Financing Through Asset Sales*

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Abstract

Most research on firm financing studies the choice between debt and equity issuance. We analyze an alternative source: selling non-core assets. We identify three new factors that drive a firm’s choice between asset sales and equity issuance. First, investors in an equity issue share in the cash raised. Since the value of cash is certain, this mitigates the information asymmetry of equity (the “certainty effect”). In contrast to Myers and Majluf (1984), even if non-core assets exhibit less information asymmetry, the firm issues equity if the financing need is high. This result is robust to using the cash to finance an uncertain investment. Second, firms can disguise the sale of a low-quality asset as instead being motivated by operational reasons (dissynergies), and thus receive a high price (the “camouflage effect”). Third, selling equity implies a “lemons” discount for not only the equity issued but also the rest of the firm, since its value is perfectly correlated. In contrast, even if an asset seller suffers a “lemons” discount on the asset, this need not lead to a low stock price as the asset is not a carbon-copy of the firm (the “correlation effect”).

Keywords: Asset sales, financing, pecking order, synergies.

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One of the most important decisions that a firm faces is how to raise financing. Most existing research focuses on the choice between debt and equity, with various theories identifying different factors that drive a firm’s security issuance decision. The pecking-order theory of Myers (1984), motivated by the model of Myers and Majluf (1984, “MM”), posits that managers issue securities that exhibit least information asymmetry. The trade-off theory argues that managers compare the benefits of debt (tax shields and a reduction in the agency costs of equity) against its costs (bankruptcy costs and the agency costs of debt). The market timing theory of Baker and Wurgler (2002) suggests that managers sell securities that are most mispriced.

While there is substantial research on financing through security issuance, another major source of financing is relatively unexplored – selling non-core assets such as a division or a plant. Asset sales are substantial in reality: in 2010, Securities Data Corporation (SDC) records $133bn in asset sales in the US\(^1\), compared to $130bn in seasoned equity issuance. Although some of these sales may have been motivated by operational reasons, capital raising is an important driver of many disposals. Major oil and gas firms (including Chevron, Shell, and Conoco) have recently sold non-core divisions to raise capital for liquidity and debt service. Most notably, in October 2011, BP set a target of $45bn in asset sales to cover the costs of Deepwater Horizon. Banks worldwide have raised billions of dollars through asset sales in the recent crisis to replenish depleted capital.\(^2\) More broadly, Hite, Owers, and Rogers (1987) examine the stated motives for asset sales and note that “in several cases, management indicated that assets were being sold to raise capital for expansion of existing lines of business or to reduce high levels of debt. In other words, selling assets was viewed as an alternative to the sale of new securities.” Borisova, John, and Salotti (2011) find that over half of asset sellers state financing motives. Campello, Graham, and Harvey (2010) report that 70% of financially constrained firms increased asset sales in the financial crisis, compared to only 37% of unconstrained firms. Ofek (1993) and Asquith, Gertner, and Scharfstein (1994) also show that firms sell assets in response to financial constraints.

However, asset sales can be costly. Pulvino (1998) finds that airlines suffer a valuation discount when selling aircraft for financing reasons, consistent with the Shleifer and Vishny (1992) fire-sale model. Why might firms choose asset sales as a means of financing, despite the cost, and when will they instead choose equity issuance? These

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\(^1\)This figure is an underestimate as 60% of asset sales in SDC have missing transaction values.

\(^2\)In September 2011, Banque Nationale de Paris and Société Générale announced plans to raise $96 billion and $5.4 billion respectively through asset sales, to create a financial buffer against contagion from other French banks. Bank of America raised $3.6 billion in August 2011 by selling a stake in a Chinese construction bank, and $755 million in November 2011 from selling its stake in Pizza Hut.
questions are the focus of this paper. In particular, it studies the conditions under which asset sales are preferable to equity issuance. We build a deliberately parsimonious model to maximize tractability; this allows for the key expressions to be solved in closed form and the economic forces to be transparent. The firm comprises a core asset and a non-core asset. The firm must raise financing to meet a liquidity need, and can either issue equity or sell part of the non-core asset. Following MM, we model information asymmetry as the principal driver of this choice. The firm’s type is privately known to its manager and comprises two dimensions. The first is quality, which determines the assets’ standalone (common) values. The value of the core asset is higher for high-quality firms. The value of the non-core asset depends on how we specify the correlation between the core and non-core assets. With a positive (negative) correlation, the value of the non-core asset is higher (lower) for high-quality firms. The second dimension is synergy, which determines the additional value that the non-core asset is worth to its current owner. This synergy dimension means that asset sales may occur for operational as well as financing reasons.

It may seem that asset sales can already be analyzed by applying the general principles of the MM model of security issuance to the sale of assets, removing the need for a new theory specific to asset sales. Such an extension would suggest that assets are preferred to equity if they exhibit less information asymmetry. While information asymmetry is indeed an important consideration, our model identifies several new distinctions between asset sales and equity issuance that also drive the financing choice, and may swamp information asymmetry considerations.

First, an advantage of equity issuance is that new shareholders obtain a stake in the entire firm. This includes not just the core and non-core assets in place (whose value is unknown), but also the cash raised through the issuance. Since the value of this cash is known, this mitigates the information asymmetry of the assets in place: the certainty effect. In contrast, an asset purchaser does not share in the cash raised, and thus bears the full information asymmetry associated with the asset’s value. Hence, in contrast to MM, even if the information asymmetry of equity is greater than for the non-core asset, the manager may sell equity if enough cash is raised that the certainty effect outweighs the difference in information asymmetry. Formally, a pooling equilibrium is sustainable where all firms sell assets (equity) if the financing need is sufficiently low (high).

Since the certainty effect strengthens as the amount of financing increases, the choice of financing depends on the amount required. This dependence contrasts standard
financing models, where the choice depends only on the inherent characteristics of the security (such as its information asymmetry (MM) or misvaluation (Baker and Wurgler (2002))) and not the amount required – unless one assumes exogenous frictions such as transactions costs or limits on the amount of financing that can be raised through a given channel (e.g., notions of debt capacity). Here, the amount of financing influences the choice of financing, even though there are no nonlinearities. Separately, since financing needs are a motive for selling assets, it may seem that a greater financing need will lead to more assets being sold. We show that it may reduce asset sales, as firms substitute into equity. Thus, a greater financial shock can improve real efficiency, as firms hold onto synergistic assets and instead sell equity.

The certainty effect applies to any use of cash whose value is uncorrelated with firm quality: for example, retaining it on the balance sheet to replenish capital, repaying debt, paying dividends, or financing an uncertain investment with expected payoffs independent of firm type. Since agents are risk-neutral, only expected values matter and so simply adding volatility has no effect on the results. We also analyze the case in which the cash is used to finance an investment whose expected payoffs are correlated with firm quality, and thus exhibit information asymmetry. It may appear that this extension should weaken the certainty effect, since the funds raised are no longer held as certain cash. This intuition turns out to be incomplete, because there is a second effect. Since investment is positive-NPV, it increases the value of the capital that investors are injecting. If the desirability of investment (for firms of both quality) is sufficiently high compared to the additional investment return generated by the high-quality firm over the low-quality firm, the second effect dominates – somewhat surprisingly, the certainty effect can strengthen when cash is used to finance an uncertain investment. Thus, equity issuance becomes easier to sustain. In contrast, if the information asymmetry of investment is sufficiently high, then this becomes the dominant force and asset sales become preferable. Thus, the source of financing depends on the use of financing, even though we have a model of pure adverse selection with no moral hazard. In almost all cases, it remains the case that asset (equity) sales are used for low (high) financing needs.

A second driver of the financing decision is the synergies between the non-core asset and the rest of the firm. High synergies drive firms towards equity financing. This effect leads to “threshold” semi-separating equilibria, in which a firm sells assets if synergies are below a cutoff, and issues equity otherwise. These equilibria are not possible in the absence of synergies as low-quality firms will mimic high-quality firms. In such
equilibria, firms can disguise the motive for their financing choices. In particular, they can disguise an asset sale driven by overvaluation reasons (the asset is of low quality and thus has a low common value) as instead being driven by operational reasons (the asset is dissynergistic and thus only has a low private value): the camouflage effect. This allows some low-quality firms to make greater profits than in the pooling equilibria, where the financing choice does not depend on synergies and so no such disguise is possible.

