers, the uninformed investors and the CRA.

Winner’s curse problem can produce even further reduction in the market coverage. When ratings are noisy, limiting market coverage to the highest type issuers reduces the underpricing costs. Intuitively, this strategy is preferable when the winner’s curse problem is so severe that the CRA is better off to forgo the surplus created by the intermediate type issuers in order to avoid the underpricing of the highest type. Next we consider two possible market outcomes depending the decision of the CRA to design a rating system that targets either both types $v_3$ and $v_2$, or only the highest type $v_3$. Then we characterize how the optimal choice of the market coverage depends on the winner’s curse problem.

For the rest of the section, we restrict attention to information structures with three signals. As we show in Proposition 1, it is without loss of generality in the market with uninformed investors. Analyzing a rating system with three signals in the market with differentially informed investors permits to have a benchmark to evaluate the effect of asymmetric information on the market outcome. Thus we consider an information
structure

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Under this information structure, in line with the result of Proposition 5, rating $s_1$ reveals perfectly type $v_1$ and issuers $v_1$ do not solicit a rating.

Suppose that the CRA targets to rate types $v_3$ and $v_2$. Then the CRA designs the rating system described in the following proposition.

**Proposition 6** If the CRA targets to rate types $v_2$ and $v_3$, it commits to an information structure

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<tr>
<td>$s_1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

where $p_{22} = \frac{\delta(\lambda_3 q_3 + \lambda_2)}{\lambda_3 q + \lambda_2} < 1$. The CRA charges the rating fee $\phi = (1 - \delta)\frac{\lambda_3 q_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}$ and gains profits $\Pi^2 = (1 - \delta)\frac{(\lambda_2 + \lambda_3)(\lambda_3 q_3 + \lambda_2 v_2)}{\lambda_3 q + \lambda_2} < (1 - \delta)E[v]$. At $t = 1$, issuers types $v_3$ and $v_2$ solicit a rating, while issuers type $v_1$ do not solicit a rating. At $t = 2$, conditional on rating $s_3$, issuers types $v_3$ and $v_2$ set the same price $b_3 = U_3 = \frac{\lambda_3 q_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 q + \lambda_2} > \delta v_3$; conditional on rating $s_2$, issuers type $v_2$ set the price $b_2 = U_2 = v_2$. At $t = 3$, all rated issuers’ offers are fully subscribed and traded. Informed investors gain a positive rent equal to $\frac{\lambda_3 \lambda_2 (1 - q)(1 - \delta)(v_3 - v_2)}{\lambda_3 q + \lambda_2}$.

Like in the market with uninformed investors described in Proposition 1, ratings are noisy and exhibit rating inflation. However, there are several important new features that arise due to the winner’s curse problem. Noisy ratings lead to the additional winner’s curse discount that issuers type $v_3$ need to offer to uninformed investors. A total discount of type $v_3$ equals to

$$v_3 - U_3 = \Pr(v_2 \mid s_3)(v_3 - v_2) = \frac{\lambda_2 (1 - \delta)}{\lambda_3 q + \lambda_2} (v_3 - v_2).$$

It is increasing as the winner’s curse problem becomes more pronounced and $q$ decreases. Importantly, the part of the discount induced by the winner’s curse reduces the profits of the CRA, as it reduces the willingness to pay of both rated types. However, the discount is necessary to compensate the uninformed investors for the potential of buying an issue rejected by informed investors.
Another implication of the winner’s curse is that informed investors gain a positive informational rent. The decision to forgo some market surplus is optimal from the CRA’s perspective. It balances the ability to equalize the willingness to pay between the rated types, and hence the need to make ratings noisy, and the size of the winner’s curse discount offered to uninformed investors. However, as the winner’s curse problem becomes more severe, the size of the winner’s curse discount increases. In the limit case of \( q = 0 \), the CRA’s profit becomes \((1 - \delta)(\lambda_3 + \lambda_2)v_2\). It means that both assets \( v_3 \) and \( v_2 \) have to be sold at price of the intermediate asset \( v_2 \), with informed investors capturing the rest of surplus. An alternative strategy of the CRA to limit the rent of informed investors is to reduce the market coverage to issuers type \( v_3 \). Next we describe the rating system under which only \( v_3 \) assets are rated, and provide the conditions when this strategy is optimal.

Suppose that the CRA designs a rating system under which ratings are solicited by one type \( v_3 \), for example, by setting \( p_{ii} = 1 \). Then the rating perfectly reveals the issuer’s type. The issuer will set the price of subscription equal to the value of the asset, \( b = v_3 \). The issuers’ willingness to pay for such a rating depends on the best outside option between holding the asset \( \delta v_3 \) or possibly selling it without a rating and thus pooling with the two lower types, \( v_2 \) and \( v_1 \). The unrated assets \( v_1 \) and \( v_2 \) are traded in the market if \( \delta v_2 < \frac{\lambda_2 q v_2}{\lambda_2 q + \lambda_1} \). It implies that the highest fee that can be charged by the CRA is

\[
\phi = \begin{cases} 
\min\{(1 - \delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2 q + \lambda_1}\}, & \delta < \frac{\lambda q}{\lambda_2 q + \lambda_1}, \\
(1 - \delta)v_3, & \delta \geq \frac{\lambda q}{\lambda_2 q + \lambda_1}.
\end{cases}
\]

Under this rating system, types \( v_2 \) and \( v_1 \) do not solicit a rating. The next proposition summarizes the market outcome under limited market coverage of type \( v_3 \) issuers.

**Proposition 7** If the CRA targets issuers type \( v_3 \), it commits to an information structure

\[
\begin{array}{ccc}
v_3 & v_2 & v_1 \\
v_3 & 1 & 0 & 0 \\
v_2 & 0 & 1 & 0 \\
v_1 & 0 & 0 & 1 \\
\end{array}
\]

The CRA charges the rating fee \( \phi = \min\{(1 - \delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2 q + \lambda_1}\} \) and gains profits \( \Pi^3 = \lambda_3 \min\{(1 - \delta)v_3, v_3 - \frac{\lambda_2 q v_2}{\lambda_2 q + \lambda_1}\} \). At \( t = 1 \), issuers types \( v_3 \) solicit a rating, while issuers type \( v_2 \) and \( v_1 \) do not solicit a rating. At \( t = 2 \), conditional on rating \( s_3 \), issuers types \( v_3 \) set the price \( b_3 = U_3 = v_3 \). If \( \delta < \frac{\lambda q}{\lambda_2 q + \lambda_1} \), issuers \( v_2 \) and \( v_1 \) set the price \( b_{1,2} = \frac{\lambda_2 q v_2}{\lambda_2 q + \lambda_1} \); otherwise, these issuers set the price \( b_{1,2} = v_2 \). At \( t = 3 \), rated issuers’ offers are fully subscribed and traded; if \( \delta < \frac{\lambda q}{\lambda_2 q + \lambda_1} \), unrated types offers are fully subscribed and traded.
The CRA chooses the market coverage that provides the highest profit under a given set of market conditions.

**Proposition 8** There exists \( \bar{q} \in [0, 1] \) such that for all \( q \geq \bar{q} \) the optimal rating system induces two issuer types \( v_3 \) and \( v_2 \) to solicit a rating; and for all \( q < \bar{q} \) it induces only type \( v_3 \) to solicit a rating.

As the winner’s curse problem becomes more severe, the CRA decreases the market coverage. Under the market coverage with two types, the CRA’s optimal information structure aims to increase the payoff of issuer type \( v_2 \) by pooling it with type \( v_3 \). Presence of informed investors leads to severe underpricing of an issue with the highest rating \( s_3 \), which ultimately reduces the fee that the CRA can charge. As the winner’s curse problem becomes substantial, the CRA is better off eliminating the underpricing by restricting market coverage to the best issuer type, \( v_3 \).

