Authority, Consensus and Governance*

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Abstract

We characterize optimal corporate boards when shareholders face a trade-off between improving information sharing between the board and management and reducing distortions in decision-making arising out of managerial agency. We show that allocating authority to management is suboptimal. Authority should be held by a supervisory board that may be imperfectly aligned with both shareholders and management. Even when management has captured all authority and the board only has an advisory role, the optimal board may be designed to withhold information from management. Optimal advisory boards must however be able to create consensus with management, making the allocation of authority irrelevant.

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Key Words: governance, consensus, supervisory boards, advisory boards, cheap talk, delegation, hierarchies.

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1 Introduction

Effective communication between the board of directors of a corporation and its management is a key determinant of shareholder value. In practice, such information exchange is often constrained by the fact that management has its own agenda. For instance, management may favor an investment motivated in part by the joy of empire-building, or may be biased towards rejecting a raider’s offer due to private benefits of retaining control. If such conflicts of interest are strong, both management and the board may withhold important information from each other during the deliberation process that precedes decision-making. From the normative perspective of maximizing shareholder value, what should the optimal board look like in such situations? Should the board be aligned with management in order to improve information flows between management and a sympathetic board? Or should the board be conflicted with management and aligned with shareholders in order to ensure that final decisions are in shareholder interest albeit at the cost of poor information transmission?

In an important paper, Harris and Raviv (2010) consider exactly this trade-off between informed decision making and distorted decision making. They show that the trade-off is often resolved in favor of an ex-ante commitment by shareholders to delegate all authority to management. Whenever management has more important information relevant for the decision, managerial control over decisions reduces the expected inefficiencies in information transformation. In this paper, we take a closer look at the role of board composition and alignment in determining shareholder value.

We share with Harris and Raviv (2005, 2008, 2010) the feature that both management and the board are likely to have expertise (private information) relevant for the decision facing the corporation. For instance, management may have private information about the operational details of an innovative new investment while the board may have technological, legal or financial expertise about future costs and benefits, likely responses of competitors and regulators, as well as potential spillover effects.\footnote{In the world of private equity (PE) for instance, the role of expert board members in value creation is well-accepted. PE firms such as KKR and Blackstone have operations groups that provide advise to PE partners who are board members in PE-sponsored firms. Blackstone for instance has 17 former CEOs who provide advice and the cost of these groups is in the order of tens of millions of dollars to these firms. PE firms form long-term relationships with these outside advisors, instead of relying exclusively on advice provided by management, presumably because these advisors have expertise not available to management. PE partners with board membership in multiple firms are also likely to have expertise about a wider set of issues compared to management and other specialized insiders.}
We differ from the previous literature in recognizing that the important decisions faced by a board are typically coarse in nature, consisting of either an approval or a rejection of a complex proposal or plan of action. For everyday matters such as inventory management, boards typically defer to managerial expertise on the details of these decisions, perhaps holding on to a right to rubber-stamp a proposal. In the more pertinent situation where the board has to evaluate a complex issue such as a poison-pill takeover defense, the board may or may not approve the proposal but it leaves the design of trigger provisions and other specific details to specialized legal entities. For other strategic decisions such as the decision to enter a new market, boards have to either reject or approve the proposal taking into account its expectations about the future contingencies that are impossible to specify in advance. The ability of the board to take finely tuned decisions may also be constrained by managerial control over implementation of approved projects. A precisely specified directive to control costs is not meaningful when the ability of management to ‘manufacture’ cost overruns in the future cannot also be precisely controlled.

We exploit the twin features of two-sided expertise and coarse decisions to show that it is never strictly optimal for shareholders to delegate all authority to management. This is true regardless of whether it is management or the board that has information more likely to be important for shareholder value. The difference with Harris and Raviv (2010) arises principally because of an extra instrument available to shareholders in our set-up, the alignment of the board. Alignment measures the extent to which the board is more or less inclined to maximize management versus shareholder welfare in taking its decisions. Alignment is the key determinant of the efficiency of information transmission within the firm and once alignment is chosen optimally it is not necessary to give all authority to management. This provides a defense of the normative point of view espoused by Bebchuk (2005) among others that a lack of board supervision over management decisions is detrimental for shareholder welfare and shareholders should try to retain at least some coarse control over ex-post decisions.

We come to this conclusion via contrasting two related information transmission problems—one where the board holds the authority to approve or reject the proposal under discussion (a supervisory board) with one where management effectively holds all authority and may take any decision it wishes with or without approval from the board. In the latter case, the only avenue by which the board can influence management is via advice based on its information (an advisory
For each of these two polar allocations of authority, the alignment of the board determines the efficiency of information transmission. For a supervisory board, the optimal alignment is a response to a trade-off between balancing the distortion in decision making against the gains in information flows similar to the one in Harris and Raviv (2010). Often this trade-off is resolved by a supervisory board that has imperfect alignment with both shareholders and management. In contrast, the problem of designing an optimal advisory board is unique to this paper and it provides our key insights on the interaction between authority and alignment in determining shareholder value.

A conflict between an advisory board and management can only hamper information flows between the board and management without affecting in any way the distortion in the final decision-making that is fully controlled by management. Management can always ignore an advisory board. We show that the shareholder value maximizing advisory board must be at least as management aligned as the optimal supervisory board. However, even though the trade-off associated with supervisory boards is absent here, the optimally aligned advisory board may also have a conflict of interest with management. This is especially likely when the board has information that is important for the decision. When the manager is uncertain about the value of pursuing his favored agenda, imprecise information from an advisory board with valuable information preserves managerial doubt about the benefits of insisting on his agenda. Management may then voluntarily forego his own favored decision, limiting the distortion arising out of managerial control over decision making. This benefits shareholders in ex-ante expected terms even after adjusting for the expected costs of relatively poor information transmission.

We show that a necessary property of the shareholder value maximizing level of conflict between an advisory board and management is that each side agrees with the final decision given its own information and the information revealed endogenously by the equilibrium play of the communication game. We call this property consensus. Consensus is not necessary for successful information exchange and because of the coarseness of decisions it may obtain even when there is a conflict of interest between board and management. When consensus obtains, the allocation of decision-making authority is irrelevant. The optimal advisory board cannot be too conflicted with management—

While we think of this case as a conceptual device that allows us to determine the value of the board’s authority, it is not entirely unrealistic. In reality, management may have decision-making authority when it has ‘captured’ key committees of the board with executive power.
it must be sufficiently aligned so that it can create consensus. In effect, management hands back authority to the optimal advisory board. In contrast, the optimal supervisory board does not need to obtain managerial consensus. The absence of this requirement of obtaining managerial consensus is exactly the source of the gain in shareholder value from being able to assign supervisory authority to the board.

Taken together, our arguments are in favor of at least some board oversight of corporate decisions in situations where management has considerable power within the firm. We strengthen and clarify this insight via a number of additional results that shed light on the source of gains from shareholder control. In a world where shareholders can commit to publicly revealing all information by expert board members, we show that they would strictly benefit from doing so, allocating all decision-making authority to a perfectly shareholder aligned board. Shareholder value can only fall either when this commitment is infeasible or when authority is held by management and cannot be allocated to the board. In such cases it may be optimal for shareholders to respond by impeding information flows between board and management at the cost of inefficiencies in decision making. For instance, when shareholders cannot commit to revealing the board’s information but can allocate effective supervisory authority to the board, they may like to only partially align the supervisory board with management. Even in this situation however, we show that it is harmless for shareholders to retain control over final approval of the decision. This is true as long as they do not observe the details of the deliberations between management and the optimally aligned board and often even when they do. In contrast, if management captures decision-making authority, shareholders may prefer coarse information transmission via an imperfectly aligned advisory board even when it is feasible to commit to revealing the board’s information. When the board has important information, coarse information transmission by the board may result in inefficient decision-making but, from the perspective of shareholders, this may also limit managerial abuse of his authority. Indeed, the optimal advisory board must be sufficiently well-aligned with management in order to create consensus and limit the damage from managerial capture of authority.

Before Harris and Raviv (2005, 2008, 2010), Dessein (2002) also made the similar point that it is often better for an uninformed party to fully delegate authority to an informed party with a conflict of interest. Similar to the present paper, Dessein (2002) allows delegation via a partially aligned intermediary but in a Crawford and Sobel (1982) model with one-sided private information and a continuum of decisions. In contrast, we consider a binary decision problem with two-sided
private information. Our results on supervisory boards are closely related to those of Dessein (2002). But the presence of two-sided private information in our set-up allows a non-trivial role for improving information flows even when all authority is held by management and the board is only an advisor. In particular, it yields the conclusion that it is never strictly optimal to allocate authority to management regardless of whether management or the board has better information, and weakly optimal to do so only when the allocation of authority is irrelevant.

Aghion and Tirole (1997) also analyze the optimal allocation of authority within organizations. They draw a distinction between formal and real authority and show that often the party with formal authority will delegate authority to another agent with information. Communication is equivalent to delegation of real authority to the informed agent since the latter will take/induce its own ideal decision in either case. Intermediation also has no role to play. In our setting in contrast, the board and management have different pieces of information but no agent has superior information. Neither party is guaranteed its own ideal decision under either communication or delegation. We also allow the board to delegate authority to management at the interim stage as a function of its own information. This has no effect on outcomes in our setting because communication at the interim stage is a perfect substitute for delegation accompanied by communication. Still, the ability to allocate formal authority ex-ante to the board is important for shareholders.

Adams and Ferreira (2007) study the tension between the advisory and monitoring role of the board. In their model the board may probabilistically force management to give up control ex-post. If the expected intervention is very likely, management will be uncooperative with an unfriendly board resulting in uninformed decision-making. Baldenius et al. (2010) consider a set-up in which the board may choose either to obtain management’s information or an independent piece of information. They show that the board may prefer to delegate control to management only under limited conflict of interest between management and shareholders. In our framework allocating formal decision-making authority completely to the board is always optimal for shareholders. If authority is held by management, the optimal advisory board must induce management to voluntarily hand back authority to the board.

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3 Harris and Raviv (2010) also allow the principal to delegate authority at the interim stage after obtaining information but since delegation by an informed principal creates an adverse selection problem, they show that such conditional allocation of authority to the agent is often ex-ante suboptimal.

4 There is also a large literature on corporate boards that do not use a strategic information transmission framework. Excellent surveys by Hermalin and Weisbach (2003) and Adams, Hermalin and Weisbach (2009) provide a detailed
Beyond corporate governance, these results further our understanding of the role of authority in organization design. Arrow (1974) suggests that governance by authority is a defining feature of organizations while governance by consensus is a “polar alternative” whose costs are likely to rise “when either interests or information differ among the members of an organization” (chapter 4, pp. 70). We pursue this intuition in a formal model of strategic communication under differences of both information and interests. We show that governance by consensus is the optimal response to a loss of authority but the costs of governance for shareholders may sometimes be lower when the interests of the board and management differ. Our focus on a difference in interests places this paper in the literature that takes an agency-theoretic approach to understanding organizations based on a conflict in preferences between owners and managers (Alchian and Demsetz (1972), Jensen and Meckling (1976)). The emphasis on information transmission reconciles our conflict-based approach in part with the approach that views firms as information processing hierarchies (Williamson (1975), Radner (1992)).

The rest of the paper is organized as follows. Section 2 contains our model and results on communication and consensus. Section 3 characterizes optimal supervisory and advisory boards, Section 4 presents results on shareholder consensus and board structure, Section 5 contains a discussion and concluding remarks and all proofs are in the Appendix.

2 Supervisory and Advisory Boards

2.1 Model

A principal or owner of a firm has to choose between a status quo or an alternative. We think of the principal as the shareholders collectively. The status quo is safe and we normalize its value to shareholders equal to zero. The alternative has uncertain value \( x - y \) for shareholders where \( x \in [0, 1] \) and \( y \in [y_L, y_H] \). If shareholders observed \( x \) and \( y \) then they would like to choose the alternative if \( x - y > 0 \) and the status quo otherwise. We assume however that shareholders are uninformed
discussion of this literature. Papers outside the literature on corporate governance that use strategic information transmission with two-sided information include Agastya et al. (2011), Chen (2009) and de Barreda (2011).

5 Beyond organizations, our analysis of advisory boards in particular sheds light on the problem of choosing a lobbyist from the perspective of an interest group, the capture of regulatory agencies by groups they are meant to regulate, as well as the value to voters from creating conflict between different branches of government.
and $x$ is the private information or expertise of management whereas $y$ is the private information or expertise of the board. Let $F_x$ and $F_y$ denote the common knowledge priors associated with the random variables $x$ and $y$ with densities $f_x$ and $f_y$. We assume that $x$ and $y$ are statistically independent and, for tractability, assume throughout that $F_x$ and $F_y$ are uniform.\(^6\)

We will often refer to the alternative as a decision to ‘invest’ in a project with the status quo being the default option of not investing. Managerial expertise $x$ then reflects specialized knowledge of the operational details of the project whereas the board’s expertise $y$ may be about legal or regulatory issues. Alternatively, the firm may be faced with choice between accepting a raider’s offer or adopting an anti-takeover measure. In such a case, management may have private information about the intrinsic value of the firm under current management whereas the board may have expertise in evaluating the legality of anti-takeover measures proposed by management or in estimating the value of keeping alive the interest of potential raiders in the market for corporate control.\(^7\)

The central conflict of interest in this paper is between management and shareholders. Management is biased in favor of the alternative and the value of the alternative to management is the sum of shareholder value $x - y$ and a private benefit $b_m$ whereas the value of the status quo is zero. Thus, management would ideally like to invest if and only if $x - y + b_m > 0$. The managerial bias parameter $b_m > 0$ reflects managerial agency in the form of private benefits of empire-building and expanded control.\(^8\)

\(^6\)None of the results in this section use this distributional assumption. We use these assumptions in the next section where we characterize optimal boards.