The threshold synergy level is different for high- and low-quality firms and this difference is driven by the certainty effect. If the amount of financing required increases, this augments the certainty effect and makes equity issuance more (less) attractive to high (low) quality firms: even if its assets are dissynergistic, a high-quality firm chooses to retain them and instead issue equity, because its suffers a smaller adverse selection discount. Thus, higher financing needs have real effects by reducing the quality of assets traded in equilibrium, and thus their price. Conversely, they increase the quality and price of equity. The model thus implies that the market reaction to equity (asset) sales should be more (less) positive for a larger sale.

A third difference is the correlation effect, which represents an advantage to selling assets. When a firm issues equity, it suffers an Akerlof (1970) “lemons” discount – the market infers that the equity is low-quality, from the firm’s decision to issue it. This leads to the market not only paying a low price for the equity issued, but also attaching a low valuation to the rest of the firm. This is because the equity issued is perfectly correlated with the firm as a whole, since the former is a carbon copy of the latter. In contrast, when a firm sells non-core assets, it receives a low price, but critically this need not imply a low valuation for the firm as the asset sold may not be a carbon copy – indeed, it may be negatively correlated. Thus, firms can sell poorly-performing assets without suffering negative inferences on the company as a whole. Asset sales are therefore preferable to equity issuance: formally, in the negative correlation model, the parameter values that support the equity-pooling equilibrium are a strict subset of those that support the asset-pooling equilibrium. An implication is that conglomerates (which are more likely to have negative correlated assets) are less likely to issue equity than firms with closely-related divisions.

Our paper can be interpreted more broadly as studying at what level to issue claims: the firm level (equity issuance) or the asset level (asset sales). Many of the effects also apply to other types of claim that the firm can issue at each level. All three effects apply to parent-company debt in the same way as parent-company equity:
since debt is also a claim to the entire firm, it also benefits from the certainty effect and is positively correlated with firm value; issuing debt does not involve the loss of synergies. Thus, our results would also apply to the choice between selling assets and risky debt. Similarly, if a firm issues collateralized debt at the asset/division level, or engages in an equity carve-out of the division, this need not imply low quality for the firm as a whole (correlation effect), and investors do not own a claim to the cash that they invest, which resides at the parent company level (certainty effect). Since issuing asset-level debt or equity does not involve a loss of (dis)synergies, a carve-out is a special case of our model where synergies are zero and so no camouflage is possible.\(^3\) Thus, the model’s forces more generally apply to the level at which to issue claims, rather than the type of claim. If these effects suggest that it is optimal to issue claims at the parent company level, then the standard pecking-order theory can be used to analyze if the type of claim should be parent-company debt or equity.


Existing theories consider asset sales as the only source of financing and do not compare it to equity issuance: a partial list includes Shleifer and Vishny (1992), De-Marzo (2005), He (2009), and Kurlat (2010). Milbradt (2012) and Bond and Leitner (2011) show that selling an asset will affect the market price of the seller’s remaining portfolio under mark-to-market accounting, changing his balance sheet constraint. We show that such correlation effects are stronger for equity issuance: while a partial asset sale may imply a negative valuation of the remaining unsold non-core assets, it need not imply a negative valuation of the whole firm. Nanda and Narayanan (1999) also consider both asset sales and equity issuance under information asymmetry, but do not feature the certainty, camouflage, or correlation effects.\(^4\)

\(^3\)For future research, it would be interesting to analyze the case in which synergies are non-zero and the firm has a choice between asset sales, carve-outs, and equity issuance.

\(^4\)Leland (1994) allows firms to finance cash outflows either by equity issuance (in the core analysis) or by asset sales (in an extension), but not to choose between the two. In Strebulaev (2007), asset sales are assumed to be always preferred to equity issuance, which is a last resort. Other papers model asset sales as a business decision (equivalent to disinvestment) and do not feature information asymmetry. In Morelec (2001), asset sales occur if the marginal product of the asset is less than its (exogenous) resale value. In Bolton, Chen, and Wang (2011), disinvestment occurs if the cost of external finance is high relative to the marginal productivity of capital. While those papers take the cost of financing
Since a partial asset sale in the no-synergies case can also be interpreted as a carve-out, our paper is also related to the literature on carve-outs. Nanda (1991) also points out that non-core assets may be uncorrelated with the core business and that this may motivate a firm to issue equity at the subsidiary level. In his model, correlation is always zero and the information asymmetry of core and non-core assets is identical. Our model allows for general correlations and information asymmetries, enabling us to generate the certainty and correlation effects.\footnote{Empirically, Allen and McConnell (1998) study how the market reaction to carve-outs depends on the use of proceeds. Schipper and Smith (1986) show that equity issuance leads to negative abnormal returns, but carve-outs lead to positive returns. Slovin, Sushka, and Ferraro (1995) find positive market reactions to carve-outs, and Slovin and Sushka (1997) study the implications of parent and subsidiary equity issuance on the stock prices of both the parent and the subsidiary.}

This paper is organized as follows. Section 1 outlines the general model. Sections 2 and 3 study the positive and negative correlation cases respectively. In Section 4, the funds raised are used to finance an uncertain investment, Section 5 discusses the model's main implications, and Section 6 concludes. Appendix A contains all proofs not in the main text. The Online Appendix contains other peripheral material.

1 The Model

The model consists of two types of risk-neutral agents: firms, which raise financing, and investors, who provide financing and set prices. The firm is run by a manager, who has private information about the firm’s type \( \theta = (q, k) \). If a firm is of type \( \theta \), we also say that the manager is of type \( \theta \). The type \( \theta \) consists of two dimensions. The first is the firm’s quality \( q \in \{H, L\} \), which measures the standalone (common) value of its assets. The prior probability that \( q = H \) is \( \pi \geq \frac{1}{2} \). The second dimension is a synergy parameter \( k \sim U[k, k] \), where \( k \leq 0 \), \( k \geq 0 \), and \( k \) and \( q \) are uncorrelated. This parameter measures the additional (private) value created by the existing owner.

The firm comprises two assets. The core business has value \( C_q \), where \( C_H > C_L \), and the non-core business has value \( A_q \).\footnote{The non-core asset can be considered either a physical asset (e.g. a plant) or a financial asset (e.g. an investment in another firm).} Where there is no ambiguity, we use the term “assets” to refer to the non-core business. We consider two specifications of the model. The first is \( A_H > A_L \), so that the two assets are positively correlated. The second is

as given, this paper microfounds the determinants of the cost of equity finance versus asset sales.
$A_L > A_H$, so the assets are negatively correlated. In both cases, we assume:

$$C_H + A_H > C_L + A_L,$$

i.e. $H$ has a higher total value. Thus, even if assets are negatively correlated, the higher value of $L$'s non-core assets is outweighed by the lower value of its core assets. In MM, the key driver of financing is the information asymmetry of the security issued. The distinction between the two cases of $A_H > A_L$ and $A_H < A_L$ shows that it is not only the information asymmetry of the non-core asset that matters ($|A_H - A_L|$), but also its correlation with the core asset ($\text{sign}(A_H - A_L)$).\footnote{If $A_H = A_L$, there is no information asymmetry surrounding the non-core asset. Thus, it is automatic that the firm will always raise financing by selling it (as shown by MM).}

We consider an individual firm, which must raise financing of $F < \min(A_L, A_H)$.$^5$\footnote{Some of the analysis in the paper will derive bounds on $F$ for various equilibria to be satisfied. We have verified that none of these bounds are inconsistent with $F < \min(A_L, A_H)$.} This need could arise from a number of sources: a liquidity need, an upcoming debt repayment, or a dividend payment. The cash raised remains on the firm’s balance sheet. This modeling treatment nests any use of cash that increases equity value by an amount $F$ in expectation, independent of firm quality. This includes all the above cases plus using the cash for an uncertain investment whose expected value is uncorrelated with firm quality. In Section 4 we allow the cash to fund an investment whose value is correlated with firm quality, so that there is information asymmetry over its expected value.

We currently treat the firm’s financing need $F$ as exogenous. In MM, the firm has the option not to raise financing and instead to forgo investment; the goal of that paper is to show that information asymmetry can deter investment by hindering financing. Since the trade-off between forgoing a desirable investment, and suffering an adverse selection discount when financing investment, is already well-known, our focus instead is to study the choice between asset sales and equity to meet a given financing need, and so we take $F$ as given. In ongoing work we are extending the model to give firms the choice of whether to raise financing and for financing needs to be privately known; preliminary results are described in the Conclusion.