The rating system with two rated types \( v_3 \) and \( v_3 \) involves rating “inflation” in the sense that while the highest type receives only the highest rating \( s_3 \), type \( v_2 \) can receive two ratings, \( s_2 \) and \( s_3 \). In other words, the CRA makes an optimistic “mistake” of assigning a higher rating \( s_3 \) to type \( v_2 \). As in the case of uninformed investors, the information structure described in Proposition 6 is may not be unique. An interesting question though is whether the rating inflation is more prevalent in markets with substantial winner’s curse problem. The next proposition shows that indeed this is the case.

**Proposition 9** Restrict attention to the set of equilibria in which only types \( v_2 \) and \( v_3 \) solicit a rating. Then for \( \delta > \bar{\delta} = \frac{\lambda_2 v_2 + q \lambda_3 v_3}{\lambda_2 v_3 + q \lambda_3 (2 v_3 - v_2)} \) equilibrium must entail rating inflation. As winner’s curse problem becomes more severe, \( q \) decreases, the set of market conditions for which rating inflation is necessary increases, \( \frac{d\delta}{dq} > 0 \).

As the winner’s curse problem increases, issuers’ \( v_3 \) have to offer a higher discount to uninformed investors. Then pooling two types in one rating leads to lower equilibrium price, which makes ex-post trade constraint of type \( v_3 \) harder to satisfy under rating deflation. It makes rating inflation necessary for a wider range of parameters. Thus markets with higher information heterogeneity among investors are more prone to exhibit rating inflation features.

When the CRA rates types \( v_2 \) and \( v_3 \), precision of ratings depends on the market conditions.
Proposition 10 In a market where issuer types \( v_3 \) and \( v_2 \) are rated and trade, \( q \in [\bar{q}, 1] \), the CRA reduces ratings precision

(i) as the share of uninformed investors increases (\( q \) increases), \( \frac{dp_{22}}{dq} < 0 \);
(ii) as the gains from trade increase (\( \delta \) decreases), \( \frac{dp_{22}}{d\delta} > 0 \);
(iii) as high quality assets become more scarce (\( \frac{\lambda_2}{\lambda_3} \) increases), \( \frac{dp_{22}}{d(\frac{\lambda_2}{\lambda_3})} < 0 \).

In the extreme case, when all investors are uninformed (\( q = 1 \)) and the issuers’ outside option is zero (\( \delta = 0 \)), ratings are uninformative, \( p_{22} = 0 \).

The comparative statics results suggest that under the conditions of booming economy, usually associated with high share of uninformed investors and high gains from trade, ratings are less informative. The results provide an explanation for the poor performance of ratings of asset backed securities. In the pre-crisis period, these assets had higher returns relative to other securities. Also the period coincided with rapid growth in several emerging economies that were eager to invest in ABS assets. Another result is that the precision of ratings depends on the distribution of investment opportunities. In an economy with a low share of highly valuable assets, a larger share of CRA’s revenue is driven by rating issuers type \( v_2 \). Then the CRA has strong incentives to increase these issuers’ willingness to pay by reducing the precision of ratings.

7 Policy implications

In this section, we apply our theory to evaluate the effect of recent CRA reform proposals on ratings precision and the market outcome. We discuss the proposals on standardization of rating symbols, regulation of the rating fees, expert liability and reducing the reliance on ratings in regulation.

7.1 Standardization of rating symbols

Major rating agencies use rating symbols to communicate the credit quality of issuers to investors. Usually CRAs employ a dozen of rating categories\(^{16}\) and distinguish between investment grade and non-investment grade securities. The common practice is that the same rating symbols are applied to different asset classes rated by the same CRA. The CRAs’ rating methodology documents emphasize that rating scales are designed to be comparable among different asset classes and time periods. However, the stark differences

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\(^{16}\)The total number of notches ranges among the nine NRSRO rating agencies from 19 notches for Fitch to 26 notches for DBRS. Moody’s and S&P use 21 and 22 notches, respectively.
of performance of ratings across asset classes during the recent crisis indicate, at least, that the comparability is imperfect. Cornaggia, Cornaggia and Hund (2012) undertake a comprehensive empirical analysis of corporate bonds, municipal and sovereign bonds and structured product ratings using a variety of risk measures, and find that rating scales differ for these asset classes.\footnote{Also see Packer and Tarashev (2011) for the analysis of bank ratings methodologies.} He, Quian and Strahan (2012) show that rating scales change over the business cycle.

The Dodd-Frank Act\footnote{Section 939(h)(1) of the Dodd-Frank Act.} requested the SEC to develop regulations to standardize the meaning of symbols for different asset classes, in particular, in terms of default probabilities and expected losses. The aim of the proposal is to eliminate the potential investors’ confusion about the credit risk of different asset classes. The European Union regulators followed a different approach and imposed the requirement that rating symbols of structured securities must have an additional “s” qualifier to distinguish this asset class.

The effect of rating symbols proposals can be evaluated within the scope of our model. As each asset class has distinct $\delta$ and $q$, our results imply that the CRA’s optimal rating scale varies across different assets. Suppose that the CRA is required to provide the same accuracy for both asset classes, or across different ratings within the same rating class. Effectively it means that the CRA is restricted to a given rating precision $\bar{p}_{22}$ for different asset classes but has flexibility to set rating fees. The policy has the following effect.

**Proposition 11** Imposing rating standardization across asset classes can decrease the market coverage and reduce market efficiency.

Rating standardization limits the CRA’s ability to fine tune the rating system. Given the required precision level $\bar{p}_{22}$, the CRA optimizes the profits by adjusting the fee. If the required level of precision in a particular rating class exceeds the optimal precision derived in Proposition 6, the CRA can choose to increase the fee so that only the highest quality issuers solicit a rating. This strategy can lead to no trade for unrated $v_2$ issuers and results in inefficiency.

The other rating standardization policy is to require CRA to provide the same precision for different ratings. In terms of our model, the CRA is required to set equal precision for the two types of issuers $v_2$ and $v_3$, $p_{22} = p_{33}$. Then the CRA’s adjustment to the policy can take one of the following forms. It can provide ratings of high precision, $p_{ii} = 1$ but reduce the market coverage to the highest quality issuers, leading to illiquidity for issuers type $v_2$. Alternatively, it can sell ratings with precision $p_{ii} < 1$ to both types. However,
it means that following a low rating, high quality issuers will refuse to trade. Thus it results in lower liquidity for high quality assets. Both outcomes reduce liquidity and lead to inefficiencies.

In the report to Congress on Rating Standardization study conducted by SEC in 2012, CRAs and market participants who submitted comments on the proposal did not favor standardization. Among other arguments, the parties suggest that the policy would lead to less diversity of rating opinions, reduce the quality of ratings, eliminate innovation and increase costs. Following the study, the SEC recommendation is not to pursue the policy and instead focus on increasing transparency of rating methodologies.

7.2 Regulation of rating fees

Rating agencies receive compensation for rating services from the issuers of securities or the parties participating in marketing the securities. Fee schedules are communicated to issuers prior to the issuance of a rating. The precise fee amounts are determined by various factors including the assets class of the rated security and the principal amount of the debt issuance that is rated. According to the code of conduct of the major NRSROs, the receipt of the compensation cannot influence the process of assigning a rating.

The rating fees have high variation across different asset classes. S&P rating fees disclosure in 2008 indicates that the price of rating corporate debt was limited at 4.25 basis points while the structured finance fees ranged up to 12 basis points. In this section we analyze the effect of imposing a cap on the fee that a CRA may charge for rating a particular class of assets.

The effectiveness of the regulation that imposes a limit on the rating fee depends on the initial equilibrium outcome. If the market coverage involves rating two types $v_2$ and $v_3$, then limiting the fee does not change the optimal information structure. Indeed, given the fee, the CRA’s objective is to maximize the market coverage. Then the CRA can choose to rate two or three types. In the former case, the trade occurs under the terms described in Proposition 6. In the later case of rating all types, the underpricing becomes more severe. It results in transfer of wealth from the CRA to informed investors. However, in either case the original information structure remains optimal, and the policy has no effect on welfare.

The policy becomes effective when the original equilibrium involves limited market coverage. According to Proposition 7, this outcome occurs in the market with a high share of uninformed investors, $q > \bar{q}$. In this case, limiting the rating fee can improve
efficiency.