\(^7\)Nothing depends on the status quo being safe and we may think of $x - y$ as the net (possibly negative) benefit of the alternative relative to the status quo. We also permit the relative importance of the two sources of information to vary, e.g., by writing $y = k z$, where $z$ is a random variable capturing the board’s information and $k \geq 0$ a scalar parameter that measures the relative importance of the two sources of information for the decision at hand.

\(^8\)The importance of managerial agency in corporate governance and performance has been extensively analyzed (See, e.g., Jensen and Meckling, 1976). In this paper, we take this agency as given and suppose that the bias $b_m$ is a necessary cost of having a manager with payoff-relevant expertise $x$. We assume as well that all private information is unverifiable and contracts are incomplete so that management is not perfectly aligned with shareholders even after an optimal provision of incentives. We also abstract away from reputational concerns. Apart from difficulties in enforcing reputational schemes when information is noisy and unverifiable, it may not also be optimal to fire a manager with valuable firm-specific capital (e.g., expertise $x$ about the firm) that her replacement will take time to build. See Section 5 for a detailed discussion of all of these issues.
We wish to determine the composition of the board and the optimal allocation of authority from the ex-ante perspective of shareholders. We capture the composition of the board simply by a bias parameter \( b_d \in [0, b_m] \) that captures the board’s “ideological” alignment with management. The value of the alternative to the board is \( x - y + b_d \) while the value of the status quo is zero. In general, the board may consist of both shareholder aligned members (such as independent directors) or management aligned members (friends or associates of management). We may think of the ratio \( b_d/b_m \) as the weight that the board puts on maximizing managerial welfare in taking its decisions, with \( 1 - b_d/b_m \) the weight on shareholder welfare.\(^9\)

Subsequently, the board learns its information \( y \) while management learns \( x \) following which the board and management communicate. Communication between the board and management is strategic and takes the form of cheap talk. Communication is followed by the final decision taken by the agent holding authority. We will contrast two alternative allocations of authority in the analysis to follow. In the first case, the board has final decision-making authority and any decision made by the firm must obtain the board’s approval. We call this the case of a supervisory board. In the second case, management has total control over the firm and can take any decision it wants even in defiance of the board’s wishes. The composition of the board is relevant in this case only because of the information held by the board. We call this the case of an advisory board. We wish to identify the shareholder value maximizing choice of \( b_d \) both in the case of a supervisory board and an advisory board. In doing so, we also determine the value of authority, i.e., the relative merits of supervisory and advisory boards after the composition of the board \( b_d \) has been optimized.

Throughout the paper we impose the parameter restrictions \( y_L < 1 \) and \( y_H > b_m \). If instead \( y_L \geq 1 \), then shareholders are not interested in obtaining information about \( x \) and can do no better than choosing perfectly shareholder aligned supervisory board that never chooses the alternative. Similarly, if \( y_H \leq b_m \), then management is not interested in obtaining information about \( y \) and an advisory board has no role to play. Even with a supervisory board, management will necessarily be uninformative since it prefers to choose the alternative regardless of the state of the world. In

\(^9\)Alternatively, if the board takes its decisions via majority voting then we may think of \( b_d \) as the bias of the median board member.

\(^{10}\)In reality, both management and shareholders have a say in board composition. In this paper we take a normative approach and focus on determining the necessary properties of shareholder value maximizing boards.
such cases, shareholders can do no better than choosing a perfectly shareholder aligned supervisory board that chooses the alternative as a function of \( y \). Our results are more interesting in the complementary case where \( y_L < 1 \) and \( y_H > b_m \) and each party has an interest in learning the other party’s information.

One last feature of our model also deserves comment. We focus on a binary decision problem with two-sided private information. This is a simple way of capturing the fact that real world board decisions are rarely about operational details such as a price or a sales level but concern broader strategic issues such as the decision to acquire a target firm. The details and contingencies that affect the value of these decisions can rarely be specified in advance and it may be suboptimal to do so if future information forces a reconsideration. The coarseness of decisions can also capture a situation where most of the effective authority within the firm is in the hands of management limiting the ability of the board to fine tune management control over the agenda-setting aspect of the firm as well as its control over the implementation of approved proposals. Both assumptions of two sided private information and the coarseness of decisions are important for our results as will be made clear in what follows.

2.2 Communication

We consider the case of a supervisory board first. We model this as a simple game where management first sends a cheap talk message \( \mu \) to the board, possibly as a function of \( x \), following which the board takes the final decision after taking into account its own information \( y \). If \( b_d = b_m \), then the management and board have perfectly aligned interests. Therefore they should be willing to perfectly share their information in order to implement the optimal decision rule given their common interests. This involves investing whenever \( x - y + b_m > 0 \). For the rest of this section we focus on determining the nature of communication in the more interesting case where \( b_d \neq b_m \) and there is a conflict of interest between the board and management.

Given any message \( \mu \) from the management, the board will prefer to invest whenever its own signal \( y \) is less than a threshold value \( t = E[x|\mu] + b_d \). From the perspective of management, the expected payoff from sending message \( \mu \) is the expected value of the investment times the probability that investment occurs, i.e., the expected payoff equals

\[
\Pr[y < t] \left[ x - E(y|y < t) + b_m \right] = q(x + b_m) - \int_0^q F_{y}^{-1}(z)dz, \tag{1}
\]
where \( q = \Pr[y < t] = F_y(t) \) is probability that the board chooses the alternative. The last expression is a supermodular function of \( x \) and \( q \) that is concave in \( q \). It follows that management’s communication strategy must take an “interval partitional” form, i.e., management will disclose an endogenously determined interval within which \( x \) lies. Proposition 1 describes the equilibrium set.\(^{11}\)

**Proposition 1** Suppose \( b_d \neq b_m \) and assume a supervisory board. In any cheap talk equilibrium, management discloses the interval \( [c_{i-1}, c_i) \) in which \( x \) lies, \( 0 = c_0 < c_1 < \ldots < c_N = 1 \), where

\[
c_i = E[y|t_i < y < t_{i+1}] - b_m, \quad i = 1, \ldots, N - 1.
\]

(2)

Following management’s message that \( x \in [c_{i-1}, c_i) \), the board chooses the alternative iff \( y < t_i \) where

\[
t_i = E[x|c_{i-1} < x < c_i] + b_d, \quad i = 1, \ldots, N.
\]

(3)

There exists \( N^* (b_d, b_m) < \infty \) that is an upper bound on the number of distinct thresholds \( t_i \) that can be induced in equilibrium.

Figure 1 illustrates Proposition 1. In the figure we take \( b_m = \frac{3}{16} \) and \( b_d = 0 \). For these parameter values, there is an informative equilibrium in which the management sends two distinct messages. The ‘low’ message \( \mu' \) reveals that \( x < c_1 = \frac{1}{8} \) whereas the ‘high’ message \( \mu \) reveals that \( x > \frac{1}{8} \).\(^{12}\)

For the low message \( \mu' \), the board chooses the alternative whenever \( y < t_1 = E[x|x < \frac{1}{8}] = \frac{1}{16} \), whereas for the high message the board chooses the alternative whenever \( y < t_2 = E[x|x > \frac{1}{8}] = \frac{9}{16} \).

\(^{11}\)The change of variables in expression (1) shows that our binary decision problem with two-sided private information has a similarity with the canonical model of cheap talk with one-sided private information and continuous actions considered in Crawford and Sobel (1982, henceforth CS). In particular, the probability \( q = \Pr[y < t] \) corresponds to the action of the receiver in the CS interpretation of (1), with \( x \) the information of the sender. The correspondence is imperfect however, in particular because the probability \( q \) is bounded and because we may have weak but not strict upward bias for the sender in some cases (Gordon, 2007). Nevertheless, we are able to exploit this imperfect correspondence for some results in the paper. At the same time, the coarseness of decisions and the presence of two-sided private information allow us to obtain many novel results that have no direct parallel in the CS framework.

\(^{12}\)In the figure, management information \( x \in [0, 1] \) while the board’s information \( y \in [0, \frac{22}{16}] \). The figure also shows the decision rule that would be used if shareholders had all the information and authority (the line \( y = x \)) and the rule that would be used if management had all the information and authority (the line \( y = x + b_m \)). Any cheap talk game also has a babbling equilibrium where the board does not attach any meaning to the management’s messages and so the management cannot do better than being uninformative.
The resulting step function depicts the decision rule that is implemented in equilibrium—below the step function the alternative is chosen by the board whereas above the step function the status quo is chosen.\textsuperscript{13} The cutoff type $c_1$ is indifferent between the two decisions, but \textit{conditional} on the event that the message sent makes a difference for the decision, i.e., on the event that the board’s information $y$ is in between the two thresholds $t_1$ and $t_2$. In general, depending on $b_d$ and $b_m$, there may be more than one informative equilibrium in the communication game between the board and management. We focus on the most informative equilibrium with $N^*(b_d, b_m)$ partition elements throughout what follows.\textsuperscript{14}

Proposition 1 characterizes all equilibria in the game where management first communicates with the board following which the board takes a decision. Call this sequence of moves $mb^*$, where we use the asterisk to denote the allocation of decision-making authority to the supervisory board. For our next result we focus on an alternative allocation of authority and characterize the equilibria of the game $bm^*$ in which the board first speaks to management following which management takes the final decision. In $bm^*$ decision-making authority is held by management and the board only

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Decision Rule under Supervision}
\end{figure}

\textsuperscript{13}A decision rule chooses a decision (status quo or alternative) as a function of $x$ and $y$. We say that a particular decision rule is an equilibrium decision rule, or implemented in equilibrium, if that mapping from states to decisions is the outcome of equilibrium play between management and the board for some equilibrium.

\textsuperscript{14}The results in this section do not rely on equilibrium selection. We need to pick a particular equilibrium for the results in subsequent sections where we compare payoffs across different organizational arrangements. Our selection rule is the same as that in the rest of the literature and can be defended either on the grounds of ex-ante Pareto dominance or via the refinement proposed by Chen, Kartik and Sobel (2008) whenever it is applicable.
has an advisory role.

**Proposition 2** Fix $b_d \neq b_m$ and assume an advisory board. In any cheap talk equilibrium, board discloses the interval $[t_{i-1}', t_i')$ in which $y$ lies, $y_L = t_0' < ... < t_i' < ... < t_M' = y_H$ where

$$t_i' = E[x|c_{i-1}' < x < c_i'] + b_d, i = 1, ..., M - 1.$$  \tag{4}

Following the board’s message that $y \in [t_{i-1}', t_i')$, management chooses the alternative iff $x > c_i'$ where

$$c_i' = E[y|t_i' < y < t_{i+1}'] - b_m, i = 1, ..., M.$$  \tag{5}

There exists $M^*(b_d, b_m) < \infty$ that is an upper bound on the number of distinct cutoffs $c_i'$ that can be induced in equilibrium.

This result is the analogue of Proposition 1 for the case where decision-making authority is held by the management instead of the board.$^{15}$ While the structure of equilibria characterized by Propositions 1 and 2 look qualitatively similar, the two games may result in quite different outcomes. The decisions that are taken as a function of $x$ and $y$ in an equilibrium of the game $mb^*$, where the board has supervisory authority, are in general not identical to the decisions that can be supported in an equilibrium of $bm^*$, where the board is an advisor. In the rest of this section we further characterize the similarities and differences between the decision rules that can be implemented in equilibria of the two games $mb^*$ and $bm^*$ for fixed $b_d$ and $b_m$.

We begin by showing that given an allocation of authority the details of the communication protocol (or game form) does not affect the equilibrium decision rules. To understand this, compare the game $mb^*$ with the game $bmb^*$. In $bmb^*$ the board first communicates with management, following which management communicates with the board, following which the board takes a final decision. In both $mb^*$ and $bmb^*$ final decision-making authority is held by the board but more extensive multi-stage communication is permitted in the latter. Since in our setting both parties are privately informed it is not clear a priori whether or not the implicit restriction on communication by the board in the game $mb^*$ has any effect on the decisions taken by the board.

Our next result shows that the equilibrium decision rules are not affected by the details of the communication protocol. In the first part of the result we compare the game $mb^*$ with the

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$^{15}$As with the previous result, we focus on the most informative equilibrium with $M^*(b_d, b_m)$ elements whenever we need to choose an equilibrium.
games $bmb^*$ as well as $b^*mb^*$. The game $b^*mb^*$ is similar to $bmb^*$ except that the board can also take a unilateral decision in the first round instead of choosing to communicate with management. If it does the latter, then management speaks to the board following which the board takes a decision. The first part of Proposition 3 shows that the equilibrium decision rules are identical in these three games in which decision-making authority is held by the board. The second part of the result provides the same result for the analogous games $bm^*$, $mbm^*$ and $m^*bm^*$ in all of which management holds decision-making authority and the board just has an advisory role.