The firm can raise funds either by selling non-core assets or by issuing equity; it

\footnote{The amount of financing $F$ does not depend on the source of financing: $F$ must be raised regardless of whether the firm sells assets or equity. In bank capital regulation, equity issuance leads to a superior improvement in capital ratios than asset sales and so $F$ does depend on the source of financing. We do not consider this effect here as the effect will be straightforward: it will encourage $H$ towards the source that reduces the amount of financing required, and thus force $L$ to follow in order to pool.}
cannot sell the core asset as it is essential for the firm. (In the Conclusion, we discuss relaxing this assumption.) Since \( F < \min(A_L, A_H) \), it is possible to raise the required financing entirely through either source. We assume that the firm raises financing from a single source. This can be motivated by transactions costs associated with multiple financing sources. (Appendix C derives conditions under which the firm will not deviate to multiple financing sources, even under no transactions costs.) There are no taxes, and any transactions costs are assumed to be the same for both sources.

The non-core asset is perfectly divisible so partial asset sales are possible. We deliberately do not feature nonlinearities as they will mechanically lead to the source of financing depending on the amount required. If a firm of quality \( \theta \) sells non-core assets with a value of \( X \), its fundamental value falls from \( C_0 + A_0 \) to \( C_0 + A_0 - X(1 + k) \). Thus, the case of \( k > (\leq) 0 \) represents synergies (dissynergies), where the non-core asset is worth more (less) to the current owner than in its next-best use. That \( k \leq 0 \) allows for asset sales to be motivated by operational reasons (dissynergies) rather than only financing reasons\(^{10} \). In addition to representing synergies, \( k > 0 \) can also arise if investment in assets is costly to reverse (e.g. Abel and Eberly (1996)).

Formally, a firm of type \( \theta \) issues a claim \( K_\theta \in \{E, A\} \), where \( K_\theta = E \) represents equity issuance and \( K_\theta = A \) an asset sale. Investors infer the firm’s type based on its choice of claim \( K_\theta \). These inferences affect both the firm’s market valuation (also referred to as its stock price) and the terms at which it raises financing (and thus its fundamental value). Investors are perfectly competitive and price the claim being sold at its expected value.\(^{11} \) The manager’s objective function is to maximize fundamental value; in the negative correlation case of Section 3, he will also care about the stock price.

A useful feature of the framework is that only the quality parameter \( q \), and not the synergy parameter \( k \), directly affects the investor’s valuation of a claim and thus the price he is willing to pay. This allows our model to incorporate two dimensions of firm type – quality and synergy – while retaining tractability. Thus, we sometimes

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\(^{10}\)One may wonder why the firm will have dissynergistic assets to begin with. Firms may acquire assets when they are synergistic, but they may become dissynergistic over time. One may still wonder why the firm has not yet disposed of the dissynergistic asset. First, the firm may retain it due to the transactions costs of asset sales: only if it is forced to raise financing and so would have to bear the transactions costs of equity issuance otherwise would it consider selling assets. Second, the market for assets is not perfectly frictionless, and so not all assets are owned by the best owner at all times. Our model allows for \( k = 0 \) in which case there are no dissynergies; more generally, our model specifies that synergy motivations are not so strong as to overwhelm the other forces in the model.

use the term “H” or “H-firm” to refer to a high-quality firm regardless of its synergy parameter, and similarly for “L” or “L-firm”. We use the terms “capital gain” and “capital loss” to refer to the gain or loss resulting from the common value component of the asset value only, and “fundamental gain” and “fundamental loss” to refer to the change in the firm’s overall value, which consists of both the capital gain/loss and any loss of (dis)synergies. For equity issuance, the capital gain/loss equals the fundamental gain/loss.

We use the Perfect Bayesian Equilibrium (“PBE”) solution concept, which involves the following: (i) Investors have a belief about which types issue which claim \(K\); (ii) The price of the claim being issued equals its expected value, conditional on investors’ beliefs in (i); (iii) Each manager type chooses to issue the claim \(K\) that maximizes his objective function, given investors’ beliefs; (iv) Investors’ beliefs satisfy Bayes’ rule. In addition to the PBE, beliefs on claims \(K\) issued off the equilibrium path satisfy the Cho and Kreps (1987) Intuitive Criterion (IC).

We first analyze the positive correlation version of the model \((A_H > A_L)\) and then move to the negative correlation version \((A_L > A_H)\).

2 Positive Correlation

We first consider pooling equilibria, which are of two types: an asset-pooling equilibrium (APE) and an equity-pooling equilibrium (EPE). We then move to semi-separating equilibria (SE). The analysis studies the conditions under which the different equilibria are sustainable, to derive predictions on what financing channels firms will use under different conditions.

2.1 Pooling Equilibrium, All Firms Sell Assets

We consider a pooling equilibrium in which all firms sell assets, supported by the off-equilibrium path belief (OEPB) that anyone who sells equity is of type \((L, \bar{k})\). Assets are valued at

\[
E[A] = \pi A_H + (1 - \pi) A_L. \tag{2}
\]

If equity is sold (off the equilibrium path), it is valued at

\[
C_L + A_L + F. \tag{3}
\]
The $F$ term arises because the cash raised from financing enters the balance sheet, and so new shareholders own a claim to this cash in addition to the two existing assets.\footnote{This is consistent with the treatment of financing in MM, although the level of financing plays no role in their analysis since both equity and debt are claims on the entire firm, which includes the financing raised.}

The fundamental values of $H$ and $L$ are respectively given by:

$$C_H + A_H - F\frac{(1 - \pi)(A_H - A_L) + kA_H}{E[A]},$$ \hspace{0.5cm} (4)

$$C_L + A_L + F\frac{\pi(A_H - A_L) - kA_L}{E[A]},$$ \hspace{0.5cm} (5)

An $L$-firm enjoys a capital gain of $\frac{\pi F(A_H - A_L)}{E[A]}$ by selling low-quality assets at a pooled price. However, it also loses the synergies from the asset. If:

$$1 + \bar{k} < \frac{E[A]}{A_L},$$ \hspace{0.5cm} (6)

then even the $L$-firm with the greatest synergies, type $(L, \bar{k})$, will prefer to sell assets than to deviate, since the capital gain from selling low-quality assets exceeds the loss of synergies. We assume that (6) holds throughout the case of positive correlation, else synergy motives are so strong that they dominate information asymmetry. Equation (6) is necessary and sufficient for all $L$-firms not to deviate, regardless of $F$.

$H$-firms suffer a capital loss of $\frac{(1 - \pi)F(A_H - A_L)}{E[A]}$ in addition to any loss of synergies. A $H$-firm may thus deviate and issue equity. If it does so, fundamental value becomes:

$$C_H + A_H - \frac{F(C_H - C_L + A_H - A_L)}{C_L + A_L + F}.$$ \hspace{0.5cm} (7)

The no-deviation ("ND") condition is that (7) $\leq$ (4). This condition is most stringent for type $(H, \bar{k})$. Thus, all $H$-firms will not deviate if:

$$F \leq F^{APE,ND,H} = \frac{(C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - E[A]}.$$ \hspace{0.5cm} (8)

Condition (8) is equivalent to the "unit cost of financing" being lower for asset sales, i.e.

$$\frac{A_H (1 + \bar{k})}{E[A]} \leq \frac{C_H + A_H + F}{C_L + A_L + F},$$ \hspace{0.5cm} (9)
the denominator is the price that investors pay for that claim.

There are three forces that determine \( H \)'s incentives to deviate. The first is whether equity or assets exhibit greater information asymmetry. This effect is a natural extension of the MM principle that high-quality firms wish to issue safe claims. Indeed, if there are no synergy considerations \( (\bar{k} = 0) \), then if \( \frac{A_H}{E[A]} > \frac{C_H + A_H}{C_L + A_L} \) i.e. assets exhibit sufficiently greater information asymmetry than equity, \( H \)-firms will deviate to sell undervalued equity rather than selling undervalued assets: the RHS of (8) is negative and so \( APE \) is unsustainable for any \( F \). Note that the values \( C_\theta \) and \( A_\theta \) represent asset values net of liabilities, and so our model also incorporates information asymmetry about a firm’s liabilities. For example, if a firm has unknown litigation liabilities at the parent company level, a purchaser of one of its factories is not exposed to them.

The second force is synergies, which are absent in MM. For \( APE \) to be sustained, the type with the greatest synergy \( \bar{k} \) must be willing to sell assets despite the loss of synergies. Thus, we require not only assets to be safe, but also the maximum synergy level to be small. If \( \bar{k} > \frac{(C_H + A_H)E[A]}{(C_L + A_L)A_H} - 1 \), i.e. synergies are very important \( (\bar{k} \) is high) and assets are risky \( (\frac{A_H}{E[A]} \) is high compared to \( \frac{C_H + A_H}{C_L + A_L} \)), then again the RHS of (8) is negative and so \( APE \) is unsustainable for any \( F \).