**Proposition 12** Consider the market in which the CRA rates only one type \(v_3\) and charges a fee \(\phi\). Imposing a fee cap \(\bar{\phi} < \phi\) induces the CRA to rate two issuer types \(v_2\) and \(v_3\) and increases efficiency.

In the market with a severe winner’s curse problem, the CRA reduces the market coverage in order to eliminate the underpricing problem that would force it to reveal a lot of information. It charges a high fee that discourages participation of \(v_2\) issuers. If a regulator can limit the fee to the level that is compatible with coverage of both types \(v_2\) and \(v_3\), the CRA’s optimal reaction to the policy is to maximize the market coverage, which leads to the information structure described in Proposition 6. The regulation is efficient because it increases market liquidity by inducing issuers \(v_2\) to trade.

### 7.3 Expert liability

Traditionally CRAs have been exempt from legal liability for inaccurate ratings under the First Amendment. The courts viewed ratings as an opinion about the credit quality. There were several cases where CRAs were sued by investors when the credit quality of highly rated securities quickly deteriorated. However, the nature of the rating business makes it hard to demonstrate that the default could have been foreseen by the CRA at the time of assigning the rating.

Dodd-Frank Act removed the protection making CRAs subject to the same expert liability as auditors or security analysts. However, in response to new rules, the major CRAs were refusing to allow citing of their ratings in prospectuses and registration statements of asset-backed securities (ABS) issues. As a result, the ABS market froze in the summer of 2010 which led SEC to exempt CRAs from expert liability in this asset class.\(^{19}\) The temporary exemption has been extended indefinitely later in 2011.

What is the effect of introducing legal liability on the equilibrium outcome? Suppose that the CRA has to pay a fine when an issue rated \(s_3\) realizes the value \(v_2\). Then the following result holds.

**Proposition 13** Consider a market in which the CRA rates issuers types \(v_2\) and \(v_3\). Imposing a fine for overrating an issue increases ratings precision but may reduce market coverage to the highest type \(v_3\).

\(^{19}\)See Request on behalf of Ford Motor Credit Company LLC to SEC, July 22, 2010; Response of the Office of Chief Counsel, Division of Corporate Finance, SEC, November 23, 2010; “IFR-ABS: Bill reversing rating agency liability advances” by Adam Tempkin, Reuters, July 20, 2011.
Imposing a fine makes rating inflation more costly, and thus increases the precision of ratings. However, more informative ratings reduce the ability of the CRA to extract the surplus as pooling type $v_2$ with the highest type $v_3$ has the liability cost. As a result, the CRA may choose to reduce the market coverage to the highest issuer type and provide precise rating to avoid the legal liability risk. If this occurs, the market outcome is inefficient because the issuers type $v_2$ are not rated and do not trade. Interestingly, a similar reaction was predicted by market participants. Financial Services Working Group has suggested\textsuperscript{20} that introducing expert liability “would raise the cost of rating bonds from approximately $100,000 per rating to approximately $1 million, and it could result in some bonds not getting a rating, which would prevent those bonds from coming to market.”

### 7.4 Reliance on ratings in regulation

The US regulators have been using credit ratings from 1930s to control the risk taking behavior of regulated financial institutions and insurance companies. The regulatory use of ratings has expanded significantly starting the 1970s when the SEC adopted the rules which use ratings as a basis for calculating capital requirements for broker-dealers. National Association of Insurance Commissioners uses ratings to assess the risk of the insurance investment portfolio that can affect the insurance company capital requirements. Department of Labor requires that the pension funds investment in asset-backed securities is restricted to securities rated A or higher. Investment grade mutual funds must sell any security rated B or below, and cannot hold more than 5\% of non-investment grade securities. Many countries use ratings in banking regulation.\textsuperscript{21}

Rating based regulatory policies effectively impose a regulatory premium on higher rated bonds and potentially decrease the liquidity on the market of lower rated bonds. The CRAs have been criticized for providing the “regulatory licence” instead of unbiased credit analysis. Following the crisis, the regulators in the US and in the EU discussed several policy options to reduce the regulatory reliance on ratings. However, the task has been challenging as there are few alternatives that can substitute ratings for a wide array of market participants.

\textsuperscript{20}IFR-ABS: Bill reversing rating agency liability advances” by Adam Tempkin, Reuters, July 20, 2011.

\textsuperscript{21}Kisgen (2007) is an excellent review on the use of credit ratings, in particular, in regulation. Kisgen and Strahan (2010) find that rating-based regulations on bond investment affect the firm’s cost of debt capital. Ellul, Jotikasthira and Lundblad (2012) provide evidence on regulation-related fire sales of downgraded corporate bonds by insurance companies.
Consistent with the observations of the investment community, our model predicts that regulatory use of ratings reduces their precision. Indeed, suppose that an issuer gains a regulatory premium \( \rho > 0 \) if an issue obtains the highest rating \( s_3 \). Then the CRA will adjust its rating system to extract the regulatory rent.

**Proposition 14** Consider a market where issuers types \( v_2 \) and \( v_3 \) are rated. Introducing a regulatory premium \( \rho \) for securities rated \( s_3 \) reduces the precision of ratings.

The economic intuition of this result is the following. In the basic model, the reason for rating inflation is that the CRA increases the willingness to pay of issuers \( v_2 \) by assigning them a high rating with probability \( 1 - p_{22} \). If the high rating value is increased by the regulatory premium, it makes rating inflation more desirable for the CRA. It is also feasible because high quality issuers value of trade is increased by regulation, and they are willing to accept less precise ratings. The regulatory premium increases the profits of the CRA. Also it permits informed investors to gain higher rent as lower rating precision aggravates the winner’s curse problem.

### 8 Conclusion

In this paper we analyze the equilibrium precision of ratings. Our results suggest that the information content of ratings depends on the market conditions and the presence of differentially informed investors. In particular, we show that ratings become less informative as the gains from trade increase, the share of high quality assets in the economy decreases, and the pool of investors becomes less informed about asset values. The results offer an explanation for heterogenous performance of ratings in different asset classes and through the cycle. We apply the model to analyze the merits of the recent reform proposals and show that some policies, in particular, rating standardization and expert liability, reduce market efficiency.

Understanding the incentives of CRAs to produce information is important for guiding policies to improve the efficiency of the financial market. Several aspect of the problem are left for future research. The focus of the paper was to evaluate the performance of ratings under the current issuer pays model. Many commentators in academic and investment communities suggest that conversion to investor pays model may increase ratings precision. It is unclear, however, whether switching the side will lead to better ratings or shift the CRAs incentives to provide ratings that are biased in favor of investors’ needs. Also our focus was on a monopoly CRA. High industry concentration has recently
led to Duopoly Relief Act of 2006 that aims to encourage competition among CRAs. The effect of competition on the information content of ratings is another area that needs further analysis.
Proof of Proposition 1. Consider an information structure with three signals. We characterize an equilibrium in which CRA chooses $p_{11} = p_{33} = 1$ and $p_{22} = \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \lambda_2} \leq 1$ and offers a fee $\phi = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$. Types $v_2$ and $v_3$ solicit rating whereas type $v_1$ does not solicit a rating. An issuer with a rating $s_i$ sets the price of the issue at $b_i$ where

$$b_3 = \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2}, \quad \text{and} \quad b_2 = v_2.$$ 

All investors subscribe to the issue. In terms of off-the-equilibrium-path-beliefs, investors believe that the issuer is of type $v_1$ if (1) an issuer solicits a rating and receives a signal $s_1$; or (2) the issue price is different than $b_i$ following rating $s_i$. At $t = 3$, investors have the following assessment of an issue following rating $s_i$ and offer price $b_i$ where $i \in \{2, 3\}$:

$$U_3 = \frac{\lambda_3}{\lambda_3 + \lambda_2(1 - p_{22})} v_3 + \frac{\lambda_2(1 - p_{22})}{\lambda_3 + \lambda_2(1 - p_{22})} v_2,$$

$$U_2 = v_2.$$ 

Inserting $p_{22} = \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta \lambda_2}$ leads to $U_3 = \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2} = b_3$. Therefore, investors are indifferent about subscribing to an issue following rating $s_i$ and offer price $b_i$ where $i \in \{2, 3\}$. Any other rating and price combination is not on the equilibrium path so investors believe that the issuer is of type $v_1$ and therefore have assessment equal to $v_1$. Consequently, they always reject any positive price offers that are not on the equilibrium path.