**Proposition 3** Fix $b_d \neq b_m$.

1. *The set of equilibrium decision rules is identical across the games mb*, $bmb^*$ and $b^*mb^*$ in which the board has authority.*

2. *The set of equilibrium decision rules is identical across the games bm*, $mbm^*$ and $m^*bm^*$ in which management has authority.*

Proposition 3 shows that, given an allocation of decision-making authority, extended communication by the two parties has no additional benefit in our setting. While the proof contains some important technical details, the intuition is relatively straightforward—the information that the two parties can bring to bear on the decision depends on their conflict of interest and each party can take this conflict (and the other party’s incentives) into account even with one round of communication by conditioning on the event when its own message makes a difference for the decision.\(^{16}\)

In the case of a supervisory board, decision-making authority is held by the board. We ask next if allowing such a supervisory board to delegate decision-making authority to management, as a function of its private information $y$, makes a difference for our results. We capture the possibility of delegation by a supervisory board with the game $b^*m^*$. In this game, the board may first take a unilateral decision or communicate with management. If the board does not take a unilateral decision and only communicates with management, it also hands over decision making authority and lets management take the final decision. The first part of our next result compares

\(^{16}\)While the result has the flavor of a revelation principle type argument, it is unrelated. This is because of the added constraint in a cheap talk games that the decision-maker must take sequentially rational decisions.
the equilibria of $b^*m^*$ with those of $mb^*$, the baseline case with a supervisory board. The second part compares the analogous game $m^*b^*$ with $bm^*$, the baseline case with an advisory board.

**Proposition 4** Fix $b_d \neq b_m$.

1. Any equilibrium decision rule of $b^*m^*$ is also an equilibrium decision rule of $mb^*$. If an equilibrium decision rule of $mb^*$ is not an equilibrium decision rule of $b^*m^*$, then there is a more informative equilibrium of $mb^*$ with a decision rule that is an equilibrium decision rule of $b^*m^*$.

2. Any equilibrium decision rule of $m^*b^*$ is also an equilibrium decision rule of $bm^*$. If an equilibrium decision rule of $bm^*$ is not an equilibrium decision rule of $m^*b^*$, then there is a more informative equilibrium of $bm^*$ with a decision rule that is an equilibrium decision rule of $m^*b^*$.

Proposition 4 shows that the possibility of delegation of authority by a supervisory board to management has no effect on outcomes at least as long as one focuses on the most efficient equilibrium. Similarly, if management has authority, delegation by management to the advisory board at the interim stage cannot alter the set of achievable outcomes. In essence, the party with authority may wish to exercise that authority without engaging in communication only when it has extreme information. In the absence of such extreme information each party is interested in the information held by the other. In such cases, communication is driven by the commonality of interest between the two parties and it does not matter for outcomes whether or not authority is also delegated as long as communication is informative.

Taken together the results in this section show that a conflict of interest between the board and management necessarily results in coarse communication between the two parties. Given an allocation of authority, it is this conflict that determines the incentives to communicate and the decision rules that can be implemented in equilibrium. The precise details of the communication protocol are not important for determining shareholder value. Neither is the possibility of transfer of authority from one agent to the other given an initial allocation of authority to the board or management. In the next section we ask if and when the initial allocation of authority is also irrelevant for shareholder value.
2.3 Consensus

As mentioned before the decision rules that can be implemented in an equilibrium for the game \( mb^* \) with a supervisory board are not in general identical to the decision rules that can be implemented in an equilibrium of \( bm^* \) where the board is an advisor. We show now that an equilibrium decision rule does not depend on the allocation of authority when the equilibrium displays a property that we call consensus. Consensus obtains when the agent without decision-making authority agrees with (i.e., does not wish to overturn) the final decision made by the agent who has such authority, given his own information and the information revealed by the equilibrium play of the game.\(^{17}\) Thus, in \( mb^* \) where the board has decision-making authority, consensus obtains when the manager agrees with every possible board decision after every possible sequence of messages sent in equilibrium and never wishes to overturn the board’s decision after it is made even if he could do so. Similarly, in \( bm^* \) where management has authority, consensus obtains when the board never wishes to overturn the management’s final decision if it had the power to do so. Notice that consensus is not a requirement of equilibrium but rather a property that a particular equilibrium may or may not possess. The next result identifies necessary and sufficient conditions for consensus to obtain in any equilibrium of \( mb^* \) with \( N \) messages (respectively, of \( bm^* \) with \( M \) messages).

**Proposition 5** Fix \( b_d \neq b_m \). When consensus obtains in an equilibrium of \( mb^* \) (respectively, \( bm^* \)), the same decision rule can be implemented in an equilibrium of \( bm^* \) (resp., \( mb^* \)) in which decision-making making authority is held by the other party.

1. In \( mb^* \), with a supervisory board, consensus obtains iff (i) \( E[y|y > t_N] \geq 1 + b_m \) or \( t_N = y_H \) and (ii) \( E[y|y < t_1] \leq b_m \) or \( t_1 = y_L \).

2. In \( bm^* \), with an advisory board, consensus obtains iff (i) \( E[x|x > c'_M] + b_d \geq y_H \) or \( c'_M = 1 \) and (ii) \( E[x|x < c'_1] + b_d \leq y_L \) or \( c'_1 = 0 \).

To gain insight into this result, consider the game \( mb^* \) in the special case where no agent has any information about \( y \), equivalently \( y_L = y_H = E[y] \), the expected value of \( y \). In this special case it is easy to see that in any informative equilibrium management can send at most two messages,\(^{17}\) Baranchuk and Dybvig (2009) propose a reduced form bargaining solution concept called consensus that abstracts away from considerations of strategic information transmission. Our notion of consensus is entirely unrelated.
a high message revealing $x > E[y] - b_m$ following which the board chooses the alternative and a low message $x < E[y] - b_m$ following which the board chooses the status quo. Such an equilibrium, when it exists, displays consensus since management agrees with the board’s decision to implement what is in fact management’s ideal decision rule. If instead of $mb^*$, the two parties played the game $bm^*$, the board’s message would trivially be uninformative and management would choose the same decision rule.

Proposition 5 extends this idea of consensus to the case where both parties have non-trivial information. When both sides are informed, an equilibrium may involve more than two messages and neither side is guaranteed its ideal decision rule in any equilibrium. Proposition 5 shows that for consensus to obtain in an equilibrium of $mb^*$ it is sufficient that the extreme types of the manager, $x = 0$ and $x = 1$, agree with every possible board decision that they may encounter on the path of play. The rough intuition is that when type $x = 1$ agrees with an observed decision to choose the status quo given his equilibrium message so must every lower type $x < 1$ given the message such a type sends in equilibrium. Similarly, when the lowest type $x = 0$ agrees with an observed decision to choose the alternative given his equilibrium message so must every higher type given the message such a type sends in equilibrium. Since the conflict of interest fully determines the incentives to reveal information in equilibrium and since every type of every player agrees with every decision observed in equilibrium when consensus obtains, the properties of the equilibrium cutoffs and thresholds provided in Propositions 1 and 2 imply that an equilibrium decision rule of $mb^*$ that displays consensus is also an equilibrium decision rule of $bm^*$ (that displays consensus). Proposition 5 shows that, for fixed $b_d$ and $b_m$, the allocation of authority matters for the decision rule only if the corresponding equilibrium does not display consensus.\(^\text{18}\)

The possibility of consensus crucially depends on the coarseness of actions. If optimal actions were continuous invertible functions of information, then management would be able to exactly infer the information held by the board from observing the board’s decisions. As long as the two

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\(^{18}\)While Proposition 5 applies to any equilibrium, given a monotonicity property of the equilibrium cutoffs and thresholds, it can be shown that the most informative equilibrium is most likely to yield consensus in the following sense: if an equilibrium with $N'$ messages yields consensus then so will an equilibrium with $N > N'$ messages. The monotonicity property that yields this conclusion shows that $t_{1:N} \leq t_{1:N'}$ and $t_{N:N} \geq t_{N':N'}$, where $t_{i:N}$ is the $i$th threshold used by the board in a $N$ message equilibrium, and $t_{i:N}$ that in a $N'$ message equilibrium, with $N > N'$. In this sense, more informative communication raises the possibility of consensus and minimizes the probability of the use of authority.
parties have conflicting interests, management would then prefer to implement at least a slightly different action than the one chosen by the board. In contrast, with coarse decisions only coarse information about the state is revealed from decisions, creating the possibility of consensus.

The previous discussion also illustrates why consensus is not a notion of ex-post agreement. Since the two parties have conflicting interests they can never agree ex-post, i.e., if all private information becomes public. Consensus is a property of the amount of information that is consistent with incentives and that can be revealed in equilibrium. Even when consensus obtains, the two parties only agree with the decision but typically do not agree on the value of the decision.\footnote{It is useful also to contrast consensus with the notion of posterior implementability introduced by Green and Laffont (1997) in a mechanism design context. Like consensus, posterior implementability is a notion of ex-post ratification given information that is revealed by the equilibrium play of a communication game or mechanism. It is not difficult to check that every equilibrium of $mb^*$ and $bm^*$ is posterior implementable. Since an equilibrium may not display consensus however, posterior implementability is a weaker notion of ex-post ratification than consensus.}

### 3 Optimal Boards and the Value of Authority

In this section we provide a characterization of the optimal alignment of a supervisory board $b^*_d$ and the optimal alignment $b^{**}_d$ of an advisory board from the ex-ante perspective of shareholders. To this end, let $V_{b\text{-authority}}(b_d; b_m)$ be the ex-ante expected shareholder value when the supervisory board controls decision-making. By definition, the optimal alignment $b^*_d$ of a supervisory board solves

$$\max_{b_d} V_{b\text{-authority}}(b_d; b_m).$$

Similarly, let $V_{m\text{-authority}}(b_d; b_m)$ be the ex-ante expected shareholder value when management controls decision-making and the board has only an advisory role. By definition, the optimal alignment of an advisory board $b^{**}_d$ solves

$$\max_{b_d} V_{m\text{-authority}}(b_d; b_m).$$

The following result provides a characterization of the optimal supervisory board in a subset of the parameter space.

**Proposition 6** Suppose $y_H > 1 + b_m$ and $y_L = 0$. With an expert supervisory board, the optimal board alignment $b^*_d = b_m$ if $b_m \leq \frac{1}{6}$. For $b_m > \frac{1}{2\sqrt{2}}$, $b^*_d = 0$. For intermediate values of $b_m \in \left(\frac{1}{6}, \frac{1}{2\sqrt{2}}\right)$,
Figure 2: Optimal Alignment $b_d^*$

$b_d^* \in (0, b_m)$ and the optimal supervisory board has a conflict of interest with both shareholders and management.

The parameter restriction on $y_H, y_L$ and $b_m$ ensures that all the thresholds $t_i$ in Proposition 1 are interior, i.e., $t_i \in [y_L, y_H]$ for all $i$. In this case the equilibrium sets, ex-ante payoffs and the value of delegation can be characterized using the analysis of Crawford and Sobel (1982) and Dessein (2002). The result then follows directly from Proposition 5 in Dessein (2002).\(^{20}\)

Figure 2 plots the alignment of the optimal supervisory board $b_d^*$ as a function of the underlying managerial bias $b_m$ under the parameter restrictions of Proposition 6. When the basic conflict of interest $b_m$ between management and shareholders is small relative to the value of information held by management ($b_m \leq \frac{1}{\delta}$) shareholders will gain by choosing a board that is perfectly aligned with management. It does not pay to impair information exchange between the board and management since managerial agency is small and distortions in decisions from a management aligned board are small relative to the gain in information flows. In contrast, when $b_m \geq \frac{1}{2\sqrt{2}}$ managerial agency is sufficiently value destructive for shareholders to entirely forego eliciting information from management. The optimal choice of $b_d$ in this case ensures that the board makes its decision without learning anything about the manager’s information $x$. In the intermediate case $b_m \in \left(\frac{1}{\delta}, \frac{1}{2\sqrt{2}}\right)$, the shareholders limit the distortion in decision-making by choosing a board that is partially management aligned and partially shareholder aligned. Although this leads to some information loss,

\(^{20}\)Similar results will obtain when $y_H < 1 + b_m$ or $y_L \neq 0$ although in such cases we may have corner solutions and the CS analysis will not immediately apply. We avoid an explicit calculation of expressions for $b_d^*$ in such cases as it does not add any further insight.
the loss is swamped by the gain from reducing distortions in decision-making away from management’s ideal and towards what is optimal for shareholders. Notice that by the envelope theorem the optimized shareholder value must be decreasing in $b_m$. Even after board composition has been optimally chosen, higher values of $b_m$ will be associated with lower firm performance all else held equal. Since optimal board alignment is a response to an agency problem $b_d^*$ may be non-monotonic in managerial agency and firm performance.\footnote{It can be shown that when $b_d^* \in (0, b_m)$, $b_d^* = \frac{N_*^2 - 1}{N_*^2 + 2} b_m$ where $N_*$ is the number of messages sent in the most informative equilibrium by management to the optimal supervisory board. Furthermore, $N_*$ is equal to either the integer least upper bound or the integer greatest lower bound of $\sqrt{\frac{2}{b_m - 1}}$ and for values of $b_m \in \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right)$ that allow one to ignore these integer constraints, $b_d^* = \frac{1}{4} - \frac{1}{2} b_m$ for $b_m \leq \frac{1}{2}$ and $b_d^* = \frac{1}{2} b_m$ for $b_m > \frac{1}{2}$. The discontinuities in Figure 2 arise from the integer constraints on $N_*$ that arise endogenously because of the strategic information transmission between the board and management.}

We turn next to the optimal alignment $b_d^{**}$ of an advisory board. The following is our central result on the optimal advisory board.