The third force is the amount of financing \( F \). This is unique to a model of asset sales and arises because the cash raised enters the firm’s balance sheet. Thus, the investor shares in the value of this cash if she purchases equity but not assets. Since the value of cash is certain, it mitigates the information asymmetry of equity: the RHS of (9) becomes dominated by the term \( F \), which is the same in the numerator and the denominator as it is known, and less dominated by the unknown assets-in-place terms \( C_q \) and \( A_q \). Thus, there is an upper bound on \( F \) to prevent deviation. If \( F \) exceeds this bound, the certainty effect is sufficiently strong that \( (H, \bar{k}) \) deviates to issuing equity – even though it is inferred as \( L \) by doing so. In particular, even if (33) holds and \( \bar{k} = 0 \), i.e. assets are safer than equity and there are no synergies, a high \( F \) can lead to (8) being violated so \( H \) issues equity. Thus, the MM result that the claim with the least information asymmetry is issued does not hold.

It may seem that, since financing is a motive for asset sales, greater financing needs should lead to more asset sales. However, if \( F \) rises sufficiently, the firm may sell fewer assets, since it substitutes into an alternative source of financing: equity issuance. Surprisingly, greater financial constraints may improve real efficiency as firms hold onto their synergistic assets.\(^{13}\) In particular, if \( \bar{k} = 0 \), all asset sales reduce total surplus

\(^{13}\)Equity issuance does not affect real efficiency as it leads to a pure wealth transfer between investors and firms; asset sales affect real efficiency due to the difference between common and private values.
since there are no dissynergies, and so higher \( F \) increases total surplus.

We now study the comparative statics on the upper bound (8). These are as follows:

\[
\frac{\partial F^{APE,ND,H}}{\partial C_H} = \frac{E[A]}{A_H(1 + \bar{k}) - E[A]} > 0,
\]

\[
\frac{\partial F^{APE,ND,H}}{\partial (-C_L)} = \frac{A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - E[A]} - 1 + \frac{\partial F^{APE,ND,H}}{\partial C_H} > 0,
\]

\[
\frac{\partial F^{APE,ND,H}}{\partial A_H} = \frac{E[A](1 - \pi)(A_H - A_L) + \bar{k}E[A] - (1 - \pi)A_L(C_H - C_L + A_H - A_L)(1 + \bar{k})}{(A_H(1 + \bar{k}) - E[A])^2} \leq 0,
\]

\[
\frac{\partial F^{APE,ND,H}}{\partial (-A_L)} = \frac{A_H(1 + \bar{k})[ (1 - \pi)(C_H - C_L + A_H - A_L) + A_H(1 + \bar{k}) - E[A]]}{(A_H(1 + \bar{k}) - E[A])^2} < 0,
\]

\[
\frac{\partial F^{APE,ND,H}}{\partial \bar{k}} = -\frac{A_H E[A](C_H - C_L + A_H - A_L)}{(A_H(1 + \bar{k}) - E[A])^2} < 0.
\]

For the values of the core business, \( C_H \) and \( C_L \), the signs of the derivatives are intuitive: increasing information asymmetry \( C_H - C_L \) (either by increasing \( C_H \) or reducing \( C_L \)) augments the loss that \( H \) makes by deviating to issue equity. This discourages deviation and the upper bound can relax, i.e., increase. The derivative with respect to \( -C_L \) is larger because reducing \( C_L \) has an additional effect: it reduces the price \( H \) receives from deviating to sell equity \( (C_L + A_L) \). He must therefore sell a greater fraction of the firm’s equity, and so he bears the capital loss over a greater base (the “base effect”).

Turning to the non-core asset, the negative sign of \( \frac{\partial F^{APE,ND,H}}{\partial (-A_L)} \) arises because lowering \( A_L \) reduces the price \( H \) receives for selling assets, and thus encourages him to deviate to equity. This requires the upper bound on \( F \) to tighten, i.e. decrease. However, the sign of \( \frac{\partial F^{APE,ND,H}}{\partial A_H} \) is ambiguous due to the base effect: an increase in \( A_L \) augments the asset price \( E[A] = \pi A_H + (1 - \pi)A_L \). This reduces the quantity of assets that the firm needs to sell and so the loss is sustained over a smaller base. The bound is also increasing in \( \pi \). As \( H \) becomes more common, the price \( E[A] \) rises, deterring deviation.

Finally, the bound is decreasing in the maximum synergy \( \bar{k} \). If \( \bar{k} \) is higher, \( (H, \bar{k}) \) is more willing to issue equity, and so \( F \) must fall to weaken the certainty effect. Viewed differently, even if \( \bar{k} \) is high (so that assets are synergistic), the firm will still be willing to sell them if \( F \) is low. One might think that the effect of synergies should be
independent of the amount of financing raised: if assets are sufficiently synergistic, the firm will not sell them, regardless of $F$. However, if $F$ is low, the firm will sell assets, even if they are synergistic. The comparative statics for the bounds henceforth are in Appendix B.

We now verify whether the OEPB, that an equity issuer is of type $(L, \bar{k})$, satisfies the IC. This is the case if $(L, \bar{k})$ would weakly prefer to issue equity if inferred as $H$, which occurs if:

$$F \leq F_{APE, IC} = \frac{(C_H + A_H)A_L(1 + \bar{k}) - E[A](C_L + A_L)}{E[A] - A_L(1 + \bar{k})}. \quad (10)$$

It may seem that the IC should be trivial since $(L, \bar{k})$ receives a high price for selling equity rather than a pooled price for selling assets. However, if $F$ is large, selling equity is less attractive since the certainty effect reduces the gains from being inferred as $H$. Thus, we have another upper bound on $F$, again due to the certainty effect.

Type $L$ enjoys such a large capital gain from pooling on assets, and loses sufficiently small synergies, that he will not deviate to selling equity even if revealed as $H$.\(^{14}\)

Lemma 1 below summarizes the equilibrium. The proof shows that, if and only if

$$1 + \bar{k} \leq \frac{E[A]}{\sqrt{A_H A_L}} (> 1),$$

the IC condition is stronger than the ND condition and thus is the relevant condition for APE to hold.

**Lemma 1.** *(Positive correlation, pooling equilibrium, all firms sell assets.)* Consider a pooling equilibrium in which all firms sell assets ($K_q = A$) and a firm that issues equity is inferred as type $(L, \bar{k})$, and assume that $\bar{k} < \frac{E[A]}{A_L} - 1$. The prices of assets and equity are $\pi A_H + (1 - \pi)A_L$ and $C_L + A_L + F$ respectively. The equilibrium is sustainable if $F \leq F_{APE}$, where

$$F_{APE} = \begin{cases} 
F_{APE, IC} = \frac{(C_H + A_H)A_L(1 + \bar{k}) - E[A](C_L + A_L)}{E[A] - A_L(1 + \bar{k})} \quad \text{if } 1 + \bar{k} \leq \frac{E[A]}{\sqrt{A_H A_L}}, \\
F_{APE, ND, H} = \frac{(C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \bar{k})}{A_H(1 + \bar{k}) - E[A]} \quad \text{if } 1 + \bar{k} > \frac{E[A]}{\sqrt{A_H A_L}}. 
\end{cases} \quad (11)$$

The upper bound $F_{APE, IC}$ is increasing in $C_H$, $-C_L$ and $\bar{k}$ and decreasing in $A_H$, $-A_L$, and $\pi$. The upper bound $F_{APE, ND, H}$ is increasing in $C_H$, $-C_L$ and $\pi$, and

\(^{14}\)To eliminate an equilibrium with $F > F_{APE, IC}$ via the IC, we also require that $H$ will deviate if he is revealed good. This will automatically be the case, as he will break even rather than suffering a capital loss and losing synergies. In all of the other equilibria that we consider, it will similarly be automatic that $H$ will deviate if he is revealed good, so we will not need to show this mathematically.
decreasing in $-A_L$ and $\bar{k}$; the effect of $A_H$ is ambiguous.

### 2.2 Pooling Equilibrium, All Firms Sell Equity

We now consider the alternative pooling equilibrium in which all firms issue equity, supported by the OEPB that an asset seller is of type $(L, \bar{k})$. Equity is valued at

$$E[C + A] + F = \pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F$$

and if assets are sold (off the equilibrium path), they are valued at $A_L$.