At $t = 2$, issuer type $v_3$ always receives rating $s_3$ and he sets the price at $b_3 = \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2}$. Given that investors always buy the issue at this price, he raises $R_3 = U_3$. Note that his outside option $\delta v_3$ is not higher. Furthermore, setting any other price leads to investors believing that he is of type $v_1$ and leads to non-positive profits. Consequently, he has no incentive to deviate. On the other hand, at $t = 2$, issuer type $v_2$ can receive ratings $s_2$ and $s_3$. If he sets the price at $b_3 = \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2}$ following rating $s_3$ then his payoff will be strictly greater than his outside option, $\delta v_2$. Similarly, if he sets the price at $b_2 = v_2$ following rating $s_2$ then his payoff will be greater than his outside option since $v_2 \geq \delta v_2$. Thus, he has no incentive to set any other price given investors beliefs. Finally, type $v_1$ is indifferent between issuing and his outside option.

\footnote{Note that we can support this equilibrium outcome with other beliefs as well. This set of beliefs simplify arguments.}
At $t = 1$, issuer type $v_3$ knows that if he pays for a rating he will raise $R_3 = U_3$ in period $t = 3$. Then his payoff after soliciting a rating will be

$$U_3 - \phi = \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2} - (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$$

$$= \delta v_3.$$

Therefore, he is indifferent between soliciting rating and his outside option. If he does not solicit a rating, then investors will believe that he is of type $v_1$ and selling the issue is worse than outside option. Consequently, there are no incentives for him to deviate.

Similarly, the issuer type $v_2$ knows that if he pays for a rating, the issue expects to raise $R_2 = U_2$ in period $t = 2$. Then his payoff due to soliciting rating will be

$$R_2 - \phi = p_{22} U_2 + (1 - p_{22}) U_3 - \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$$

$$= \frac{\delta (\lambda_3 + \lambda_2)}{\lambda_3 + \delta \lambda_2} v_2 + (1 - \frac{\delta (\lambda_3 + \lambda_2)}{\lambda_3 + \delta \lambda_2}) \frac{\lambda_3 v_3 + \lambda_2 v_2 + \delta \lambda_2 (v_3 - v_2)}{\lambda_3 + \lambda_2} - (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}$$

$$= \delta v_2.$$

Therefore, he is indifferent between soliciting a rating and his outside option. If he does not solicit a rating, then investors will believe that he is of type $v_1$ and selling the issue is worse than outside option. Consequently, there are no incentives for him to deviate either.

Finally, type $v_1$ is indifferent between soliciting rating and his outside option because he knows that he will get the $s_1$ rating if he solicits one.

At $t = 0$, the CRA aims to sell rating to issuers with a positive willingness to pay, $v_3$ and $v_2$. Then an optimal information structure solves

$$\max (\lambda_3 + \lambda_2) \phi$$

$$R_i - \delta v_i - \phi \geq 0, \ i = 2, 3.$$

Constraints (3) imply that

$$R_3 - \delta v_3 = R_2 - \delta v_2 = \phi,$$

which simplifies to

$$p_{22} (U_3 - U_2) = \delta (v_3 - v_2),$$

and consequently,

$$p_{22} = \frac{\delta (\lambda_3 + \lambda_2)}{\lambda_3 + \delta \lambda_2} < 1.$$
Then the CRA sets the fee

\[ \phi = U_3 - \delta v_3 = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2} \]

and gains the market surplus, \((\lambda_3 + \lambda_2)\phi = (1 - \delta)(\lambda_3 v_3 + \lambda_2 v_2)\). Thus the information structure achieves the first best for the CRA and he has no incentive to deviate. ■

**Proof of Proposition 2.** Proposition 1 establishes that there exists a rating system that entails inflation, that is an equilibrium outcome. Without loss of generality, let \(p_{22} = p < 1\) characterize this rating system with an optimal fee \(\phi = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}\). One can verify the symmetric information structure, i.e., \(p_{33} = p < 1\) and \(p_{22} = 1\), can also achieve \(R_3 - \delta v_3 = R_2 - \delta v_2 = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}\). This leads to \(U_2 = \frac{\lambda_3 (1-p) v_3 + \lambda_2 v_2}{\lambda_3 (1-p) + \lambda_2}\) and substituting \(p = \frac{\delta(\lambda_3 + \lambda_3)}{\lambda_3 + \delta \lambda_3}\) results in \(U_2 = \frac{\lambda_3 v_3 + \lambda_2 v_2 - \delta \lambda_3 (v_3 - v_2)}{\lambda_3 + \lambda_2} = \overline{U}\). Furthermore, one can show that for all \(p_{33} \in (p, 1)\) there exists \(p_{22} \in (p, 1)\) such that \(R_3 - \delta v_3 = R_2 - \delta v_2 = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}\) is satisfied. From Bayesian updating, it is immediate that for all \((p_{22}, p_{33}) \in (p, 1)^2\), \(U_2 < \overline{U}\). Therefore, if \(\delta v_3 > \overline{U}\) then we must have rating inflation as the only rating system that can achieve \(R_3 - \delta v_3 = R_2 - \delta v_2 = (1 - \delta) \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 + \lambda_2}\) and also satisfy the participation constraint of type \(v_3\), i.e., \(\delta v_3 \geq U_2\). Solving for \(\delta v_3 = \overline{U}\) leads to \(\delta = \frac{\lambda_3 v_3 + \lambda_2 v_2}{\lambda_3 (2v_3 - v_2)}\). ■

**Proof of Proposition 3.** From Proposition 1,

\[ \frac{dp_{22}}{d\delta} = (\lambda_3 + \lambda_2) \frac{\lambda_3}{(\lambda_3 + \delta \lambda_2)^2} > 0. \]

■

**Proof of Proposition 4.** First, we show that there exists an information structure such that \(R_j - \delta v_j = \phi\) for all \(j \in \{1, 2, 3\}\). Suppose that \(R_j - \delta v_j = \phi\) for all \(j \in \{1, 2, 3\}\) does not hold. Without loss of generality, there must exist types \(k, j\) such that \(R_k - \delta v_k > R_j - \delta v_j = \phi\), and yet the CRA cannot increase its profit by increasing \(R_j\) and decreasing \(R_k\). Then we claim that we must have \(k > j\). To see this, first note that \(R_k\) is a continuous function of \(p_{ij}\) for all \(k, i\) and \(j\). Furthermore, perfectly informative signals, lead to \(R_j = v_j\) so that \(R_k - \delta v_k > R_j - \delta v_j\) implies \(k > j\). However, if \(k > j\), then we can make the signals more uninformative until \(R_k - \delta v_k = R_j - \delta v_j\) since (i) \(R_k\) is a continuous function of \(p_{ij}\) for all \(k, i\) and \(j\); and (ii) fully noisy signals result in \(R_k - \delta v_k < R_j - \delta v_j\) as long as \(k > j\). Therefore, there exists an information structure such that \(R_j - \delta v_j = \phi\) for all \(j \in \{1, 2, 3\}\). Characterization of the equilibrium in which CRA uses an information structure such that \(R_j - \delta v_j = \phi\) for all \(j \in \{1, 2, 3\}\) and rates all three types capturing all of the surplus is similar to the equilibrium characterization in Proposition 1, and therefore is omitted. Consequently, we proceed with payoff calculations.
If all three types are rated, then
\[
\sum_{j} \lambda_j R_j = \sum_{j} \lambda_j v_j = E[v].
\]