**Proposition 7** Fix $b_m$. The optimal advisory board $b_d^{**}$, which is the solution to the program in (7), is also the solution to the program in (6) but subject to the additional constraint on $b_d$ that the board and management reach consensus.

If consensus does not obtain with an advisory board then shareholder value can be raised by raising $b_d$ slightly. This either lowers the loss from imperfect communication or raises the number of distinct messages used in the most informative equilibrium. As a result, in solving the program (7), it is harmless to impose the constraint on $b_d$ that consensus obtains. But if consensus obtains, the allocation of authority is irrelevant and consensus also obtains in the game $mb^*$ with a supervisory board that has the same alignment. It follows that $b_d^{**}$ must in fact also solve the program in (6) subject to the additional constraint of consensus.\footnote{For Proposition 7 it is necessary that communication be unrestricted and that the most informative equilibrium is played. The result does not obtain if, for instance, players can only send binary messages, e.g., by voting on the decision.}

The intuition behind Proposition 7 is as follows. If the alignment $b_d$ of the advisory board is such that there is no consensus, there is no trade-off between informed decision making and distorted decision that is relevant for the case of a supervisory board. Consequently, shareholder value must rise from increasing $b_d$ since this only improves information flows. In contrast, if the alignment $b_d$
is such that consensus obtains, it is as if the advisory board is also a supervisory board and there is a trade-off between informed decision making and distorted decision making. The alignment of the optimal advisory board optimizes this trade-off but given consensus is reached.

Proposition 7 contains our key insight on optimal corporate governance from the perspective of shareholders. Relative to allocating authority to management and optimizing over the alignment of the resulting advisory board, allocating authority to an optimally aligned supervisory board avoids the requirement that the board and management reach consensus. The optimal supervisory board may sometimes use the force of its authority to over-rule management. This is beneficial for shareholders. Shareholders can never find it strictly optimal to allocate authority to management and the value of a supervisory board’s authority for shareholders is exactly equal to the cost of imposing the constraint that the board and management reach consensus.

If management somehow captures authority, Proposition 7 shows that an optimal response for shareholders is not to allow management to use the force of its authority with positive probability. Consequently, shareholders should choose a board that is more aligned with management and less aligned with shareholders. This makes management unwilling to take decisions in defiance of the board’s wishes and limits the damage from managerial capture of decision-making authority. The next result makes precise the relationship between authority and optimal alignment.

**Proposition 8** The shareholder value maximizing advisory board is weakly more management aligned than the shareholder value maximizing supervisory board, \( b^*_a \geq b^*_s \) with equality iff consensus obtains at \( b^*_d \).

Proposition 8 shows that a shareholder value maximizing board must be weakly more aligned with shareholders when the board holds authority and weakly more aligned with the management when management holds authority. It follows immediately from Proposition 7 via observing that if consensus does not obtain when \( b_d = b^*_d \), to obtain consensus it is necessary to raise the alignment \( b_d \) of the advisory board closer to \( b_m \). In contrast, if consensus obtains even under a supervisory board the optimal alignment of the board does not depend on the allocation of authority. In such cases, the allocation of authority is irrelevant for shareholder value.

Consensus obtains at \( b^*_d \) if for instance \( b^*_d = b_m \), in which case we must also have \( b^*_a = b^*_d = b_m \). Even when \( b^*_d < b_m \), consensus can obtain if for instance \( y_H \) is large enough. In such cases \( b^*_a = b^*_d < b_m \) and the optimal advisory board is conflicted with management. However, the allocation of authority
is irrelevant for shareholder value. If consensus does not obtain with a supervisory board with alignment \( b_d^* \), the allocation of authority matters for shareholder value and \( b_d^{**} > b_d^* \). We may still have \( b_d^{**} < b_m \) so that the optimal advisory board is conflicted with management. For this to be the case it is necessary that the board’s information be sufficiently important relative to the information held by management. For the parameter values of the example depicted in Figure 1 with \( y_L = 0, y_H = \frac{22}{15} \) and \( b_m = \frac{3}{10} \), the alignment of the optimal supervisory board is equal to \( b_d^* = \frac{5}{32} \). Consensus does not obtain at this point and the supervisory board must use its authority with positive probability. It follows from Proposition 8 that the alignment of the optimal advisory board \( b_d^{**} > b_d^* \). Computations verify that the optimal advisory board has alignment \( b_d^{**} = \frac{17}{100} < b_m \) at which point consensus is just obtained.\(^{23}\) By choosing such an advisory board shareholders in effect commit to withhold information from management. The board provides coarse or imprecise advice to management resulting in losses arising from imperfect communication. However, coarse advice from the board also preserves managerial uncertainty about the value of his favored decision even when \( x \) is large. This leads the manager to sometimes voluntarily forego the investment, tempering his agency and enabling shareholders to partially overcome the value loss created by imperfect communication.

4 Shareholder Consensus and Board Structure

So far we have shown that shareholders should optimally allocate supervisory authority to the board, choosing the alignment of the board optimally as well. In doing so we have assumed that the shareholders can commit to respecting themselves the authority of the board. Our next result shows that such commitment is not necessary when the supervisory board’s alignment is optimally chosen since in such cases shareholders will agree with every possible final decision of the board. The optimal supervisory board need only be an intermediary.

**Proposition 9** With the optimal board and at the optimal allocation of authority, shareholders will agree with the final decision, provided they do not observe the deliberations between the board and management.

\(^{23}\)In general, the consensus constraint may hold with slack at \( b_d = b_d^{**} \) because \( V_{b-authority}(b_d; b_m) \) is not globally concave in \( b_d \).
Proposition 9 shows that the optimal board must obtain ‘shareholder consensus’. It follows from the following two observations. First, shareholders will necessarily agree with a board decision to choose the status quo because both the management and the board are biased against the status quo relative to shareholders and a recommendation from the board against the direction of its own bias will meet with shareholder approval. Furthermore, shareholders will also agree with the optimal board’s decision to choose the alternative. This is because the optimal board is weakly better in expected terms than a perfectly shareholder aligned board and shareholders always agree ex-post with the decision of a perfectly shareholder aligned board. Proposition 9 can be interpreted as saying that shareholder ‘outrage’ with a board decision is inconsistent with an optimally aligned board that maximizes shareholder value.24

The possibility of shareholder consensus sheds light both on features of one-tier boards that are commonly observed in the U.S., Canada and the U.K. as well as those of two-tier boards that are more common in continental Europe (see, e.g., Cadbury (1995), Maassen (2002)). One tier boards usually have both executive as well as independent members and integrate the advisory function of a corporate board with its supervisory role. In contrast, two-tier boards tend to separate these two functions. The lower tier provides advice on decision management and typically has executive members. The upper tier is concerned exclusively with a supervisory role and typically contains only founding family members with controlling interests as well as independent directors who are presumably more aligned with shareholders. Proposition 9 shows that the optimal board may be designed as a two-tier structure with a perfectly shareholder aligned upper tier that has final decision-making authority. As long as the lower tier is also chosen optimally, and as long as the upper tier does not observe the details of the deliberations between the lower tier and management, shareholder aligned decision-making authority within the board does not jeopardize shareholder welfare. In this sense, shareholder board membership and activism is consistent with shareholder value maximization provided the activist belongs to the supervisory upper tier and there is a lack of transparency between the upper and lower tiers.25

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24 As with consensus between the board and management, the possibility of shareholder consensus also depends on the coarseness of actions. Notice however that not all supervisory boards will generate shareholder consensus and the argument needs the optimal choice of the supervisory board’s alignment.

25 The two-tier structure is not necessary for this interpretation since we may always label the tiers as different mutually opaque subcommittees of a one-tier board. Moreover, the lack of transparency between the two tiers is
To get more insight into the effect of transparency and shareholder authority on shareholder value, we turn finally to the question of identifying the optimal decision rule when shareholders can commit to reveal $y$. Using the correspondence of the our model with the CS framework, we can use well-known results established in that context to obtain the following result.$^{26}$

**Proposition 10** When the board has decision-making authority, shareholders benefit from revealing $y$ perfectly to management and the board, allocating authority to a perfectly shareholder aligned board.

When shareholders can commit to reveal $y$, it is optimal for them to allow management to take its own optimal action unless $y$ is higher than a cut-off value equal to $1 - b_m$ in which case the alternative is not chosen. This can be implemented by a two-tier board where all board members observe $y$, with the lower perfectly management aligned tier first communicating privately with management and then with the perfectly shareholder aligned upper tier that holds final decision making rights.

If the information of the board arrives to all board members from some expert outside auditor or consultant then it may be feasible to commit to reveal it to all parties and Proposition 10 applies.$^{27}$ In this ideal situation shareholders would like complete transparency about the board’s information $y$. Moreover, they do not need to choose a board that is even partially aligned with management. Instead they can keep final decision-making authority themselves. Only when committing to reveal $y$ is not feasible do shareholders need to consider aligning the board with management. By choosing an expert tier of the supervisory board that is not perfectly aligned with management, shareholders in effect create a lack of transparency between the board and management. Even in such cases, as Proposition 9 reveals, shareholders may hold final decision-making authority without any loss sufficient but not necessary for obtaining shareholder consensus. Even if shareholders observe the messages (but not the signals) of the board and management, shareholder consensus may obtain, especially if communication takes place according to the extended protocol $bmb^*$. Decision-irrelevant changes in the communication protocol may therefore have a bearing for third-party (shareholder) consensus.

$^{26}$We use the optimal arbitration mechanism identified by Goltman et al (2009) in the Crawford and Sobel context. See also Holmstrom (1984), Melumad and Shibano (1991), Dessein (2002).

$^{27}$Proposition 10 also implies that shareholders will benefit if, for instance, a management aligned board member privately “leaks” the board’s information to management. If this is anticipated, shareholders should allocate authority to a perfectly shareholder aligned board.
of welfare, provided the shareholder aligned upper tier does not observe the deliberations between management and the lower expert tier of the board with alignment $b_d^*$. 

Proposition 10 does not apply when it is not possible to freely share the board’s information with all parties. This may be the case if for instance the signal $y$ is privately observed by a particular member or subcommittee imperfectly aligned with both shareholders and management. In such cases only coarse information about $x$ and $y$ can be brought to bear upon the decision and the optimal alignment of the expert supervisory board is determined by the ex-ante trade-off faced by shareholders between information transmission and distortions in decision-making in the communication game between board and management.

Proposition 10 is also limited to the case where the board has supervisory authority. It does not apply to situations where management has captured authority and the board only has an advisory role. As shown in the previous section, when $b_d^{**} < b_m$ shareholders prefer a lack of transparency between an advisory board and management created via a conflict between the two. But, in sharp contrast to the case with a supervisory board, this is optimal for shareholders even when it is feasible for them to commit to reveal the board’s information $y$ via choosing $b_d = b_m$. Since management holds authority it cannot be prevented from choosing the alternative when $y > 1 - b_m$ and shareholders may then prefer imperfect transparency and coarse information transmission.

5 Discussion and Concluding Remarks

We characterize shareholder value maximizing boards in situations where the board and the manager have independent expertise about the value of accepting a plan or rejecting it in favor of an alternative. In such situations, effective information transmission between the board and management is a key determinant of shareholder welfare. We allow shareholders to choose both the alignment of the board (that affects information flows) as well as the allocation of decision-making authority (that affects distortions in decision making). Our objective is to find the normative properties of shareholder value maximizing governance structures in these environments.

In contrast to the broad message of the closely related literature, the main thrust of our arguments is in favor of at least some shareholder-aligned oversight of corporate decisions. Allocating all decision-making authority to management is never strictly optimal for shareholders. When it is feasible to publicly reveal all information that is not held by management, shareholders would ben-
efit from a commitment to such transparency and allocate decision-making authority to a perfectly shareholder aligned board. If it is not feasible either to publicly reveal the board’s information (because its unavoidably private information) or to allocate authority to the board (because management has captured authority), then shareholders are typically hurt. In such cases, if shareholders can allocate authority to a supervisory board, they would like to do so, choosing the board’s alignment to optimize information flows. Such an optimally aligned supervisory board need only be an intermediary and shareholders can still hold final decision-making authority without any loss in welfare. In contrast, if management has captured authority, shareholders may actually like to impede information flows via an imperfectly aligned advisory board even when they can commit to publicly revealing the board’s information. The optimally aligned advisory board must create consensus with management limiting the damage from managerial capture of authority.