As in APE, $L$ makes a capital gain; however, he will deviate and sell assets if they are sufficiently disynergistic. Type $(L, \bar{k})$ has the greatest incentive to deviate and its no-deviation condition is given by:

$$F \leq F^{EPE,ND,L} = \frac{E[C + A](1 + \bar{k}) - (C_L + A_L)}{-\bar{k}}. \tag{12}$$

A necessary condition for (12) to be satisfied is that

$$\bar{k} > \frac{C_L + A_L}{E[C + A]} - 1, \tag{13}$$

i.e. the disynergies cannot be sufficiently large to outweigh the capital gain on selling low-quality assets. Equation (13) is analogous to (6) in APE which also limits the size of synergies; we similarly assume that (13) holds throughout the case of positive correlation. However, while (6) in APE is both necessary and sufficient for $L$ not to deviate, (13) is only a necessary condition. We also require $F$ to be low for (12) to be satisfied. If $F$ is high, $(L, \bar{k})$ only makes a small capital gain from equity issuance, due to the certainty effect. Thus, it will deviate in order to get rid of a disynergistic asset.

In sum, unlike in APE, satisfying $L$’s ND condition imposes a bound on $F$.

Firms of quality $H$ will not deviate if:

$$F \geq F^{EPE,ND,H} = \frac{A_L(C_H + A_H) - A_H E[C + A](1 + \bar{k})}{A_H(1 + \bar{k}) - A_L}. \tag{14}$$

In contrast to Section 2.2, the ND condition now imposes a lower bound on $F$. This also results from the certainty effect. If $F$ is high, $H$ suffers a small loss from equity issuance due to the certainty effect, and so will not deviate.
The OEPB, that an asset seller is of type \((L, k)\), satisfies the IC if

\[
F \geq F_{EPE,IC}^{EPE} = \frac{A_L \left[ C + A \right] (1 + k) - A_H (C_L + A_L)}{A_H - A_L (1 + k)}.
\]  \(15\)

The denominator is always positive, and so the lower bound can always be satisfied for some \(F\): unlike in \(APE\), there is no necessary condition that we require for the IC condition to be achievable. The condition is a lower bound, since \(F\) must be sufficiently high that the certainty effect reduces the capital gain from pooling on equity, so that \(L\) prefers to deviate and sell assets.

Lemma 2 below summarizes the equilibrium.

**Lemma 2.** (Positive correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium in which all firms sell equity \((K_0 = E)\) and a firm that sells assets is inferred as type \((L, k)\). The prices of assets and equity are \(A_L\) and \(\pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F\) respectively. This equilibrium is sustainable if the following conditions are satisfied:

(i) \(F \leq F_{EPE,ND,L}^{EPE} = \frac{E[C + A](1 + k) - (C_L + A_L)}{\overline{k}}\),

(ii) \(F \geq F_{EPE}^{EPE}\), where

\[
F_{EPE}^{EPE} = \begin{cases} 
F_{EPE,IC}^{EPE} = \frac{A_L \left[ C + A \right] (1 + k) - A_H (C_L + A_L)}{A_H - A_L (1 + k)} & \text{if } 1 + \overline{k} \geq \frac{A_H A_L}{E[A]} \\
F_{EPE,ND,H}^{EPE} = \frac{A_L (C_H + A_H) - A_H E[C + A](1 + k)}{A_H (1 + \overline{k}) - A_L} & \text{if } 1 + \overline{k} < \frac{A_H A_L}{E[A]}.
\end{cases}
\]  \(16\)

The lower bound \(F_{EPE,IC}^{EPE}\) is increasing in \(\overline{k}\); the lower bound \(F_{EPE,ND,H}^{EPE}\) and the upper bound \(F_{EPE,ND,L}^{EPE}\) are both decreasing in \(\overline{k}\). The comparative statics with respect to the other parameters are ambiguous.

### 2.3 Comparing the Pooling Equililibria

We now study the conditions under which each pooling equilibrium is sustainable. The results are given in Proposition 1 below:

**Proposition 1.** (Positive correlation, comparison of pooling equilibria.)

(i) An asset-pooling equilibrium is sustainable if \(F \leq F_{APE}^{APE}\), where \(F_{APE}\) is given by \((11)\),

(ii) An equity-pooling equilibrium is sustainable if \(F_{EPE}^{EPE} \leq F \leq F_{EPE,ND,L}^{EPE}\), where \(F_{EPE,ND,L}^{EPE}\) and \(F_{EPE}^{EPE}\) are given by \((13)\) and \((16)\) respectively,

(iii) If synergies are sufficiently small \((\underline{k} \text{ and } \overline{k} \text{ are low in absolute value})\), then \(F_{APE}^{APE} > F_{EPE}^{EPE}\). Thus, if \(F_{EPE}^{EPE} < F \leq F_{APE}^{APE}\), both pooling equilibria are sustainable.
If synergies are sufficiently large, then \( F^{APE} < F^{EPE} \). Thus, if \( F^{EPE} < F \leq F^{APE} \), neither pooling equilibrium is sustainable.

Proposition 1 shows that, when the amount of financing required increases (but remains below \( F^{EPE,ND,L} \)), firms switch from selling assets to issuing equity, since the certainty effect becomes stronger. Thus, the type of claim issued depends not only on the inherent characteristics of the claim (its information asymmetry and synergies) but also the amount of financing required – firms only issue equity to raise large amounts of financing, even though we have no fixed costs of equity issuance. In standard theories, the type of security issued only depends on the security’s inherent characteristics (information asymmetry or overvaluation) unless one assumes nonlinearities such as limited debt capacity. Here, there are no limits as the amount of financing required can be fully raised by either source. When financing needs become too high (\( F > F^{EPE,ND,L} \)), no pooling equilibrium is sustainable. Due to the certainty effect, the information asymmetry of equity becomes second-order, and so firms with synergies (dissynergies) will sell equity (assets).

Part (iii) states that, if synergies are sufficiently large, the bounds \( F^{APE} \) and \( F^{EPE} \) do not overlap and there is a range of \( F \) values in which neither pooling equilibrium is sustainable. The intuition is as follows. If \( k \) is high, it is difficult to sustain \( APE \) as a firm with \( k = k^* \) will deviate from asset sales to retain its synergistic asset. If \( k \) is low, it is difficult to sustain \( EPE \) as a firm with \( k = k^- \) will deviate from equity issuance to get rid of its dissynergistic asset.

Finally, one interesting case is that of a single-segment firm, which corresponds to \( C_\theta = A_\theta \), i.e. core and non-core assets are one and the same. Since the information asymmetry of the core asset is exactly the same as that of the firm, \( APE \) is never sustainable in the absence of synergies, due to the certainty effect. Thus, single-segment firms sell only equity.

### 2.4 Semi-Separating Equilibrium

In a semi-separating equilibrium, the financing choice depends on the synergy parameter \( k \): there is a cutoff \( k^*_s \) so any firm below (above) the cutoff will sell assets (equity). \( H \) and \( L \) can use different cutoff rules, so separation will be along both type dimensions.

While investors do not directly care about \( k \) (as it only affects private values), the synergy cutoffs matter since they affect the expected quality (common value) of the claims. Thus, investors will infer \( q \) from the seller’s choice of claim. Using Bayes’ rule,
the prices paid for sold assets and issued equity are, respectively:

\[
E[A|k < k_q^*] = \pi \frac{k_H - k}{E[k_q^*] - k} A_H + (1 - \pi) \frac{k_L - k}{E[k_q^*] - k} A_L \tag{17}
\]

\[
E[C + A|k > k_q^*] + F = \pi \left( \frac{\kappa - k_H}{\kappa - E[k_q^*]} \right) (C_H + A_H) + (1 - \pi) \left( \frac{\kappa - k_L}{\kappa - E[k_q^*]} \right) (C_L + A_L) + F. \tag{18}
\]

where

\[
E[k_q^*] = \pi k_H^* + (1 - \pi) k_L^*.
\]

A type \((q, k)\) will prefer equity if and only if its unit cost of financing is less:

\[
\frac{C_q + A_q + F}{E[C + A|k > k_q^*] + F} < \frac{A_q(1 + k)}{E[A|k < k_q^*]}, \tag{19}
\]

where the denominators are given by (17) and (18), respectively. The cutoff \(k_q^*\) for a particular \(q\) is that which allows (19) to hold with equality. Thus, it is defined by:

\[
1 + k_q^* = \frac{C_q + A_q + F}{A_q} \frac{E[A|k < k_q^*]}{E[C + A|k > k_q^*] + F}. \tag{20}
\]

Although \(k_q^*\) is not attainable in closed form, we can study whether \(k_H^* \lesssim k_L^*\). Since only the \(\frac{C_q + A_q + F}{A_q}\) term on the RHS depends on \(q\), the higher cutoff \(k_q^*\) will belong to the quality \(q\) for which this term is higher. Thus, \(k_H^* > k_L^*\) if and only if

\[
\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L}. \tag{21}
\]

Condition (21) is intuitive. It requires that the “certainty-effect” adjusted information asymmetry is higher for equity than assets. \(H\) dislikes information asymmetry as it increases his capital loss; conversely, \(L\) likes information asymmetry. Thus, if equity is riskier, it becomes less (more) attractive to \(H (L)\); therefore, the threshold synergy below which \(H\) sells assets is higher.