In equilibrium, the CRA equates the issuers willingness to pay, i.e., \(R_j - \delta v_j = \phi\) for all \(j \in \{1, 2, 3\}\), implying
\[
\sum_{j} \lambda_j R_j = \lambda_3 (R_1 + \delta v_3) + \lambda_2 (R_1 + \delta v_2) + R_1 = R_1 + \delta (\lambda_3 v_3 + \lambda_2 v_2) = E[v].
\]

Thus,
\[
\phi = R_1 = E[v] - \delta (\lambda_3 v_3 + \lambda_2 v_2) = (1 - \delta) E[v],
\]
\[
R_j = (1 - \delta) E[v] + \delta v_j,
\]
\[
\Pi^3 = (1 - \delta) E[v].
\]

If two types \(v_3\) and \(v_2\) are rated, we know from Proposition 1 that
\[
R_2 - \delta v_2 = R_3 - \delta v_3 = \phi,
\]
\[
\lambda_3 R_3 + \lambda_2 R_2 = E[v].
\]

Thus,
\[
(\lambda_3 + \lambda_2) R_2 = E[v] - \lambda_3 \delta (v_3 - v_2),
\]
\[
\Pi^2 = (\lambda_3 + \lambda_2) \phi = (\lambda_3 + \lambda_2) R_2 - (\lambda_3 + \lambda_2) \delta v_2 = (1 - \delta) E[v].
\]

Hence, \(\Pi^2 = \Pi^3\) and the CRA is indifferent between rating two or three types.

Finally, we show that rating one type will lead to lower profits for the CRA. If only one type is rated, then it is most profitable to rate type \(v_3\). The fee, \(\phi\), is bounded above by the gains from trade, \(v_3 - \delta v_3\), and the spread between trading with and without rating, \(v_3 - \frac{\lambda_2 v_2}{\lambda_2 + \lambda_1}\). Therefore, the profit is
\[
\Pi^1 = \lambda_3 \min\{ (1 - \delta) v_3, v_3 - \frac{\lambda_2 v_2}{\lambda_2 + \lambda_1} \} < \Pi^2 = \Pi^3.
\]

\[\blacksquare\]
Proof of Proposition 5. First note that the issue price must be equal to the expected value of the issue conditional on an uninformed investor’s demand being fulfilled. We omit the characterization of the equilibria in which uninformed investors make zero expected profit in order to avoid the repetition of arguments in Proof of Proposition 1. Instead, we show that in such equilibria the CRA makes strictly higher expected profit by rating only two highest types, i.e., $v_3$ and $v_2$. We consider a general information structure and let $\beta_{ij} = \Pr(v_j|s_i)$ stand for the posterior belief. Recall that $\gamma_{ij}$ denotes the belief of an uninformed investor who is offered an issue that the issuer rated $s_i$ is of type $v_j$.

We consider two cases where the CRA rates (i) all types, or (ii) two highest types $v_3$ and $v_2$.

Case (i). Suppose the CRA rates all issuers. Denote $b^*_i$ the equilibrium price of a security rated $s_i$. Two cases are possible, $b^*_i \in (v_1, v_2)$ and $b^*_i \in (v_2, v_3)$.

If $b^*_i \in (v_1, v_2)$, the posterior beliefs of uninformed investors are

$$
\gamma_{i3} = \frac{q\beta_{i3}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}},
$$

$$
\gamma_{i2} = \frac{q\beta_{i2}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}},
$$

$$
\gamma_{i1} = \frac{\beta_{i1}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}}.
$$

Then

$$
b^*_i = \sum_j \gamma_{ij} v_j = \frac{q\beta_{i3} v_3 + q\beta_{i2} v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}},
$$

and $b^*_i \in (v_1, v_2)$ holds when

$$
\frac{q\beta_{i3} v_3 + q\beta_{i2} v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < v_2,
$$

which simplifies to

$$
\frac{q\beta_{i3}}{\beta_{i1}} < \frac{v_2}{v_3 - v_2}.
$$

The issue raises

$$
\frac{q\beta_{i3} v_3 + q\beta_{i2} v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}}.$$
If \( b_i^* \in (v_2, v_3) \), then the posterior beliefs of uninformed investors are

\[
\begin{align*}
\gamma_{i3} &= \frac{q \beta_{i3}}{q \beta_{i3} + \beta_{i2} + \beta_{i1}}, \\
\gamma_{i2} &= \frac{\beta_{i2}}{q \beta_{i3} + \beta_{i2} + \beta_{i1}}, \\
\gamma_{i1} &= \frac{\beta_{i1}}{q \beta_{i3} + \beta_{i2} + \beta_{i1}}.
\end{align*}
\]

Then

\[
b_i^* = \sum_j \gamma_{ij} v_j = \frac{q \beta_{i3} v_3 + \beta_{i2} v_2}{q \beta_{i3} + \beta_{i2} + \beta_{i1}},
\]

and \( b_i^* \in (v_2, v_3) \) holds when

\[
\frac{q \beta_{i3} v_3 + \beta_{i2} v_2}{q \beta_{i3} + \beta_{i2} + \beta_{i1}} > v_2
\]

which simplifies to

\[
\frac{q \beta_{i3}}{\beta_{i1}} > \frac{v_2}{v_3 - v_2}.
\]

In this case the issue raises

\[
\frac{q \beta_{i3} v_3 + \beta_{i2} v_2}{q \beta_{i3} + \beta_{i2} + \beta_{i1}}.
\]

**Case (ii).** When only two higher types are rated, the price must satisfy \( b_i^* \in (v_2, v_3) \).

The posterior beliefs are

\[
\begin{align*}
\gamma_{i3} &= \frac{q \beta_{i3}}{q \beta_{i3} + \beta_{i2}}, \\
\gamma_{i2} &= \frac{\beta_{i2}}{q \beta_{i3} + \beta_{i2}}.
\end{align*}
\]

Then

\[
b_i^* = \sum_j \gamma_{ij} v_j = \frac{q \beta_{i3} v_3 + \beta_{i2} v_2}{q \beta_{i3} + \beta_{i2}}.
\]

The issue raises

\[
(\lambda_3 + \lambda_2) b_i^* = \frac{(\lambda_3 + \lambda_2)(q \beta_{i3} v_3 + \beta_{i2} v_2)}{q \beta_{i3} + \beta_{i2}}.
\]

The price satisfies \( b_i^* \in (v_2, v_3) \) for all parameter values.

Next, we consider any signal \( s_i \) and show that the expected revenue with two ratings is higher than that with three ratings for each signal.
If \( b_i^* \in (v_2, v_3) \), then we need to show that
\[
\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q\beta_{i3} + \beta_{i2} + \beta_{i1} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}}}
\]

\[\Rightarrow \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2} + \beta_{i1})}{q\beta_{i3} + \beta_{i2}} > 1.\]

Denote \( A = q\beta_{i3} + \beta_{i2} + \beta_{i1} \). Then the condition becomes
\[
\frac{(1 - \beta_{i1})A}{A - \beta_{i1}} > 1 \Rightarrow A < 1,
\]

which holds for all parameter values as long as \( q < 1 \).

If \( b_i^* \in (v_1, v_2) \), then we need to show that
\[
\frac{q\beta_{i3}v_3 + \beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2} + \beta_{i1})} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}}
\]
holds for all parameter values such that \( \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} < \frac{v_3}{v_3 - v_2} \) and \( v_3 > v_2 \), or equivalently, \( v_3 \in (v_2, \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2) \). When we have \( v_3 = \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2 \), then \( b_i^* = v_2 \). In other words, for \( v_3 = \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2 \) we have the left hand side (LHS) is equal to \( v_2 \). For \( v_3 = \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2 \) the right hand side (RHS) is
\[
\frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2} + \beta_{i1}v_2)}{q\beta_{i3} + \beta_{i2}}
\]
Simplifying leads to
\[
\frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2} + \beta_{i1})v_2}{q\beta_{i3} + \beta_{i2}}
\]
Let \( B = q\beta_{i3} + \beta_{i2} \). Then the condition becomes
\[
\frac{(1 - \beta_{i1})(B + \beta_{i1})v_2}{B}.
\]
Given that \( \frac{(1 - \beta_{i1})(B + \beta_{i1})v_2}{B} > 1 \) simplifies to \( q\beta_{i1} + \beta_{i2} + \beta_{i3} < 1 \), we have RHS greater than \( v_2 \).