Our approach effectively assumes that contracting solutions and career or reputational concerns do not perfectly eliminate managerial agency. This is likely to be the case when the decisions taken within the firm and the information held by the different agents cannot easily be verified by outside courts. Commonly observed forms of incentive provision are of course completely consistent with our approach. If a possibly unbiased replacement will take time to acquire comparable expertise, it will not be optimal to fire a biased but experienced manager. The managerial bias parameter $b_m$ then reflects managerial rents from possessing superior expertise about firm prospects. Similarly, attempts by shareholders to contractually align management by providing an equity share $\alpha \in (0, 1)$ will keep the essence of our problem unchanged. With an equity share $\alpha$, managerial payoffs from choosing the alternative is equal to $\alpha(x - y) + b_m$ whereas the payoff to shareholders is $(1 - \alpha)(x - y)$. Our model extends to this setting but with the effective managerial bias now equal to $b_m/\alpha$. Our results on optimal board design then apply given a choice of $\alpha$, including the optimal $\alpha$. Notice in this respect that contractual solutions are expensive in that they force shareholders to pay part of the surplus to management and/or the board, in contrast to the mechanisms of “ideological” alignment that we focus on. Krishna and Morgan (2008) show that contracts, even when they are feasible, may not be used at all or used in conjunction with the type of alignment mechanisms that we use.

28 A full analysis of the interaction between the optimal choice of incentive contracts $\alpha$ and governance $b_d$ as a function of the underlying agency $b_m$ is an interesting research question but it is beyond the scope of the present paper.
Even within the class of alignment mechanisms, we focus on a particular class of delegation mechanisms, i.e., mechanisms where the supervisory board has complete control over decisions. We do not consider the possibility of more elaborate delegation schemes (see, e.g., Alonso and Matouschek 2008) on the grounds that such schemes may be difficult to specify for shareholders. We also do not consider intermediation schemes where shareholders use an agent without decision-making power to negotiate with management. Goltsman et al. (2009), Blume et al. (2009), Ivanov (2009), Ambrus et al. (2011) develop results for intermediation schemes in the CS framework, showing in particular that delegation schemes of the type we use dominate intermediation schemes from the perspective of the principal. Our framework with two-sided private information and binary decisions is different from the CS setting. As Proposition 9 shows, the optimal delegation scheme corresponding to a supervisory board is in fact an intermediation scheme in our framework. The presence of two-sided private information allows us to develop the notion of advisory boards and compare with supervisory boards, an exercise unique to our set-up. A full treatment of intermediation mechanisms with two-sided private information promises to yield new theoretical insights, although this is likely to be beyond the scope of a single paper.

Finally, alternative specifications of managerial agency and governance may also give rise to forces similar to those analyzed in this paper. While we have interpreted the managerial bias parameter \( b_m \) in non-pecuniary terms such as private benefits of empire-building or control, it could equally be interpreted in terms of behavioral biases such as hubris or overconfidence. Similarly, the board’s bias \( b_d \) has so far been interpreted in terms of ideological sympathy for management that summarizes in a simple way the net effect of the structure and composition of the board. In an alternative specification, we may think of \( b_m \) in terms cash or other resources that the manager directly steals from shareholders while implementing his favored decision after it is approved by the board. In such cases, the board may also be able to obtain a cut \( b_d \) of the manager’s share of

\[29\] See however Proposition 10 which is an application of the optimal delegation scheme of Holmstrom (1984) developed for the CS framework to our binary decision problem with two-sided private information.

\[30\] Intermediation in the CS setting is also related to the construction in Krishna and Morgan (2004) (see also Aumann and Hart, 2003) that shows that simultaneous communication in the CS setting may expand the equilibrium set by allowing the players to create jointly controlled lotteries via simultaneous use of randomized messages.

\[31\] It is also possible to think of a delegate or an intermediary choosing another delegate or intermediary. In the present paper we allow the board to delegate to management as a function of its information, implicitly ruling out the possibility of longer chains of delegation and intermediation.
the loot leaving only \( b_m - b_d \) for management. In contrast to the set-up of this paper, such a self-serving board has an added effect on shareholder welfare. It may mitigate the agency problem not only by improving communication but also via directly forcing the manager to share his pecuniary gains with the board, reducing management’s incentives to push his biased agenda in the first place. In contrast, if the board’s bias \( b_d \) arises out of additional resources that the board also steals from shareholders, then shareholders have to trade off the gains from improved communication (as measured by \( b_m - b_d \)) against not only the distortion in decision-making but also the loss from increased looting by both the management and the board (as measured by \( b_m + b_d \)). We leave a full investigation of these model variations for future research.

6 Appendix

6.1 Proofs

Proof of Proposition 1. For any message \( \mu \) sent by the manager the board chooses the alternative if \( y < t(\mu) = E[x|\mu] + b_d \) and the status quo otherwise. Let \( q(\mu) = \Pr[y < t(\mu)] \in [0,1] \) and notice that the manager’s expected payoff from sending the message \( \mu \) and inducing \( t(\mu) \) is \( q(\mu)(x + b_m - E[y|y < t(\mu)]) \). The last expression can be rewritten as \( U(x, q) = q(x + b_m) - \int_0^q F_y^{-1}(z)dz. \) This is supermodular in \( q \) and \( x \) and concave in \( q \).

We show first that only a finite number of distinct \( q(\mu) = F_y(t(\mu)) \) may be induced in equilibrium. In a babbling equilibrium only one \( q \) is induced in equilibrium. So consider an informative equilibrium with two distinct \( q \) and \( q' \) with \( q < q' \) and \( q, q' \in [0,1] \). The corresponding thresholds are \( t \) and \( t' \) with \( t < t' \). Since the manager types who prefers to induce \( q \) (resp., \( q' \)) reveals a weak preference for inducing it, by continuity and supermodularity of \( U(x, q) \) there exists an unique \( x^* \) who is indifferent between \( q \) and \( q' \). By the concavity of \( U(x, q) \) it follows that \( F_y(x^* + b_m) \in (0,1) \) and so

\[
t < x^* + b_m < t'.
\]

Furthermore, by the supermodularity of \( U(x, q) \), it follows that \( q \) cannot be induced by any type \( x > x^* \) (they prefer \( q' \)) and \( q' \) cannot be induced by any type \( x < x^* \) (they prefer \( q \)). Since \( t(\mu) = E[x|\mu] + b_d \), it follows that

\[
t \leq x^* + b_d \leq t'.
\]
But then there exists $\varepsilon > 0$ such that $t' - t > b_m - b_d > \varepsilon$. Since $y$ lies in a compact set, it follows that at most $\varepsilon(y_H - y_L)$ thresholds ($t'$s or $q'$s) can be induced in equilibrium.

Consider an equilibrium with $N < \infty$ distinct thresholds $t_1, \ldots, t_N$. A cutoff type $x = c_i$ is indifferent between two successive thresholds $t_i, t_{i+1}$ with $t_i < t_{i+1}$, $i = 1, \ldots, N - 1$, iff

$$
\Pr[y < t_{i+1}]|c_i + b_m - E(y|y < t_{i+1})] = \Pr(y < t_i)\{c_i + b_m - E(y|y < t_i)],
$$

which can be rewritten as

$$
c_i + b_m = \frac{\int_{t_i}^{t_{i+1}} zF_y(z)dz}{\Pr[t_i < y < t_{i+1}]} = E[y|t_i < y < t_{i+1}].
$$

The result now follows. ■

Proof of Proposition 2. Analogous to the proof of Proposition 1. ■

Proof of Proposition 3. We prove part 1 of the result. The proof of part 2 is analogous.

We start by comparing the games $mb^*$ and $bmb^*$. Notice that the decision rule implemented by any equilibrium of $mb^*$ can be replicated by an equilibrium of $bmb^*$ in which the board is uninformative in stage 1. We wish to prove that any equilibrium of $bmb^*$ results in a decision rule that is also an equilibrium decision rule of $mb^*$.

The proof proceeds as follows. We find necessary properties of an equilibrium of $bmb^*$. These properties imply that the board simply wishes to know from management if $x$ is above or below a cutoff value $\tilde{c}(y)$ that depends on the board’s information $y$. The board then chooses the alternative if and only if $x > \tilde{c}(y)$. Further, the set of types $y$ that wish to use the same decision rule, corresponding to a particular value of the cutoff $\tilde{c}$, must be an interval i.e., such $y$ must lie between two threshold values. The necessary relationship between these cutoffs and thresholds imply that the same decision rule is part of an equilibrium of $mb^*$ as well.

Consider an equilibrium of $bmb^*$. Let $\mu_d$ be a message received by the manager from the board in the first stage and let $\mu_m$ be a message received by the board from management in the second stage. Let $\mu = (\mu_d, \mu_m)$ and $x(\mu) = E[x|\mu_m, \mu_d]$. In the third decision-making stage, the board would like to choose the alternative iff

$$
y < x(\mu) + b_d \equiv t(\mu).
$$

Consider now the manager’s problem in the second stage given a message $\mu_d$ from the board. Let $F_y(\cdot|\mu_d)$ be the updated beliefs about $y$ from the manager’s perspective given $\mu_d$. The expected
payoff to the management from sending a message $\mu_m$ is

$$F_y(t(\mu)|\mu_d)[x + b_m - E(y|y < t(\mu), \mu_d)],$$

provided $F_y(t(\mu)|\mu_d) > 0$, and zero otherwise. The manager of type $x$ is then weakly prefers a message $\mu_m$ to a message $\mu'_m$, $t(\mu_m, \mu_d) < t(\mu'_m, \mu_d)$ iff

$$x + b_m \leq E[y|t(\mu_m, \mu_d) < y < t(\mu'_m, \mu_d)],$$

with equality in the case of indifference. Using arguments identical to those in the proof of Proposition 1 it follows that the manager’s strategy after each $\mu_d$ can be represented by a finite partition of $[0, 1]$ where each partition element is an interval. Let the cutoffs $\{c_i(\mu_d)\}_{i=0}^{N(\mu_d)}$ represent the partition with $N(\mu_d)$ elements where $[c_{i-1}(\mu_d), c_i(\mu_d)]$ is the $i$th element. Let $\mu'_m$ denote the manager’s message corresponding to the event that $x \in [c_{i-1}(\mu_d), c_i(\mu_d)]$ with $x_i(\mu_d) = E[x|\mu'_m, \mu_d]$ and $t_i(\mu_d) = x_i(\mu) + b_d$. In what follows we identify a $\mu_d$ with the managerial partition that it leads to $\{c_i(\mu_d)\}_{i=0}^{N(\mu_d)}$.

We turn now to the board’s problem in stage 1 of $bmb^*$. If after sending a message $\mu_d$, a board type $y$ learns no decision relevant information from management if $N(\mu_d) = 1$ or if $y \leq t_1(\mu_d)$ or if $y \geq t_{N(\mu_d)}(\mu_d)$. Otherwise, type $y$ learns decision relevant information from management; in particular, $N(\mu_d) > 1$ and there exists $i$ with $1 \leq i < N(\mu_d)$ with

$$t_i(\mu_d) \leq y \leq t_{i+1}(\mu_d),$$

where the first inequality is strict if $i = 1$ and the second if $i = N - 1$. For $y$ satisfying (10), if the board sends the message $\mu_d$ in stage 1, then in stage 3 the board chooses the status quo when it learns from management that $x < c_i(\mu_d)$ where

$$c_i(\mu_d) + b_m = E[y|t_i(\mu_d) < y < t_{i+1}(\mu_d)],$$

and

$$E[x|x < c_i(\mu_d)] + b_d \leq x_i(\mu_d) + b_d = t_i(\mu_d) \leq y,$$

with at least one strict inequality. And the board chooses the alternative if it learns that $x > c_i(\mu_d)$ since

$$E[x|x > c_i(\mu_d)] + b_d \geq x_{i+1}(\mu_d) + b_d = t_{i+1}(\mu_d) \geq y,$$
with at least one strict inequality. Call such a stage 1 message from the board type \( y \), a decision relevant message for type \( y \).

Observe first that every type \( y \) must in equilibrium strictly prefer to send a decision relevant message, if one exists, to sending a decision irrelevant one. A board type \( y \) may not have any decision relevant message \( \mu_d \) if \( y \leq t_1(\mu_d) \) for all \( \mu_d \). Let \( Y_L \) be the (possibly empty) set of types who choose the alternative in equilibrium regardless of the message they send and the message sent back by the management. If \( y \in Y_L \) then so is all \( y' < y \). Similarly, types \( y \) with \( y \geq t_N(\mu_d) \) for all \( \mu_d \) also do not have a decision relevant message since such types choose the status quo in equilibrium regardless of the message they send and the message sent back by the management. Let \( Y_H \) be the (possibly empty) set of such types. If \( y \in Y_H \) then so is all \( y > y \). Notice from (9) that the behavior of board types in \( Y_L \cup Y_H \) does not affect the communication incentives of the manager since their decision does not vary with management’s message. Let \( Y^* \) be the complementary set of board types who send a decision-relevant message in equilibrium. If \( Y^* \) is empty, then the board learns no information from the management in the second stage. Such equilibria implement decision rules corresponding to the babbling equilibrium of \( mb^* \) and there is nothing left to prove. Accordingly we focus on the case where the set \( Y^* \) is non-empty in what follows.