The effect of the different cutoffs is to skew the valuations. If \(k_H^* > k_L^*\), \(H\)-firms are more willing to sell assets than \(L\)-firms. Thus, asset sales are a positive signal of quality, and so the asset price (17) is higher than in the asset pooling equilibrium (2). As a result, \(L\)-firms make an even greater capital gain than under APE. Their sale of assets is motivated by overvaluation: the assets are low-quality and thus have a low
common value. However, they are able to disguise them as being motivated instead by operational reasons (the assets are dissynergistic and thus have a low private value, but are actually high quality and thus have a high common value) by pooling with $H$-firms who are indeed selling for operational reasons. Thus, they receive a higher price than under a pooling equilibrium where financing choices are independent of synergies and no such disguise is possible. We call this the “camouflage effect”. In the absence of synergy motives (i.e., if $\bar{K} = \bar{k} = 0$), $SE$ is unsustainable. It is impossible for firms to disguise their financing decisions as being motivated by operational reasons rather than overvaluation.

Moreover, the camouflage effect interacts with the certainty effect. The amount of financing required $F$ changes the cutoffs and thus the quality of assets and equity sold in equilibrium, in turn affecting their prices. If $F$ rises so that (21) is violated, the certainty effect is sufficiently strong that equity issuance is more attractive to $H$. Thus, more $H$ firms sell equity, increasing the quality and price of equity issued and decreasing the quality and price of assets sold. Indeed, when $k^*_H < k^*_L$ (i.e. (21) is violated), we have $k^*_H < 0$: since assets exhibit greater information asymmetry, $H$ will retain them even if they are mildly dissynergistic (and conversely, when $k^*_H > k^*_L$, we have $k^*_H > 0$). Now, $L$-firms achieve camouflage by selling equity: they pool with $H$-firms who issue equity not because their equity is of low quality, but for the operational reason that they do not wish to part with synergistic assets.

The results of this subsection are summarized in Proposition 2 below. Since $F$ does not affect the sustainability of this equilibrium\textsuperscript{15}, but only the cutoffs, we do not include $SE$ in the comparison of equilibria in Section 2.3. In Appendix D, we compare the $F$-bounds implied by (21), which determine whether $k^*_H \leq k^*_L$, to those required for $APE$ and $EPE$ to be sustained, for the case of no synergies.

**Proposition 2.** (Positive correlation, semi-separating equilibrium): A semi-separating equilibrium is sustainable in which quality $q$ sells assets if $k \leq k^*_q$ and sells equity if $k > k^*_q$, where $k^*_q$ is defined by (20). The prices of assets and equity are given by (17) and (18) respectively.

\begin{enumerate}
  \item If $\frac{C_H + A_H + F}{C_L + A_L + F} < \frac{A_H}{A_L}$ (i.e. $F > \frac{C_H A_L - A_H C_L}{A_H - A_L}$), $k^*_H < k^*_L$ and $k^*_H < 0$.
  \item If $\frac{C_H + A_H + F}{C_L + A_L + F} = \frac{A_H}{A_L}$ (i.e. $F = \frac{C_H A_L - A_H C_L}{A_H - A_L}$), $k^*_L = k^*_H = 0$.
  \item If $\frac{C_H + A_H + F}{C_L + A_L + F} > \frac{A_H}{A_L}$ (i.e. $F < \frac{C_H A_L - A_H C_L}{A_H - A_L}$), $k^*_H > k^*_L$ and $k^*_H > 0$.
\end{enumerate}

\textsuperscript{15}In a later draft we will provide a formal analysis of the conditions under which $SE$ is sustainable for all $F$. 

19
3 Negative Correlation

We now turn to the case of negative correlation. Since $A_L > A_H$, we now use the term “high (low)-quality assets” to refer to the assets of $L$ ($H$). Note that negative correlation is a rather mild condition: it only means that high-quality firms are not universally high-quality, as they may have some low-quality assets. It does not require the values of the divisions covary in opposite directions to each other (e.g. that a market upswing helps one division and hurts the other). It is reasonable for the market to know the correlation of the asset with the core business (even if it does not observe quality) simply by observing the type of asset traded. For example, the value of airplanes in a bank’s leasing division is unlikely to be highly correlated with the bank as a whole, but the the value of mortgages will be.

In this section, the manager’s objective function places weight $\omega$ on the firm’s stock price and $1 - \omega$ on fundamental value. The manager’s stock price concerns can stem from a number of sources introduced in earlier work, such as takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990)), or the manager expecting to sell his shares before fundamental value is realized (Stein (1989)). We introduce stock price concerns because, with negative correlation, there is now a trade-off involved in selling assets: being inferred as $H$ maximizes the market value, but being inferred as $L$ maximizes proceeds and thus fundamental value. We also make the following technical assumption:

$$1 + \frac{k}{A_L} > \frac{A_H}{A_L}. \quad (22)$$

Analogous to (6) and (13) in the positive correlation case, this assumption ensures that synergies are not sufficiently strong that they swamp information asymmetry considerations. As with Section 2, we first study the pooling equilibria and then the semi-separating equilibrium.

3.1 Pooling Equilibrium, All Firms Sell Assets

As in Section 2.1, we consider a pooling equilibrium in which all firms sell assets, supported by the OEPB that an equity issuer is of type $(L, k)$. As before, sold assets are valued at $E[A] = \pi A_H + (1 - \pi) A_L$ and issued equity is valued at $C_L + A_L + F$. An asset seller has a stock price of $E[C + A]$, and an equity issuer is priced at $C_L + A_L$.

By deviating, an $L$-firm avoids the capital loss from selling highly-valued assets at
a pooled price as well as any loss of synergies, but suffers a low stock price. Thus, he will only cooperate if its concern for the stock price is high. Since \((L, \bar{k})\) is most likely to deviate, all \(L\)-firms will cooperate if:

\[
\omega \geq \omega^{APE,ND,L} = \frac{F\left(\frac{A_L}{E[A]}(1 + \bar{k}) - 1\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\frac{A_L}{E[A]}(1 + \bar{k}) - 1\right)}.
\]  (23)

If (23) is satisfied, it is automatic that all \(H\)-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by pooling on asset sales. Thus, (23) is necessary and sufficient for all firms not to deviate.

The lower bound given by (23) is relatively loose. It is relatively easy to rule out a deviation to equity issuance. Issuing equity not only leads to a low price (of \(C_L + A_L\)) on the equity being sold (as in MM), but also implies a low valuation (of \(C_L + A_L\)) for the rest of the firm. This is because the correlation between the equity being sold and the rest of the firm is necessarily 1. The second effect is absent in MM, since the manager only cares about fundamental value and not the stock price. As we will see in \(EPE\), this will not be the case when considering deviations to asset sales, since the asset being sold is not a carbon copy of the firm.

It is automatic that the OEPB satisfies the IC. Type \((L, \bar{k})\) will indeed deviate to equity if revealed \(H\): his stock price will rise, he will receive a capital gain by selling equity for a high price (compared to his current loss for selling high-quality assets at a pooled price) and he avoids the loss of synergies \(\bar{k} \geq 0\).