The other extreme condition under \( v_3 \in (v_2, \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2) \) is \( v_3 = v_2 \). Then LHS becomes \( \frac{(q\beta_{i3} + \beta_{i2})v_2}{q(\beta_{i3} + \beta_{i2} + \beta_{i1})} \) and RHS becomes \( \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2})v_2}{q\beta_{i3} + \beta_{i2}} \). For our result to hold for this extreme point, we must have
\[
\frac{q\beta_{i3} + \beta_{i2}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3} + \beta_{i2})}{q\beta_{i3} + \beta_{i2}}
\]
This term simplifies to
\[
\frac{q}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} < 1
\]
Further iteration leads to \(q < q(\beta_{i3} + \beta_{i2}) + \beta_{i1}\). Subtracting \((1 - q)\beta_{i1}\) from both sides we have \(q - (1 - q)\beta_{i1} < q(\beta_{i3} + \beta_{i2} + \beta_{i1})\). Since \(\beta_{i3} + \beta_{i2} + \beta_{i1} = 1\), we must have LHS less than RHS for \(v_3 = v_2\).

Next we show that the difference between LHS and RHS is monotonic, and thus LHS is less than RHS for all interior points, \(v_3 \in (v_2, \frac{q\beta_{i3} + \beta_{i1}}{q\beta_{i3}} v_2)\).

Define
\[
F(v_3, a) = \frac{q\beta_{i3}v_3 + q\beta_{i2}v_2}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} - \frac{(\beta_{i3} + \beta_{i2})(q\beta_{i3}v_3 + \beta_{i2}v_2)}{q\beta_{i3} + \beta_{i2}},
\]
where \(a\) denotes all the parameters, \(a = (\lambda_H, \lambda_M, q)\). Then
\[
F'(v_3, a) = \frac{q\beta_{i3}}{q(\beta_{i3} + \beta_{i2}) + \beta_{i1}} \frac{q\beta_{i3}}{q\beta_{i3} + \beta_{i2}} - \frac{\beta_{i3} + \beta_{i2}}{(q(\beta_{i3} + \beta_{i2}) + \beta_{i1})(q\beta_{i3} + \beta_{i2})} (q\beta_{i3} + \beta_{i2} - (\beta_{i3} + \beta_{i2})(q(\beta_{i3} + \beta_{i2}) + \beta_{i1}))
\]
The first term is positive, thus the sign of \(F'(v_3, a)\) is defined by the sign of
\[
q\beta_{i3} + \beta_{i2} - (\beta_{i3} + \beta_{i2}) (q(\beta_{i3} + \beta_{i2}) + \beta_{i1})
\]
\[
= (1 - q)(\beta_{i3} - 2(\frac{1}{2} - \beta_{i2})\beta_{i3} + \beta_{i2}^2).
\]
Solve the inequality with respect to \(\beta_{i3}\),
\[
\beta_{i3}^2 - 2(\frac{1}{2} - \beta_{i2})\beta_{i3} + \beta_{i2}^2 > 0.
\]
The roots are
\[
\frac{1}{2} - \beta_{i2} \pm \sqrt{(\frac{1}{2} - \beta_{i2})^2 - \beta_{i2}^2} = \frac{1}{2} - \beta_{i2} \pm \sqrt{\frac{1}{4} - \beta_{i2}^2}.
\]
If \(\beta_{i2} > \frac{1}{4}\), then (4) holds for any \(a \in A_1 = \{\beta_{i3} \in (0, 1), \beta_{i2} \in (\frac{1}{4}, 1), q \in (0, 1)\}\).

If \(\beta_{i2} \leq \frac{1}{4}\), then note that the low root is positive, \(\frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}} > 0\) and the high root is less than 1, \(\frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}} < 1\). Therefore, if \(a \in A_2\), where
\[
A_2 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (0, \frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}}) \cup (\frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}}, 1), q \in (0, 1)\},
\]
then \(F' > 0\). If \(a \in A_3\), where
\[
A_3 = \{\beta_{i2} \in (0, \frac{1}{4}), \beta_{i3} \in (\frac{1}{2} - \beta_{i2} - \sqrt{\frac{1}{4} - \beta_{i2}}, \frac{1}{2} - \beta_{i2} + \sqrt{\frac{1}{4} - \beta_{i2}}), q \in (0, 1)\},
\]


then $F' < 0$.

Hence, for $a \in A_1 \cup A_2$, $F' > 0$, and $F(\frac{q_{33} + \beta_1 v_2}{q_{33}} a) < 0$ implies the result. For $a \in A_3$, $F' < 0$ and $F(v_2, a) < 0$ implies the result.

Given that the expected revenue with two ratings is higher than that with three ratings, the price the issuer sets in the two types rated case must be greater than $\frac{1}{\lambda_2 + \lambda_2}$ times that of the other case. Considering the fact that this holds for any signal, the CRA can charge a higher fee that enables it to extract more expected profit when it rates two highest types rather than all types.

Consequently, we are left to show that if we restrict attention to equilibria in which uninformed investors make zero expected profit, rating all types is never an equilibrium. Suppose that such an equilibrium exists. Then consider the following deviation by the CRA. CRA increases the fee while retaining the same information structure and makes it unattractive for type $v_1$ to solicit a rating. By above arguments, this deviation leads to higher expected profit for the CRA as long as he sets a sufficiently high fee. Therefore, for any candidate equilibrium with all types rated, there exists a profitable deviation. ■

**Proof of Proposition 6.** Conditional on observing ratings $s_3$ and $s_2$, the uninformed investors hold beliefs $\gamma_{ij} = \Pr(v_j | s_i)$ with

\[
\begin{align*}
\gamma_{33} &= \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2 (1 - p_{22})}, \\
\gamma_{22} &= \frac{\lambda_2 p_{22}}{\lambda_2 q (1 - p_{33}) + \lambda_2 p_{22}}, \\
\gamma_{32} &= 1 - \gamma_{33} \text{ and } \gamma_{23} = 1 - \gamma_{22}, \\
\gamma_{31} &= \gamma_{31} = \gamma_{13} = \gamma_{12} = 0, \gamma_{11} = 1
\end{align*}
\]

The resulting uninformed investors’ assessment of the assets rated $s_3$ and $s_2$ are

\[
\begin{align*}
U_3 = \gamma_{33} v_3 + \gamma_{32} v_2 &= \frac{\lambda_3 q p_{33} v_3 + \lambda_2 (1 - p_{22}) v_2}{\lambda_3 q p_{33} + \lambda_2 (1 - p_{22})}, \\
U_2 = \gamma_{23} v_3 + \gamma_{22} v_2 &= \frac{\lambda_2 q (1 - p_{33}) v_3 + \lambda_2 p_{22} v_2}{\lambda_2 q (1 - p_{33}) + \lambda_2 p_{22}}.
\end{align*}
\]

After receiving a rating $s_i$, an issuer $v_j$ can either sell the issue at price $U_i$ or hold the asset and realize the value $\delta v_j$. Then the expected payoff of soliciting a rating for types $v_3$ and $v_2$ are

\[
\begin{align*}
R_3 &= p_{33} \max\{U_3, \delta v_3\} + (1 - p_{33}) \max\{U_2, \delta v_3\}, \\
R_2 &= (1 - p_{22}) \max\{U_3, \delta v_2\} + p_{22} \max\{U_2, \delta v_2\}.
\end{align*}
\]
Consider the information structure

<table>
<thead>
<tr>
<th></th>
<th>$v_3$</th>
<th>$v_2$</th>
<th>$v_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_3$</td>
<td>$p_{33}$</td>
<td>$1 - p_{22}$</td>
<td>0</td>
</tr>
<tr>
<td>$s_2$</td>
<td>$1 - p_{33}$</td>
<td>$p_{22}$</td>
<td>0</td>
</tr>
<tr>
<td>$s_1$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Under this information structure, rating $s_1$ perfectly reveals the types $v_1$, $\gamma_{13} = \gamma_{12} = 0$, $\gamma_{11} = 1$. Ratings $s_2$ and $s_3$ lead to beliefs updating $\gamma_{ij} = \Pr(v_j|s_i)$, with

\[
\gamma_{33} = \frac{\lambda_3 q p_{33}}{\lambda_3 q p_{33} + \lambda_2 (1 - p_{22})},
\gamma_{32} = \frac{\lambda_3 q (1 - p_{33})}{\lambda_3 q (1 - p_{33}) + \lambda_2 p_{22}},
\gamma_{23} = \frac{\lambda_3 q(1 - p_{33})}{\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22}},
\gamma_{31} = 0,
\gamma_{21} = 0.
\]

Issuers soliciting a rating obtain expected payoff equal to

\[
R_3 = p_{33} \max\{U_3, \delta v_3\} + (1 - p_{33}) \max\{U_2, \delta v_3\},
R_2 = (1 - p_{22}) \max\{U_3, \delta v_2\} + p_{22} \max\{U_2, \delta v_2\},
R_1 = v_1 = 0.
\]

The payoff of type $v_1$ implies that it does not solicit a rating.