Consider type \( y \in Y^* \) who in equilibrium sends the decision relevant message \( \mu_d \), inducing the decision rule of choosing the alternative if it learns from management that \( x > c_i(\mu_d) \in (0, 1) \). The expected payoff to \( y \in Y^* \) from inducing the cutoff \( c_i(\mu_d) \) is

\[
(1 - F_x(c_i(\mu_d)))(E[x|x > c_i(\mu_d)] + b_d - y) = \int_{q'}^{1} F_x^{-1}(z)dz - (1 - q')(y - b_d), \tag{14}
\]

where \( q' \equiv F_x(c_i(\mu_d)) \). The last expression is concave in \( q' \) and supermodular in \( q', y \). For any two distinct cutoffs \( c \) and \( c' \) where \( c < c' \) (possibly corresponding to different decision relevant messages \( \mu_d \) and \( \mu'_d \)), type \( y \in Y^* \) weakly prefers to induce a cutoff \( c \) iff, using (14),

\[
y \leq E[x|c < x < c'] + b_d, \tag{15}
\]

with indifference in case of equality. It follows that any \( y \in Y^* \) can be indifferent between inducing at most two distinct cutoffs \( c \) and \( c' \).

For every \( y \in Y^* \) that induces a unique cutoff in equilibrium denote by \( \widehat{c}(y) \in (0, 1) \) this cutoff. For \( y \in Y^* \) who are indifferent between and induce multiple distinct cutoffs, choose \( \widehat{c}(y) \in [0, 1] \) to be the higher of the two. For \( y \in Y_0 \) who prefer to always choose the status quo in equilibrium
define \( \hat{c}(y) = 0 \) whereas for \( y \in Y_1 \) who prefer to always choose the alternative in equilibrium define \( \hat{c}(y) = 1 \). Using (15) and the supermodularity and concavity of the expression in (14), \( \hat{c}(y) \) must be an increasing step function of \( y \) with a finite number of steps. Therefore, the inverse image of \( \hat{c} \) is a finite partition of \( Y \) where each partition element is an interval.

For the case where \( Y_0 \) and \( Y_1 \) are both non-empty, let \( \{\hat{c}_i\}_i=0 \) be value of \( \hat{c} \) along each of its \( N < \infty \) horizontal segments, with \( \hat{c}_0 = 0 < c_1 < \ldots < \hat{c}_N = 1 \) and let \( [\hat{t}_i, \hat{t}_{i+1}] \) be the inverse image of \( \hat{c}_i \), \( \hat{t}_0 = y_L < \hat{t}_1 < \ldots < \hat{t}_{N+1} = y_H \). By construction, \( y = \hat{t}_i \) indifferent between inducing the cutoffs \( \hat{c}_i \) and \( \hat{c}_{i-1} \), any \( y \in (\hat{t}_i, \hat{t}_{i+1}) \) strictly prefers to induce \( \hat{c}_i \) in equilibrium, any \( y \notin [\hat{t}_i, \hat{t}_{i+1}] \) strictly prefers to induce some \( \hat{c}_j \neq \hat{c}_i \) and, furthermore,

\[
\hat{c}_i + b_m = E[y|\hat{t}_i < y < \hat{t}_{i+1}], \quad i = 1, \ldots, N - 1,
\]

and

\[
\hat{t}_i = E[x|\hat{c}_{i-1} < x < \hat{c}_i] + b_d, \quad i = 1, \ldots, N.
\]

From Proposition 1, it follows that the cutoffs \( \{\hat{c}_i\}_i=0 \) together with the thresholds \( \{\hat{t}_i\}_i=1 \) is a \( N \) message equilibrium of \( mb^* \).

The arguments for the case where either \( Y_0 \) or \( Y_1 \) is empty is entirely analogous and we omit the details. The equivalence of \( b^* mb^* \) with \( bmmb^* \) follows from observing that the management’s communication incentives does not depend on the behavior of types in \( Y_0 \) or \( Y_1 \) and so it does not matter if these types take their decision at stage 1 or 3.

**Proof of Proposition 4.** We prove part 1 of the result. The proof of part 2 is analogous.

We show first that an equilibrium decision rule of \( b^* m^* \) is also an equilibrium decision rule of \( bmmb^* \) and so, by Proposition 3, also of \( mb^* \). Consider an equilibrium of \( b^* m^* \). We construct an strategy profile that implements the same decision in an equilibrium of . Let \( Y_L \) be the set of \( y \) who choose the alternative in \( b^* m^* \) and \( Y_H \) the set that choose the status quo, with \( Y^* \) the set who send a message and let management make the final decision. If \( Y^* \) is empty, then the same decision rule can be implemented by the equilibrium of \( bmb^* \) where the first two rounds involve babbling. So suppose \( Y^* \) is non-empty.

Let all \( y \in Y^* \) use the same communication strategy in stage 1 of \( bmb^* \) as in \( b^* m^* \). Pick an arbitrary \( y' \in Y^* \) and let \( y \in Y_L \cup Y_H \) use the communication strategy of \( y' \) in stage 1 of \( bmmb^* \). Suppose that after a message \( \mu_d \), management chooses the alternative in \( b^* m^* \) iff \( x > c(\mu_d) \). In \( bmmb^* \), let management recommend the alternative if \( x > c(\mu_d) \) and the status quo otherwise. Suppose all

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$y \in Y^*$ follow the management’s recommendation in stage 3 of $bmb^*$ while all $y \in Y_L$ choose the alternative and all $y \in Y_H$ choose the status quo regardless of management’s recommendation.

Using arguments that are detailed in the proof of Proposition 3, we show now that such behavior is an equilibrium of $bmb^*$. In stage 3, types $y \in Y^*$ will find it in their interest to follow management’s recommendation, using (12) and (13) in particular; while $y \in Y_L \cup Y_H$ will prefer to take their specified decisions since they preferred to take an unilateral decision without knowledge of $x$ in the original equilibrium of $b^*m^*$. Given this, management will infer that its own message is irrelevant for the behavior of $y \in Y_L \cup Y_H$. In determining the cutoff $c(\mu_d)$ of their recommendation, management will only condition on the communication strategy of $y \in Y^*$ and will therefore arrive at the same cutoff as they did in $b^*m^*$. Because the strategy profile in $bmb^*$ leads to the same outcome as in $b^*m^*$, no board type has an incentive to deviate in the first stage either. This shows that an equilibrium decision rule of $b^*m^*$ is an equilibrium decision rule of $bmb^*$ and so of $mb^*$.

In the other direction, consider first any informative equilibrium of $mb^*$ with $N > 1$. Construct the following strategy profile for $b^*m^*$. Types $y > t_N$ take the status quo in stage 1 while $y < t_1$ take the alternative. All other $y \in [t_i, t_{i+1}]$ reveal the interval they belong to, $1 \leq i < N$, following which management chooses the alternative iff $x > c_i$. Using the definitions of the cutoffs $c_i$ and thresholds $t_i$ in Proposition 1, it is straightforward to show that this is an equilibrium of $b^*m^*$ that implements the same decision rule.

To complete the proof, consider the babbling equilibrium of $mb^*$. We show that whenever the same decision rule corresponding the babbling equilibrium of $mb^*$ cannot be implemented in $b^*m^*$, then there is another informative equilibrium of $bm^*$ with $N = 2$. By the arguments of the previous paragraph, the decision rule of this informative equilibrium can be implemented as an equilibrium of $b^*m^*$. We proceed through a number of different cases.

**Case 1:** $y_H \geq 1 + b_m$ or $y_L \leq b_m$.

Suppose first $y_H \geq 1 + b_m$. Consider the strategy profile of $b^*m^*$ where in stage 1 board types $y$ take the alternative if

$$y < \hat{t} \equiv E[x] + b_d,$$

choosing the status quo otherwise, and no type delegates the decision to management. If any type deviates and delegates the decision to management with a message $\mu$, then management forms beliefs that $y_\mu \equiv E[y|\mu] = y_H$. Since $y_H \geq 1 + b_m$, management takes the status quo regardless
of $x$ following any such message. Since management’s decision does not depend on $x$, all board types prefer to take their decision in the first stage as a function of whether or not $y < \hat{t}$. As a result, the decision rule corresponding to the babbling equilibrium of $mb^*$ is also an equilibrium decision rule of $b^*m^*$. A similar argument obtains where $y_L \leq b_m$ in which case we keep the board’s behavior unchanged but give management the option to take the alternative regardless of $x$, implying in turn that no board type wants to deviate from its specified behavior in stage 1 of $b^*m^*$.

**Case 2:** $y_H < 1 + b_m$ and $y_L > b_m$.

Consider next the case where $y_H < 1 + b_m$ and $y_L > b_m$. For any beliefs $y_\mu \in [y_L, y_H]$ of management in the second stage, management is indifferent between the two decisions iff $x = \hat{c}_\mu = y_\mu - b_m \in (0, 1)$. Therefore, management will never take the same decision for all $x$. The expected payoff for board type $y$ from sending message $\mu$ and inducing beliefs $y_\mu$ is

$$(1 - F_x(\hat{c}_\mu))(E[x|x > \hat{c}_\mu] + b_d - y).$$

(17)

There are three subcases to consider depending on the value of $\hat{t}$ relative to $y_L$ and $y_H$.

**Case 2a:** $y_L \leq \hat{t} \leq y_H$

Suppose that $y_L \leq \hat{t} \leq y_H$ with $y_H < 1 + b_m$ and $y_L > b_m$. Then type $\hat{t}$, in particular, will prefer to send any message $\mu$ that induces a cutoff $\hat{c}_\mu \in (0, 1)$ to taking a decision unilaterally. If it does the latter, by the definition of $\hat{t}$ in (16), type $\hat{t}$ earns an expected payoff of zero. If $\hat{t}$ sends the message $\mu$ instead, by (17) the alternative is chosen with strictly positive probability, and conditional on this the expected payoff is strictly positive since $\hat{t} < E[x|x > \hat{c}_\mu] + b_d$ for $\hat{c}_\mu \in (0, 1)$. In such cases the babbling equilibrium decision rule of $mb^*$ is not an equilibrium of $b^*m^*$.

We show now that in such cases a $N = 2$ message equilibrium exists in $mb^*$. For any $c \in [0, 1]$, consider the two element partition given by the left element $[0, c]$ and the right element $[c, 1]$. For $c$ sufficiently small but positive, by disclosing the left element, type $c$ will induce the board to choose the status quo and this is the most preferred decision for $c$ sufficiently small, using $y_L > b_m > b_d$. If instead $c$ discloses the right element, then the board will choose the alternative iff $y \leq \hat{t} + \varepsilon \in (y_L, y_H)$ for some $\varepsilon > 0$ and small. It follows that such a type $c$ strictly prefers to disclose the left element. Now consider $c$ sufficiently close to 1. If $c$ discloses the left interval, then the board will be induced to choose the alternative iff $y \leq \hat{t} - \varepsilon \in [y_L, y_H]$ for some $\varepsilon > 0$ and small. If instead, type $c$ discloses the right interval, the the board will choose the alternative iff $y < 1 + b_d - \varepsilon'$ for some
\( \varepsilon' > 0 \) and small. Since \( 1 + b_m > y_H \) and \( 1 + b_d > \hat{\ell} \), a cutoff type \( c \) sufficiently close to 1 strictly prefers to induce the higher threshold \( 1 + b_d - \varepsilon' \) to \( \hat{\ell} - \varepsilon \), i.e., to disclose the right interval. Since the preference of the cutoff type \( c \) switches from the left to the right interval as \( c \) moves from zero to one, by continuity and the intermediate value theorem it follows that there exists \( c^* \in (0, 1) \) who is indifferent between the two intervals. Such a partition constitutes a \( N = 2 \) message equilibrium of \( mb^* \).

**Case 2b: \( \hat{\ell} > y_H \)**

It remains to consider the cases where \( y_H < 1 + b_m \) and \( y_L > b_m \) but either \( \hat{\ell} > y_H \) or \( \hat{\ell} < y_L \). We consider the case \( \hat{\ell} > y_H \) first. In this case, if the board takes its decision unilaterally in stage 1 of \( b^*m^* \), it always chooses the alternative. However (17) obtains and so a board type \( y \) weakly prefers to unilaterally choose the alternative over sending a message \( \mu \) and inducing beliefs \( y_\mu \) and the cutoff \( \tilde{\mu} \in (0, 1) \) iff \( y \leq E[x|x < \tilde{\mu}] + b_d \). Since \( \tilde{\mu} = y_\mu - b_m \) and \( y_\mu \in [y_L, y_H] \), we conclude that the decision rule corresponding to the babbling equilibrium of \( mb^* \) can be implemented in an equilibrium of \( b^*m^* \) if

\[
y_H \leq t' \equiv E[x|x < y_H - b_m] + b_d < \hat{\ell}.
\]

In such an equilibrium, all \( y \) take the alternative in stage 1 of \( b^*m^* \). If any type delegates the decision to management together with a message \( \mu \), then management forms the belief \( y_\mu = y_H \) and takes the alternative iff \( x > y_H - b_m \). However, when \( y_H \leq t' \) all board types \( y \) weakly prefer to choose the alternative unilaterally in stage 1 of \( b^*m^* \).