The results of this subsection are summarized in Lemma 3 below:

**Lemma 3.** (Negative correlation, pooling equilibrium, all firms sell assets.) Consider a pooling equilibrium in which all firms sell assets \((K_H = K_L = A)\) and a firm that sells equity is inferred as type \((L, \bar{k})\). The prices of assets and equity are \(\pi A_H + (1 - \pi)A_L\) and \(C_L + A_L + F\) respectively. This equilibrium is sustainable if

\[
\omega \geq \omega^{APE,ND,L} = \frac{F\left(\frac{A_L}{E[A]}(1 + \bar{k}) - 1\right)}{\pi(C_H - C_L - (A_L - A_H)) + F\left(\frac{A_L}{E[A]}(1 + \bar{k}) - 1\right)}.
\]

The lower bound \(\omega^{APE,ND,L}\) is increasing in \(F\), \(\pi\), \(\bar{k}\), and \(A_L - A_H\), and decreasing in \(C_H - C_L\).

The bound is increasing in \(F\), so again the choice of financing depends on the amount required. However, \(F\) plays a different role here than in the positive correlation.
model, where it was important due to the certainty effect. Here, a greater \( F \) means that \( L \)'s capital loss from pooling is sustained over a larger base. It increases the fundamental value motive relative to the market value motive, and requires a higher weight on the market value to maintain indifference. This “base effect” was absent from the positive correlation section, as there was no trade-off between stock price and fundamental value. Put differently, if \( F \) is high, \( L \) suffers such a large capital loss from selling assets that it prefers to “bite the bullet” and issue equity even though this leads to a low stock price. The bound is also increasing in \( \overline{k} \): higher \( \overline{k} \) increases the fundamental loss that \((L, \overline{k})\) suffers from selling assets, and so we require a lower weight on fundamental value (a higher \( \omega \)) for him not to deviate.

### 3.2 Pooling Equilibrium, All Firms Sell Equity

We next consider a pooling equilibrium in which all firms sell equity, supported by the OEPB that anyone who sells assets is of type \((L, k)\).\(^{16}\) As before, issued equity is valued at \( E[C + A] + F \) and sold assets are valued at \( A_L \). The stock price is \( E[C + A] \) for an equity issuer and \( C_L + A_L - F k \) for an asset seller.

By deviating, an \( H \)-firm avoids the capital loss from equity issuance and gets rid of a disynergistic asset, but suffers a low stock price from being inferred as \( L \). Since \((H, k)\) is most likely to deviate, all \( H \)-firms will cooperate if:

\[
\omega \geq \omega^{EPE,ND,H} = \frac{F \left( \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H(1+k)}{A_L} \right)}{\pi \left( C_H - C_L - (A_L - A_H) \right) + F \left( \frac{k + \frac{C_H + A_H + F}{E[C + A] + F} - \frac{A_H(1+k)}{A_L}}{A_L} \right)}. \tag{24}
\]

If (24) is satisfied, it is automatic that all \( L \)-firms will not deviate: their incentives to deviate are weaker as they are making a capital gain by pooling on equity issuance.

\(^{16}\)For all equilibria, we specify the OEPB that anyone who deviates is of quality \( L \), and are free to choose whichever synergy parameter makes the equilibrium most likely to hold. In all equilibria considered thus far, the synergy parameter only affected the IC condition and so the choice was straightforward: we choose the synergy parameter which makes the IC condition easier to satisfy. Thus, for example, the OEPB for \( APE \) under positive correlation was that a deviator is \((L, \overline{k})\) rather than \((L, k)\). Here, the synergy parameter affects both IC and ND and so the choice is not straightforward. A lower \( k \) makes IC easier to satisfy (as \((L, k)\) is more willing to deviate to asset sales to get rid of a disynergistic asset) but increases the stock price of a deviator (as it is deemed to be losing a disynergistic asset) and makes ND harder to satisfy. We follow the earlier equilibria and choose the \( k \) that makes the IC easiest to satisfy. This is because the goal of this section is to show that \( APE \) is sustainable for a greater range of parameters than \( EPE \). In Section 3.3 we show that the IC condition for \( EPE \) is tighter than the ND condition for \( APE \), so if we chose a different \( k \) (which would make the IC condition for \( EPE \) harder to satisfy), this would still hold.
Thus, (24) is necessary and sufficient for all firms not to deviate. Comparing (24) with (23), the condition for no deviation in $APE$, we can see that the $EPE$ condition is relatively harder to satisfy. In $APE$, deviation to equity issuance leads to a low price of $C_L + A_L$ not only on the equity being sold, but also on the rest of the firm. Here, deviation to asset sales leads to a low price of $C_L + A_L - F_k$ on the rest of the firm, but a high price of $A_L$ on the asset being sold, since it is not a carbon copy. This difference is due to the correlation effect.

Unlike in Section 3.1, it is not automatic that the OEPB satisfies the IC, since deviation to asset sales causes $(L, k)$ to suffer a capital loss. The IC condition is satisfied if and only if:

$$\omega \geq \omega^{EPE, IC} = \frac{F \left( \frac{A_L(1+k)}{A_H} - \frac{C_L+A_L+F}{E[C+A]+F} \right)}{(1 - \pi)(C_H - C_L - (A_L - A_H)) + F \left( k + \frac{A_L(1+k)}{A_H} - \frac{C_L+A_L+F}{E[C+A]+F} \right)}.$$  \hspace{1cm} (25)

This is also a lower bound. From (22), the first term in the numerator exceeds 1: if $L$ deviates to asset sales and is inferred as $H$, he suffers a fundamental loss because the capital loss from selling good assets at a low price is greater than the gain from getting rid of a dissynergistic asset. Thus, only if his stock price concerns are high will he deviate. The IC was trivially satisfied in $APE$ where the deviation involved issuing equity – if the deviator is inferred as $H$, he receives both a high stock price and a high value for the equity being sold: both are valued at $C_H + A_H$ since the former is a carbon copy of the latter. Here, the deviation is to assets, which are not a carbon copy of the firm and so can be priced differently: even though the deviator enjoys a high market valuation (of $C_H + A_H - F_k$), he suffers a loss on the assets being sold (which fetch only $A_H$).

The IC condition (25) is stronger than the ND condition (24) if and only if:

$$F_k(N_2 - N_1) < [(C_H - C_L) - (A_L - A_H)] [\pi N_1 - (1 - \pi)N_2]$$  \hspace{1cm} (26)

where $N_1$ and $N_2$ are the parenthetical terms in the numerators of (25) and (24), i.e.:

$$N_1 = \frac{A_L}{A_H} (1 + k) - \frac{C_L + A_L + F}{E[C+A] + F}$$
$$N_2 = \frac{C_H + A_H + F}{E[C+A] + F} - \frac{A_H}{A_L} (1 + k).$$

Lemma 4 below summarizes the equilibrium.
Lemma 4. (Negative correlation, pooling equilibrium, all firms sell equity.) Consider a pooling equilibrium in which all firms sell assets \((K_H = K_L = A)\) and a firm that sells assets is inferred as type \((L, \bar{k})\). The prices of assets and equity are given by \(A_L\) and \(\pi (C_H + A_H) + (1 - \pi) (C_L + A_L) + F\) respectively. This equilibrium is sustainable if \(\omega \geq \omega^{EPE}\), where

\[
\omega^{EPE} = \begin{cases} 
\omega^{EPE, IC} = \frac{F \left( \frac{A_L(1+k)}{A_H} \right)}{(1-\pi)(C_H-C_L-(A_L-A_H))+F \left( \frac{A_L(1+k)}{A_H} \right)} & \text{if (26) holds} \\
\omega^{EPE, ND,H} = \frac{F \left( \frac{C_H + A_H + F}{C_L + A_L + F} \right)}{\pi(C_H-C_L-(A_L-A_H))+F \left( \frac{C_H + A_H + F}{C_L + A_L + F} \right)} & \text{if (26) does not hold.}
\end{cases}
\]

(27)

3.3 Comparing the Pooling Equilibria

We now study the conditions under which each pooling equilibrium is sustainable. The goal of the comparison is to illustrate that the correlation effect leads to asset sales being preferred to equity issuance. Since this concept does not depend on synergies, we undertake the comparison for \(k = k = 0\). The results are given in Proposition 3 below:

Proposition 3. (Negative correlation, comparison of pooling equilibria.) An asset-pooling equilibrium is sustainable if \(\omega \geq \omega^{APE, ND,L}\) and an equity-pooling equilibrium is sustainable if \(\omega \geq \omega^{EPE}\), where \(\omega^{APE, ND,L}\) and \(\omega^{EPE}\) are given by (23) and (27), respectively and \(\omega^{APE, ND,L} < \omega^{EPE}\). Thus, if:

(i) \(0 < \omega < \omega^{APE, ND,L}\), neither pooling equilibrium is sustainable,
(ii) \(\omega^{APE, ND,L} \leq \omega \leq \omega^{EPE}\), only the asset-pooling equilibrium is sustainable,
(iii) \(\omega^{EPE} \leq \omega < 1\), both the asset-pooling and equity-pooling equilibria are sustainable.