The investor’s assessment of the asset values under this information structure is

\[
U_3 = \gamma_{33} v_3 + \gamma_{32} v_2 = \frac{\lambda_3 q p_{33} v_3 + \lambda_2 (1 - p_{22}) v_2}{\lambda_3 q p_{33} + \lambda_2 (1 - p_{22})},
U_2 = \gamma_{23} v_3 + \gamma_{22} v_2 = \frac{\lambda_3 q (1 - p_{33}) v_3 + \lambda_2 p_{22} v_2}{\lambda_3 q (1 - p_{33}) + \lambda_2 p_{22}}.
\]

Consider the case when all rated issuers trade, $U_i \geq \delta v_j$ for all $i, j = 2, 3$. (Note that if $U_i < \delta v_j$ for all $i, j$, then types $v_j$ do not solicit a rating.) Then the optimal information structure solves

\[
\max (\lambda_3 + \lambda_2) \phi
\]
\[ \rho_3 : p_{33}U_3 + (1 - p_{33})U_2 - \phi \geq 0, \]
\[ \rho_2 : (1 - p_{22})U_3 + p_{22}U_2 - \phi \geq 0, \]
\[ \mu_3 : p_{33}(U_3 - \delta v_3) \geq 0, \]
\[ \mu_2 : (1 - p_{33})(U_2 - \delta v_3) \geq 0, \]
\[ \tau_3 : 1 - p_{33} \geq 0, \]
\[ \tau_2 : 1 - p_{22} \geq 0, \]
\[ \kappa_3 : p_{33} \geq 0, \]
\[ \kappa_2 : p_{22} \geq 0. \]

Note that we did not include the constraints \( U_i - \delta v_2 \geq 0 \) because these are implied by the other two constraints.

The first order conditions of the problem are:

with respect to \( p_{33} \)
\[
\rho_3(U_3 - U_2 + p_{33}\frac{dU_3}{dp_{33}} + (1 - p_{33})\frac{dU_2}{dp_{33}}) + \rho_2((1 - p_{22})\frac{dU_3}{dp_{33}} + p_{22}\frac{dU_2}{dp_{33}})
+ \mu_3(U_3 - \delta v_3 + p_{33}\frac{dU_3}{dp_{33}}) + \mu_2(-(U_2 - \delta v_3) + (1 - p_{33})\frac{dU_2}{dp_{33}}) - \tau_3 + \kappa_3 = 0,
\]

with respect to \( p_{22} \)
\[
\rho_3(p_{33}\frac{dU_3}{dp_{22}} + (1 - p_{33})\frac{dU_2}{dp_{22}}) + \rho_2(U_2 - U_3 + (1 - p_{22})\frac{dU_3}{dp_{22}} + p_{22}\frac{dU_2}{dp_{22}})
+ \mu_3p_{33}\frac{dU_3}{dp_{22}} + \mu_2(1 - p_{33})\frac{dU_2}{dp_{22}} - \tau_2 + \kappa_2 = 0,
\]

with respect to \( \phi \)
\[
(\lambda_3 + \lambda_2) - \rho_3 - \rho_2 = 0.
\]

In terms of the information structure, the values \( \frac{dU_i}{dp_{kk}}, i, k = 2, 3 \) write
\[
\frac{dU_3}{dp_{33}} = \frac{\lambda_3\lambda_2 q (1 - p_{22})(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2 (1 - p_{22}))^2} > 0,
\]
\[
\frac{dU_3}{dp_{22}} = \frac{\lambda_3\lambda_2 q p_{33}(v_3 - v_2)}{(\lambda_3 q p_{33} + \lambda_2 (1 - p_{22}))^2} > 0,
\]
\[
\frac{dU_2}{dp_{33}} = -\frac{\lambda_3\lambda_2 q p_{22}(v_3 - v_2)}{(\lambda_3 q (1 - p_{33}) + \lambda_2 p_{22})^2} < 0,
\]
\[
\frac{dU_2}{dp_{22}} = -\frac{\lambda_3\lambda_2 q (1 - p_{33})(v_3 - v_2)}{(\lambda_3 q (1 - p_{33}) + \lambda_2 p_{22})^2} < 0.
\]

Rated issuers must have the same willingness to pay, and thus \( \rho_i > 0, i = 2, 3 \). Then
\[
(p_{33} - (1 - p_{22}))(U_3 - U_2) = \delta(v_3 - v_2). \quad (5)
\]
Case (a). Suppose that $\mu_i > 0$ for $i = 2, 3$. Then three cases are possible, (a1) $p_{33} = 0$ and $U_2 = \delta v_3$; (a2) $U_3 = \delta v_3$ and $p_{33} = 1$; (a3) $U_3 = U_2 = \delta v_3$. Case (a1) is not feasible due to (5) and $U_3 \geq \delta v_3$. Case (a2) is not feasible due to (5) and $p_{22} \leq 1$. Case (a3) is not feasible due to (5). Thus $\mu_i > 0$ for $i = 1, 2$ is not a solution.

Case (b). Suppose that $\mu_3 = 0$ and $\mu_2 > 0$. Thus the optimal information structure is determined by the conditions

$$(1 - p_{33})(U_2 - \delta v_3) = 0,$$

$$(p_{33} - (1 - p_{22}))(U_3 - U_2) = \delta(v_3 - v_2).$$

We obtain

$$U_3 - U_2 = \frac{\lambda_3\lambda_2 q(p_{33}p_{22} - (1 - p_{33})(1 - p_{22}))}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})}(v_3 - v_2),$$

and thus

$$\frac{\lambda_3\lambda_2 q(p_{33} - (1 - p_{22}))^2}{(\lambda_3 q p_{33} + \lambda_2(1 - p_{22}))(\lambda_3 q(1 - p_{33}) + \lambda_2 p_{22})} = \delta.$$

Two alternatives are possible for $(1 - p_{33})(U_2 - \delta v_3) = 0$. Consider first the case $p_{33} = 1$. Then

$$\frac{\lambda_3 q p_{22}}{\lambda_3 q + \lambda_2(1 - p_{22})} = \delta,$$

$$\lambda_3 q p_{22} - \delta(\lambda_3 q + \lambda_2(1 - p_{22})) = 0$$

$$p_{22}(\lambda_3 q + \delta \lambda_2) - \delta(\lambda_3 q + \lambda_2) = 0$$

$$p_{22} = \frac{\delta(\lambda_3 q + \lambda_2)}{\lambda_3 q + \delta \lambda_2}.$$