In contrast, when \( t' < y_H < \hat{\ell} \) the babbling decision rule of \( mb^* \) cannot be supported as an equilibrium of \( mb^* \). We show now that in such cases a \( N = 2 \) message equilibrium exists in \( mb^* \). To see this, once again consider a two element partition \([0, c] \) and \([c, 1] \) with cutoff \( c \in (0, 1) \) and use the intermediate value theorem. For \( c \) small enough, the board will choose the status quo when management discloses the left element of the partition (since \( y_L > b_m > b_d \)) and the alternative when management discloses the right element (since \( y_H < \hat{\ell} \)) and the cutoff type \( c \) strictly prefers to disclose the left element. On the other hand, for \( c = y_H - b_m \), the board will choose the alternative iff \( y < t' \) when management discloses the left element and the alternative for sure when management discloses the right element. The last fact follows from observing that \( y_H > t' = E[x|x < y_H - b_m] + b_d \) and that \( y_H < \hat{\ell} = E[x] + b_d \) implies \( y_H < E[x|x > y_H - b_m] + b_d \). Furthermore, the cutoff type \( c = y_H - b_m \) strictly prefers the right interval, i.e., to induce the board
to choose the alternative for sure over using the interior threshold $t' < y_H$ since $c + b_m = y_H$. It follows that the cutoff type $c$ switches its preference from the left to the right interval as $c$ goes from zero to $y_H - b_m$. Continuity and the intermediate value theorem then guarantee the existence of a $N = 2$ message equilibrium.

Case 2c: $\hat{t} < y_L$

Finally, consider the case where $y_H < 1 + b_m$ and $y_L > b_m$ with $\hat{t} < y_L$. In this case, if the board takes its decision unilaterally in stage 1 of $b^*m^*$, it always chooses the status quo. But (17) obtains and so a board type $y$ weakly prefers to choose the status quo unilaterally over sending a message $\mu$ and inducing beliefs $\hat{\mu}$ and the cutoff $c_{\mu} \in (0,1)$ iff $y \geq E[x|x > \hat{\mu}] + b_d$. Using arguments analogous to that for the previous case it follows that when

$$y_L \geq t'' = E[x|x > y_L - b_m] + b_d > \hat{t},$$

the decision rule corresponding to babbling equilibrium of $mb^*$ can be implemented in $b^*m^*$. In this equilibrium, the board always chooses the status quo. On the other hand, when $\hat{t} < y_L < t''$, then this decision rule cannot be implemented in $b^*m^*$. However, arguments identical to the previous case show that a $N = 2$ message equilibrium of $mb^*$ exits with a decision rule that can be implemented in $b^*m^*$.

Proof of Proposition 5. Consider first the case of a supervisory board and the game $mb^*$. Pick any equilibrium with $N$ messages and consider $x \in [c_{i-1}, c_i]$, $i = 1, ..., N$. If following the (equilibrium) message sent by such a type the board chooses the status quo, the payoff to type $x$ is 0. Given the board’s decision, the expected payoff from the alternative is instead

$$x + b_m - E[y|t_i < y] \leq c_i + b_m - E[y|t_i < y].$$

Since for $i < N$,

$$c_i + b_m = E[y|t_i < y < t_{i+1}] \leq E[y|t_i < y],$$

we conclude that type $x$ agrees with the board’s decision to choose the status quo for all $x \in [c_{i-1}, c_i]$ for $i < N$ and also for $i = N$ as long as $1 + b_m < E[y|y > t_N]$ and $t_N < y_H$. Notice that if $t_N = y_H$, the board never chooses the status quo after the message sent by $x \in (c_{i-1}, c_i]$. Similarly, if following the (equilibrium) message sent by $x \in [c_{i-1}, c_i]$, $i = 1, ..., N$, the board chooses the alternative, the expected payoff to type $x$ is

$$x + b_m - E[y|y < t_i] \geq c_{i-1} + b_m - E[y|y < t_i].$$
Since for \(i > 1\),
\[
c_{i-1} + b_m = E[y|t_{i-1} < y < t_i] \geq E[y|y < t_i],
\]
we conclude that type \(x\) is better off with the board’s decision to invest (gets at least as much as the payoff of 0 from the status quo) for \(x \in [c_{i-1}, c_i]\) for \(i > 1\) and also for \(i = 1\) as long as \(b_m > E[y|y < t_1]\) and \(t_1 < y_L\). Notice that if \(t_1 = y_L\), the board never chooses the alternative after the message sent by \(x \in [c_{i-1}, c_i]\). Identical arguments cover the case of an advisory board and the game \(bm^*\).

To complete the proof suppose consensus obtains in a \(N\) message equilibrium of the game \(mb^*\). Consider first the case where \(t_N < y_H\) and \(t_1 > y_L\). It is straightforward to verify that the thresholds \(t'_i = t_i\), \(i = 1, ..., N\), \(t'_0 = y_L\) and \(t'_{N+1} = y_H\) with corresponding cutoffs \(c'_{i+1} = c_i\) for \(i = 0, ..., N\) and \(c'_{N+1} = 1\) define a \(N + 1\) message equilibrium of the game \(bm^*\). Analogous arguments obtain in the case where either \(t_N = y_H\) or \(t_1 = y_L\), in which case the equivalent equilibrium in \(bm^*\) has either \(N\) or \(N - 1\) messages.

**Proof of Proposition 6.** In the case where \(y_L = 0\) and \(y_H > 1 + b_m\), \(t_i \in (y_L, y_H)\) for all \(i = 1, ..., N\) and so the CS equations apply. The result follows from Proposition 5 in Dessein (2002).

**Proof of Proposition 7.** Suppose the manager has authority and \(b_d < b_m\). Consider the most informative equilibrium with \(M \geq 1\) messages with thresholds \(y_L = t'_0 < ... < t'_M = y_H\) and cutoffs
\[
c'_i = \frac{t'_{i-1} + t'_i}{2} - b_m, \quad i = 1, ..., M.
\]
Let \(V_m(b_d, b_m)\) be shareholder value in this equilibrium and let \(\widehat{V} = V_m(b_m, b_m)\) be shareholder value at \(b_d = b_m\). Letting \(\Delta = y_H - y_L\), we can write
\[
V_m(b_d, b_m) = \widehat{V} + \sum_{i=1}^M T_i,
\]
where, for \(i = 1, ..., M\),
\[
T_i = \int_{c'_i}^{\max[0, \min[t'_{i-1} - b_m, 1]]} \int_{x+b_m}^{t_i} (x-y) \frac{1}{\Delta} dydx - \int_{\max[0, t'_{i-1} - b_m]}^{c_i} \int_{x+b_m}^{x+b_m} (x-y) \frac{1}{\Delta} dydx.
\]
suffices to show that the second term, $\sum_{i=1}^{M} T_i$, on the rhs of (19) is either negative or has a non-zero derivative with respect to $b_d$ when $b_d < b_m$.

**Case 1**: $M \geq 2$.

Using (18) it is easy to verify that whenever $i > 1$ and $i < M$

$$T_i = -\frac{1}{24\Delta} l_i^3,$$

(21)

where $l_i' = l_i' - l_{i-1}'$ is the length of the $i$th interval in the board’s strategy. For $i = M$, notice using Proposition 2 that $t_{M-1}' < 1 + b_m$ so that

$$T_M = \begin{cases} \\
\frac{1}{6\Delta} \left( t_{M-1}' - (1 + b_m) \right)^2 \left( t_{M-1}' + 2b_m - 1 \right) & \text{if } c_M' \geq 1, \\
-\frac{1}{24\Delta} l_M^3 + 1_{y_H > 1 + b_m} \frac{1}{6\Delta} \left(y_H - (1 + b_m)\right)^2 \left(y_H + 2b_m - 1\right) & \text{if } c_M' < 1.
\end{cases}$$

(22)

where $1\{\cdot\}$ is the indicator function. For $i = 1$, notice using Proposition 2 that $t_1' > 0$ and $y_L < b_m$ whenever $c_1' \leq 0$ and $t_1' > b_m$ whenever $c_1' > 0$. Using this,

$$T_1 = \begin{cases} \\
-\frac{1}{6\Delta} \left(b_m - t_1'\right)^2 \left(t_1' + 2b_m\right) & \text{if } c_1' \leq 0, \\
-\frac{1}{24\Delta} l_1^3 - 1_{y_L < b_m} \frac{1}{6\Delta} (b_m - y_L)^2 (y_L + 2b_m) & \text{if } c_1' > 0.
\end{cases}$$

(23)

Further, since the equilibrium does not display consensus, from Proposition 5 we must either have $c_M' < 1$ or $c_1' > 0$.

**Case 1a**: $c_M' < 1$, $c_1' > 0$.

In this case

$$V_m = \hat{V} - \frac{1}{24\Delta} \sum_{i=1}^{M} l_i^3 - 1_{y_L < b_m} \frac{1}{6\Delta} (b_m - y_L)^2 (y_L + 2b_m)$$

$$+ 1_{y_H > 1 + b_m} \frac{1}{6\Delta y} \left(y_H - (1 + b_m)\right)^2 \left(y_H + 2b_m - 1\right).$$

(24)

The first and last two terms do not depend on $b_d$ and so it suffices to show that $\sum_{i=1}^{M} l_i^3$ is strictly decreasing in $b_d$, which follows from the results in CS (see in particular Theorem 4 and expression (25) on pp. 1442 of CS).

**Case 1b**: $c_M' < 1$, $c_1' \leq 0$.  

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In this case
\[
V_m = \hat{V} - \frac{1}{24\Delta} \sum_{i=2}^{M} t_i^3 - \frac{1}{6\Delta} (b_m - t'_1)^2(t'_1 + 2b_m) \\
+ 1_{y_H > 1 + b_m} \frac{1}{6\Delta} (y_H - (1 + b_m))^2 (y_H + 2b_m - 1).
\]  
(25)

It follows immediately that if \( y_H \leq 1 + b_m \), then \( V_m < \hat{V} \) and choosing a perfectly management aligned board is better than choosing \( b_d < b_m \). So suppose \( y_H > 1 + b_m \). Using expression 25 on pp. 1442 in CS,
\[
\sum_{i=2}^{M} t_i^3 = \sum_{j=1}^{M-1} t_j^3 = \frac{(y_H - t'_1)^3}{(M-1)^2} + 4B(y_H - t'_1)((M-1)^2 - 1),
\]
where \( B = b_m - b_d \). Then
\[
\frac{\partial}{\partial b_d} \left( \sum_{i=2}^{M} t_i^3 \right) = - \left[ \frac{3(y_H - t'_1)^2}{(M-1)^2} + 4M(M-2)B^2 \frac{\partial t'_1}{\partial b_d} + 8M(M-2)B(y_H - t'_1) \right] < 0,
\]  
(26)
since \( \frac{\partial t'_1}{\partial b_d} = \frac{2M(M-1)}{2M-1} > 0 \) using the equilibrium formulas in Section 6.2. Furthermore,
\[
\frac{\partial}{\partial b_d} ((b_m - t'_1)(t'_1 + 2b_m)) = 3(t'_1 - b_m)^2 \frac{\partial t'_1}{\partial b_d}.
\]  
(27)

Using (25) through (27) it follows that \( \frac{\partial V_m}{\partial b_d} > 0 \) iff
\[
- \frac{M(M-2)B(y_H - t'_1)}{3} + \left[ \frac{1}{2}(t'_1 - b_m)^2 - \frac{1}{24} \frac{3(y_H - t'_1)^2}{(M-1)^2} + 4M(M-2)B^2 \right] \frac{\partial t'_1}{\partial b_d} < 0.
\]  
(28)

Notice that the last expression is increasing in \( t'_1 < y_H \) and therefore in \( b_d \) since \( \frac{\partial t'_1}{\partial b_d} = \frac{2M(M-1)}{2M-1} > 0 \).

Furthermore, using the equilibrium equations in Section 6.2, it can be verified that for there to be no \( M + 1 \) partition equilibrium with \( c'_1 \leq 0 \) it is necessary and sufficient that \( b_d < b_d^{M+1} = \frac{2M^2b_m - y_H}{2M(M+1)} \).

It suffices now to show that the expression in (28) is negative when evaluated at \( b_d = b_d^{M+1} \). Using the expressions for \( \frac{\partial t'_1}{\partial b_d} \) and \( t'_1 \) some manipulation yields the equivalent inequality
\[
y_H^2 (4M^3 - 3M + 7) + 4b_m y_H (2M^4 - M^3 - 9M^2 - M - 1) + 4b_m^2 M (4M^3 + 3M + 1) > 0.
\]  
(29)

But this is easily verified to be true using \( M \geq 2 \) and \( y_H > 1 + b_m \).