Then the values $\frac{dU_i}{dp_{kk}}$ write

$$\frac{dU_3}{dp_{33}} = \frac{\lambda_3\lambda_2 q(1 - p_{22})(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$

$$\frac{dU_3}{dp_{22}} = \frac{\lambda_3\lambda_2 q(v_3 - v_2)}{(\lambda_3 q + \lambda_2(1 - p_{22}))^2},$$

$$\frac{dU_2}{dp_{33}} = \frac{\lambda_3 q(v_3 - v_2)}{\lambda_2 p_{22}} < 0,$$

$$\frac{dU_2}{dp_{22}} = 0.$$
The values $U_i$ write

\[
U_3 = \frac{\lambda_3 q v_3 + \lambda_2 (1 - p_{22}) v_2}{\lambda_3 q + \lambda_2 (1 - p_{22})},
\]

\[
U_2 = v_2.
\]

We need to verify that this solution satisfies ex-ante and ex-post participation constraints. Constraints with respect to $i > 0$ for $i = 2, 3$ imply

\[
U_3 - \delta v_3 - \phi = 0,
\]

\[
(1 - p_{22}) U_3 + p_{22} U_2 - \delta v_2 - \phi = 0,
\]

Thus

\[
\phi = U_3 - \delta v_3
\]

\[
= \frac{\lambda_3 q v_3 + \lambda_2 (1 - p_{22}) v_2}{\lambda_3 q + \lambda_2 (1 - p_{22})} - \delta v_3,
\]

and then

\[
p_{22} (U_3 - U_2) - \delta (v_3 - v_2) = 0,
\]

which holds. The ex-post constraints are

\[
\lambda_3 q v_3 + \lambda_2 v_2 \frac{\lambda_3 q (1 - \delta)}{\lambda_3 q + \delta \lambda_2} - \delta v_3 (\lambda_3 q + \lambda_2 (1 - \delta(\lambda_3 q + \lambda_2))) > 0,
\]

\[
\lambda_3 q v_3 + \lambda_2 v_2 > 0.
\]

The CRA charges the fee

\[
\phi = U_3 - \delta v_3 = (1 - \delta) \frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.
\]

it gains profit

\[
(\lambda_2 + \lambda_3) \phi = (1 - \delta) (\lambda_2 + \lambda_3) \frac{\lambda_3 q v_3 + \lambda_2 v_2}{\lambda_3 q + \lambda_2}.
\]

The surplus of informed investors is equal to the difference between the ex-ante market surplus and the CRA’s profits,

\[
\lambda_2 v_2 + \lambda_3 v_3 - (\lambda_2 + \lambda_3) \phi = \frac{\lambda_3 \lambda_2 (1 - q) (1 - \delta) (v_3 - v_2)}{\lambda_3 q + \lambda_2}.
\]

Type $v_3$ obtains the rating $s_2$ with zero probability, thus the ex-post constraint is satisfied,

\[
(1 - p_{33})(U_2 - \delta v_3) = 0.
\]
The ex-post constraint for the type $v_2$ following rating $s_3$ is implied by the constraint for type $v_3$. The ex-post constraint for type $v_2$ following rating $s_2$ is $(1 - \delta)v_2 > 0$.

It is immediate to verify that any rating system that permits no trade following the rating reduces the CRA’s profits. ■

**Proof of Proposition 7.** Proof follows from discussion in text and is omitted. ■

**Proof of Proposition 8.** We know from Proposition 4 that $\Pi^2 > \Pi^1$ for $q = 1$. Furthermore, recall that $\Pi^2 = (1 - \delta)\left(\frac{\lambda_2 + \lambda_3(\lambda_3v_3 + \lambda_2v_2)}{\lambda_3q + \lambda_2}\right)$ for $q \in [0, 1]$. First, we show that $\Pi^2$ is an increasing function of $q$ for all $q \in [0, 1]$:

$$\frac{d\Pi^2}{dq} = (1 - \delta)(\lambda_2 + \lambda_3)\frac{\lambda_3(\lambda_3q + \delta\lambda_2) - \lambda_3(\lambda_3v_3 + \lambda_2v_2)}{(\lambda_3q + \delta\lambda_2)^2}$$

$$= (1 - \delta)(\lambda_2 + \lambda_3)\frac{\lambda_3\lambda_2(v_3 - v_2)}{(\lambda_3q + \delta\lambda_2)^2} > 0.$$ 

Therefore, $\Pi^2$ reaches its lowest value at $q = 0$:

$$\lim_{q \to 0} \Pi^2 = (1 - \delta)(\lambda_2 + \lambda_3)v_2$$

Recall that the profits for rating only type 3, $\Pi^1$, is $\lambda_3 \min\{(1 - \delta)v_3, v_3 - \frac{\lambda_2v_2}{\lambda_3}q\}$. Both arguments in $\Pi^1$ can be greater or less than $(1 - \delta)(\lambda_2 + \lambda_3)v_2$ depending on prior probabilities, $\lambda_1$, $\lambda_2$, and $\lambda_3$, and values, $v_2$ and $v_3$. Therefore, $\eta \in [0, 1)$. ■

**Proof of Proposition 9.** Using the identical argument in Proposition 2, one gets

$$\bar{\delta} = \frac{\lambda_2v_2 + q\lambda_3v_3}{\lambda_2v_3 + q\lambda_3(2v_3 - v_2)}.$$ 

Taking the derivative of $\bar{\delta}$ with respect to $q$, we obtain

$$\frac{\lambda_3v_3(\lambda_2v_3 + q\lambda_3(2v_3 - v_2)) - \lambda_3(2v_3 - v_2)(\lambda_2v_2 + q\lambda_3v_3)}{(\lambda_2v_3 + q\lambda_3(2v_3 - v_2))^2}$$

Note that the denominator is always positive. The numerator simplifies to $\lambda_2\lambda_3(v_3 - v_2)^2$ which is also positive. Therefore, as the winner’s curse increases, $\bar{\delta}$ decreases leading to a larger set of $\delta$ that requires inflation. ■

**Proof of Proposition 10.** We obtain the following comparative statics results.

As the share of uninformed investors increases, the ratings become less informative.

$$\frac{dp_{22}}{dq} = -\frac{\lambda_3\lambda_2(1 - \delta)}{(\lambda_3q + \delta\lambda_2)^2} < 0.$$ 

As the aggregate value of liquidity increases, that is $\delta$ decreases, the ratings become less informative.

$$\frac{dp_{22}}{d\delta} = \frac{\delta\lambda_2(\lambda_3q + \lambda_2)}{(\lambda_3q + \delta\lambda_2)^2} > 0.$$
As high quality assets become more scarce, \( \frac{\lambda_2}{\lambda_3} \) increases, ratings become less informative. Define \( s = \frac{\lambda_2}{\lambda_3} \). Then \( p_{22} = \frac{\delta(q+s)}{q+s} \) and

\[
\frac{dp_{22}}{ds} = -\frac{\delta(1 - \delta)q}{(q + \delta s)^2} < 0.
\]

If all investors are uninformed, \( q = 1 \), then

\[
p_{22} = \frac{\delta(\lambda_3 + \lambda_2)}{\lambda_3 + \delta \lambda_2} < 1.
\]

If the value of liquidity is very high, \( \delta = 0 \) (Lizzeri’s case), then \( p = 0 \) and ratings are uninformative. ■

**Proof of Proposition 11.** The results follow from Propositions 7 and 6. ■

**Proof of Proposition 12.** Consider a market with a high share of informed investors, \( q > \bar{q} \) and the rating system described in Proposition 7. Also assume that \( \delta > \frac{q \lambda_2}{\lambda_2 + \lambda_1} \), and thus unrated types \( v_2 \) and \( v_1 \) prefer to hold the asset. Now suppose that a regulator imposes a fee cap \( \delta = \frac{\lambda_2 q (1 - \delta) (q \lambda_3 v_3 + \lambda_2 v_2)}{\lambda_2 q + \delta \lambda_2} \) which is less than the fee \( (1 - \delta)v_3 \) charged to type \( v_3 \). Then the CRA profit is maximized under the information structure that induces rating two types \( v_2 \) and \( v_3 \) described in Proposition 6. The regulation is efficient as it allows issue type \( v_2 \) to trade. However, the regulation reduces the profits of the CRA. Informed investors gain positive rent. ■

**Proof of Proposition 13.** The results follow from Propositions 7 and 6. ■

**Proof of Proposition 14.** The result follows from Proposition 6. ■
References