**Case 1c.** \( c'_M \geq 1, c'_1 > 0 \).
In this case

\[ V_m = \frac{1}{24\Delta} \sum_{i=1}^{M-1} t_i^3 + \frac{1}{6\Delta} (t'_{M-1} - (1 + b_m))^2 (t'_{M-1} + 2b_m - 1) \]

\[ -1_{y_L < b_m} \frac{1}{6\Delta} (b_m - y_L)^2 (y_L + 2b_m). \] (30)

This is the mirror-image of the previous case. Using arguments analogous to the previous case, one can show that \( \frac{\partial V_m}{\partial \delta_d} > 0 \) for all \( \delta_d \) such that there exists no \( M + 1 \) partition equilibrium with \( c'_{M+1} \geq 1 \) and we suppress the details.

**Case 2.** \( M = 1 \).

There are three subcases to consider for this case in which the board plays babbling and management uses a cutoff \( c'_1 = E[y] - b_m \).

**Case 2a.** \( c'_1 \leq 0 \).

In this case management always chooses the alternative so that shareholder value in the babbling equilibrium can be written as

\[ E[x - y] = \Pr[x - y < -b_m]E[x - y|x - y < -b_m] + \Pr[x - y > -b_m]E[x - y|x - y > -b_m], \] (31)

using the law of iterated expectations. Since \( y_H > b_m \), \( \Pr[x - y < -b_m] > 0 \) and the first term on the rhs of (31) strictly negative so that the second term is strictly greater than the lhs. But since the second term on the rhs of (31) is shareholder value from a perfectly management aligned board, it follows that \( \delta_d \) has not been chosen optimally.

**Case 2b.** \( c'_1 \geq 1 \).

In this case management never chooses the alternative so that shareholder value in the babbling equilibrium is equal to zero. Since the equilibrium does not display consensus, type \( y_L \) must prefer the alternative, i.e., \( y_L < E[x] + \delta_d \). Moreover, \( c'_1 \geq 1 \) implies \( y_H \geq 1 + b_m \). Straightforward arguments now show that there exists a \( M = 2 \) partition equilibrium with \( c'_2 \geq 1 \) and either \( c'_1 \leq 0 \) with \( t'_1 = E[x] + \delta_d \) or \( c'_1 \in (0, 1) \) with \( t'_1 = \frac{2}{3} + \frac{1}{3}y_L - \frac{2}{3}b_m \). Using the formulas derived in Section 6.2.

**Case 2c.** \( c'_1 \in (0, 1) \).

The expected payoff is, using arguments similar to those for case 1,

\[ V_m = \frac{1}{24\Delta} (y_H - y_L)^3 \]

\[ -1_{y_H > 1 + b_m} \frac{1}{6\Delta} (y_H - (1 + b_m))^2 (y_H + 2b_m - 1). \] (32)
It follows that if \( y_H \leq 1 + b_m \), \( b_d \) has not been optimally chosen. So suppose henceforth that \( y_H > 1 + b_m \).

Construct a 2 partition equilibrium with threshold \( t'_1 \) and cutoffs \( c'_1 \) and \( c'_2 \) as follows. For \( t'_1 \) close to \( y_H > 1 + b_m \), \( c'_1 \) is close to \( E[y] - b_m \in (0, 1) \) while \( c'_2 \) is close to \( y_H - b_m \), implying type \( t'_1 \) prefers sending the higher message. On the other hand, for \( t'_1 \) close to \( y_L \), \( c'_1 \) is close to \( y_L - b_m \) while \( c'_2 \) is close to \( E[y] - b_m \in (0, 1) \). Type \( t'_1 \) now prefers to send the lower message iff

\[
y_L < E[x|y_L - b_m < x < E[y] - b_m] + b_d.
\] (33)

If the last inequality holds, then by the intermediate value theorem we have a two partition equilibrium. Otherwise, consider raising the alignment of the board to \( b'_d > b_d \) till (33) holds. From (32), raising the alignment to \( b'_d \) keeps shareholder value unchanged as long as there is no two partition equilibrium. Furthermore, there must exist \( b'_d < b_m \) for which (33) holds since at \( b_d \) equal to \( b_m \) the rhs of (33) is strictly greater than \( y_L \). For \( b_d \) close to the value at which (33) holds with equality, \( c'_2 < 1 \) and since \( y_H > 1 + b_m \) such a two partition equilibrium cannot have consensus. From case 1 we know that shareholder value is strictly increasing in board alignment from this point, yielding the desired result.

**Proof of Proposition 8.** The result follows from Proposition 7, using the formulas for the equilibrium cutoffs and thresholds in Section 6.2, and observing that given a \( b_m \) if consensus does not obtain for a given \( b_d \) it can only obtain for a sufficiently higher (and not lower) value of \( b_d \).

**Proof of Proposition 9.** If the optimal supervisory board chooses the status quo after a message \( \mu \) from management, then \( y > E[x|\mu] + b_d > E[x|\mu] \), implying shareholders agree with such a decision. On the other hand, the expected shareholder value from the optimal supervisory board with alignment \( b'_d \) equals \( Pr[alt.|b'_d]E[x - y|alt., b'_d] \). The last expression is weakly greater than the shareholder value from a shareholder aligned board and the latter must yield non-negative expected payoff to shareholders. But then the expectation \( E[x - y|alt., b'_d] \) must be non-negative whenever \( Pr[alt.|b'_d] > 0 \), implying shareholders must also agree with a decision to choose the alternative at the optimum.

**Proof of Proposition 10.** Because of the ex-ante payoff equivalence with the CS framework, the decision-rule implemented by the optimal arbitration mechanism in Goltsman et al (2009) (see also Dessein, 2002) is better for shareholders compared to the decision rule implemented by the optimal supervisory board of Proposition 6. In our context, this decision rule consists of allowing
management to choose the alternative as long as \( x - y + b_m > 0 \) and \( y < 1 - b_m \) and to choose the status quo otherwise.

### 6.2 Formulas for Equilibrium Cutoffs and Thresholds

#### 6.2.1 Supervisory Board

Assume a supervisory board. Using Proposition 1, for each \( N \geq 1 \) we provide an explicit characterization of the unique \( N \) message equilibrium given by the cutoff managerial types \( \{c_i\}_{i=0}^N \) with \( c_0 = 0 \) and \( c_N = 1 \) and the threshold board types \( \{t_i\}_{i=1}^N \). Let \( l_i = c_i - c_{i-1} \) so that \( c_i = \sum_{j=1}^i l_j \).

Define \( B = b_m - b_d > 0 \) and for \( N \geq 2 \) let

\[
\begin{align*}
y_L^L(N) &= \frac{1}{2N} + [Nb_d - (N - 1) b_m], \\
y_H^L(N) &= 1 - \frac{1}{2N} + [Nb_d - (N - 1) b_m], \\
y_L^H(N) &= \frac{1}{2N - 1} y_H + (1 - \frac{1}{2N - 1})[Nb_d - (N - 1) b_m], \\
y_H^H(N) &= (1 - \frac{1}{2N - 1}) + \frac{1}{2N - 1} y_L + (1 - \frac{1}{2N - 1})[Nb_d - (N - 1) b_m].
\end{align*}
\]

**Case 1** \( (t_1 \geq y_L, t_N \leq y_H) \). Such an equilibrium with \( N \geq 2 \) messages exists iff \( y_L \leq y_L^L(N) \), \( y_H \geq y_H^L(N) \) and \( l_1 > 0 \), where

\[
\begin{align*}
l_1 &= \frac{1}{N} - 2B(N - 1), \\
l_i &= l_1 + 4B(i - 1), \ i = 2, ..., N.
\end{align*}
\]

**Case 2** \( (t_1 < y_L, t_N \leq y_H) \). Such an equilibrium with \( N \geq 2 \) messages exists iff \( y_L > y_L^L(N) \), \( y_H \geq y_H^L(N) \) and \( l_1 > 0, l_2 > 0 \) where

\[
\begin{align*}
l_1 &= \frac{1}{2N - 1} + (1 - \frac{1}{2N - 1}) y_L + (1 - \frac{1}{2N - 1})[(N - 1)b_d - Nb_m], \\
l_2 &= \frac{1 - l_1}{N - 1} - 2B(n - 2), \\
l_i &= l_2 + 4B(i - 2), \ i = 3, ..., N.
\end{align*}
\]

**Case 3** \( (t_1 \geq y_L, t_N > y_H) \). Such an equilibrium with \( N \geq 2 \) messages exists iff \( y_L \leq y_L^H(N) \),
\( y_H < y_H^*(N) \) and \( l_1 > 0, l_N > 0 \) where

\[
1 - l_N = (1 - \frac{1}{2N-1})y_H + (1 - \frac{1}{2N-1})[(N-1)b_d - N b_m],
\]

\[
l_1 = \frac{1 - l_N}{N-1} - 2B(N-2),
\]

\[
l_i = l_1 + 4B(i-2), \quad i = 2, ..., N-1.
\]

**Case 4** \((t_1 < y_L, t_N > y_H)\). Such an equilibrium with \(N \geq 2\) messages exists iff \(y_L > y_L^*(N)\), \(y_H < y_H^*(N)\) and \(l_1 > 0, l_2 > 0, l_N > 0\) where

\[
l_1 = \frac{1}{2(N-1)}y_H + (1 - \frac{1}{2(N-1)})y_L + [(N-2)b_d - (N-1)b_m],
\]

\[
1 - l_N = (1 - \frac{1}{2(N-1)})y_H + \frac{1}{2(N-1)}y_L + [(N-2)b_d - (N-1)b_m],
\]

\[
l_2 = \frac{y_H - y_L}{N-1} - 2B(N-3),
\]

\[
l_i = l_2 + 4B(i-2), \quad i = 3, ..., N-1.
\]

### 6.2.2 Advisory Board

Assume an advisory board. Using Proposition 2, for each \(M \geq 1\), we provide an explicit characterization of the unique \(M\) message equilibrium given by the threshold board types \(\{t'_i\}_{i=0}^M\) with \(t'_0 = y_L\) and \(t'_M = 1\) and the cutoff managerial types \(\{c'_i\}_{i=1}^M\). Let \(l'_i = t'_i - t'_{i-1}\) so that \(t'_i = y_L + \sum_{j=1}^{i} l'_j\).

Define \(B = b_m - b_d > 0\) and for \(M \geq 2\) let

\[
x_L^*(M) = \frac{1}{2M}y_H + (1 - \frac{1}{2M})y_L - [M b_m - (M-1)b_d],
\]

\[
x_H^*(M) = (1 - \frac{1}{2M})y_H + \frac{1}{2M}y_L - [M b_m - (M-1)b_d],
\]

\[
x_L^*(M) = y_L + \frac{1}{2(M-1)} - [M b_m - (M-1)b_d],
\]

\[
x_H^*(M) = y_H - \frac{1}{2(M-1)} + [M b_m - (M-1)b_d].
\]

**Case 1** \((c'_1 \geq 0, c_M \leq 1)\). Such an equilibrium with \(M \geq 2\) messages exists iff \(0 \leq x_L^*(M)\), \(1 \leq x_H^*(M)\) and \(l'_1 > 0\), where

\[
l'_1 = \frac{1}{M}(y_H - y_L) - 2B(M-1),
\]

\[
l'_i = l'_1 + 4B(i-1), \quad i = 2, ..., M.
\]
Case 2 ($c'_1 < 0$, $c'_M \leq 1$). Such an equilibrium with $M \geq 2$ messages exists iff $0 > x^*_L(M)$, $1 \geq x^*_H(M)$ and $l'_1 > 0$, $l'_2 > 0$ where

\[ l'_1 + y_L = \frac{1}{2M-1} y_H - (1 - \frac{1}{2M-1})[(M-1)b_m - Mb_d], \]
\[ l'_2 = \frac{y_H - y_L - l'_1}{M-1} - 2B(M-2), \]
\[ l'_i = l'_2 + 4B(i-2), \quad i = 3, \ldots, M. \]

Case 3 ($c'_1 \geq 0$, $c'_M > 1$). Such an equilibrium with $M \geq 2$ messages exists iff $0 \leq x^*_L(M)$, $1 < x^*_H(M)$ and $l'_1 > 0$, $l'_M > 0$ where

\[ y_H - l'_M = (1 - \frac{1}{2M-1}) + \frac{1}{2M-1} y_L - (1 - \frac{1}{2M-1})[(M-1)b_m - Mb_d], \]
\[ l'_1 = \frac{y_H - y_L - l'_M}{N-1} - 2B(M-2), \]
\[ l'_i = l'_1 + 4B(i-2), \quad i = 2, \ldots, M-1. \]

Case 4 ($c'_1 < 0$, $c'_M > 1$). Such an equilibrium with $M \geq 2$ messages exists iff $0 > x^*_L(M)$, $1 < x^*_H(M)$ and $l'_1 > 0$, $l'_2 > 0$, $l'_M > 0$ where

\[ l'_1 + y_L = \frac{1}{2(M-1)} - [(M-2)b_m - (M-1)b_d], \]
\[ y_H - l'_M = 1 - \frac{1}{2(M-1)} - [(M-2)b_m - (M-1)b_d], \]
\[ l'_2 = \frac{1}{M-1} - 2B(M-3), \]
\[ l'_i = l'_2 + 4B(i-2), \quad i = 3, \ldots, M-1. \]

References