The thresholds \(\omega^{APE, ND,L}\) and \(\omega^{EPE}\) are both increasing in \(F\).

Proposition 3 shows that, for the case of negative correlation, asset sales are more common than equity issuance. The range of \(\omega\)'s over which \(EPE\) is sustainable is a strict subset of the range of \(\omega\)'s over which \(APE\) is sustainable. Note that the preference for asset sales exists even though assets may be riskier than equity. If \(\frac{A_L}{A_H} > \frac{C_H + A_H}{C_L + A_L}\), then assets exhibit higher information asymmetry and the MM principle would suggest that equity issuance should be preferred. We show that asset sales are

\[17\]In contrast, if we allowed for very high \(k\) and \(\bar{k}\) close to zero, trivially \(APE\) would be difficult to sustain, despite the correlation effect, as firms will not wish to part with synergistic assets.
may be preferred due to the correlation effect. Note that this correlation effect is absent in a standard financing model of security issuance, because both debt and equity are positively correlated with firm value. Thus, the issuance of debt may imply that debt is low-quality, and thus the remainder of the firm is also low-quality.

3.4 Semi-Separating Equilibrium

As in Section 2.4, we have a semi-separating equilibrium characterized by a cutoff \( k_q^* \). The prices paid for assets and equity are given by (17) and (18). Since the manager now places weight on the firm’s stock price, we need to calculate the stock prices of assets seller and equity issuers. These are, respectively:

\[
E[C + A|k < k_q^*] = \pi \left( \frac{k_H^* - k}{E[k_q^*] - \bar{k}} \right) (C_H + A_H) + (1 - \pi) \frac{k_L^* - k}{E[k_q^*] - \bar{k}} (C_L + A_L),
\]

\[
E[C + A|k > k_q^*] = \pi \left( \frac{\bar{k} - k_H^*}{\bar{k} - E[k_q^*]} \right) (C_H + A_H) + (1 - \pi) \left( \frac{\bar{k} - k_L^*}{\bar{k} - E[k_q^*]} \right) (C_L + A_L).
\]

The cutoff \( k_q^* \) for a particular quality \( q \) is defined by:

\[
\omega \left( E[C + A|k < k_q^*] - E[C + A|k > k_q^*] \right) < (1 - \omega) F \left( \frac{A_q(1 + k_q^*)}{E[A|k < k_q^*]} - \frac{C_q + A_q + F}{E[C + A|k > k_q^*] + F} \right).
\]

Only the parenthetical term on the RHS differs according by quality \( q \). Ignoring \( k \), this term will be higher for \( L \), and so \( k_H^* > k_L^* \). This is intuitive: since \( H \) has lower-quality assets but higher-quality equity, it is more willing to sell assets. Under positive correlation, \( k_H^* > k_L^* \) only if assets are safer than equity (adjusted for the certainty effect), as this means that the capital loss from asset sales is less than from equity issuance. With negative correlation, the capital loss from asset sales is always less since it is negative (i.e., a capital gain), so we always have \( k_H^* > k_L^* \). From (28) and (29), \( k_H^* > k_L^* \) implies that asset (equity) sales lead to a positive (negative) stock price reaction.

The amount of financing \( F \) has two effects on the cutoffs. To illustrate, consider \( L \)’s decision. On the one hand, an increase in \( F \) augments the certainty effect and makes equity issuance less attractive, because \( L \) enjoys a smaller capital gain. This tends to make \( L \) prefer asset sales and increase \( k_L^* \). On the other hand, an increase in \( F \) augments fundamental value considerations as they are now off a higher base. Thus,
the stock price boost from asset sales is less important, which tends to decrease \( k_L^* \), so the overall effect is ambiguous. Thus, this semi-separating equilibrium combines both effects of \( F \): the certainty effect which is present in \( APE \) under positive correlation, and the base effect which is present in \( APE \) under negative correlation.

A second contrast with the positive correlation case is that it is possible to have separation purely by quality and not by synergy, i.e. \( k_H^* = \bar{k} \) and \( k_L^* = \bar{k} \), where all high-quality firms sell assets and all low-quality firms sell equity. We use \( SE^q \) to denote a semi-separating equilibrium by quality only. In the positive correlation case, \( SE^q \) is unsustainable as \((L, \bar{k})\) will deviate to sell assets and enjoy a capital gain plus a loss of dissynergies. Here, \( SE^q \) may be sustainable as, even though deviation allows \((L, \bar{k})\) to get rid of a dissynergetic asset and enjoy a higher stock price, he will suffer a capital loss. Indeed, \( SE^q \) is sustainable if both of the following conditions are satisfied:

\[
\omega \geq \omega^{SE^q,H} = \frac{F \left( 1 + \bar{k} \right) - \frac{C_H + A_H + F}{C_L + A_L + F}}{(C_H - C_L - (A_L - A_H)) + F \left( 1 + \bar{k} \right) - \frac{C_H + A_H + F}{C_L + A_L + F})} \tag{31}
\]

\[
\omega < \omega^{SE^q,L} = \frac{F \left( \frac{A_L (1 + k)}{A_H} - 1 \right)}{(C_H - C_L - (A_L - A_H)) + F \left( \frac{A_L (1 + k)}{A_H} - 1 \right)}. \tag{32}
\]

The lower bound on \( \omega \) ensures that \( H \) will not deviate. Type \((H, \bar{k})\) may wish to deviate to retain its synergistic asset; a high \( \omega \) is needed for the stock price motive to be strong enough to deter deviation. If \( 1 + \bar{k} < \frac{C_H + A_H + F}{C_L + A_L + F} \), i.e. synergies are less important than information asymmetry, then the loss of synergies is less than the capital loss that \((H, \bar{k})\) would suffer by issuing equity. Thus, \( H \)’s fundamental value and stock price are both higher under asset sales, and the lower bound on \( \omega \) is trivially satisfied. There are two effects on \( F \) on the lower bound – it tightens it due to the certainty effect but loosens it due to the base effect – so the overall impact is ambiguous.

The upper bound ensures that \( L \) will not deviate. By deviating, type \((L, \bar{k})\) suffers a fundamental loss: from (22), the capital loss from selling high-quality assets exceeds the benefits of getting rid of a dissynergetic asset. Deviating also leads to a stock price increase. Thus, \( \omega \) must be low so that the stock price motive is sufficiently weak to deter deviation. Unlike the lower bound, the upper bound on \( \omega \) is unambiguously increasing in \( F \) as only the base effect exists.

The range of \( \omega \)’s that satisfy (31) and (32) is increasing in \( \bar{k} \) and decreasing in \( \bar{k} \): the weaker the synergy motive, the easier it is to sustain \( SE^q \). Indeed, if \( \bar{k} = 0 \), \( H \)
will never deviate and so the lower bound is always satisfied. If \( \omega \) falls outside the bounds given by (31) and (32), a semi-separating equilibrium may be sustainable but will involve \( k_H < \bar{k} \) and/or \( k_L > \bar{k} \) (rather than \( SE^q \)).

In \( SE^q \), assets are sold for the lowest possible price of \( A_H \) and equity is issued at the lowest price of \( C_L + A_L \), so there are no capital gains or losses. Type \( H \)'s assets are correctly assessed by the market as being a “lemon”, and so the market timing motive for financing (e.g. Baker and Wurgler (2002)) does not exist. However, the low valuation on the assets sold does not imply a low valuation for the rest of the firm, and so \( H \) is willing to sell assets despite receiving a low price. Thus, the correlation effect – and its implications for the desirability of financing through asset sales – manifests in two ways. First, \( APE \) is sustainable over a greater range of parameters than \( EPE \). Second, \( SE^q \) is sustainable, unlike in the positive-correlation model.\(^{18}\)

This analysis points to an interesting benefit of diversification. Stein (1997) notes that an advantage of holding assets that are not perfectly correlated is that a conglomerate can engage in “winner-picking”, i.e. increase investment in the division with the best investment opportunities at the time. Our model suggests that an advantage of diversification is “loser-picking”: a firm can sell a low-quality asset, thus raising financing, without implying a low value for the rest of the firm. Non-core assets may thus be a form of financial slack and may even be preferable to debt capacity: the value of debt is typically positively correlated with firm value, so a debt issue may lead the market to infer that both the debt being sold and the remainder of the firm are low-quality. In contrast, as discussed previously, single-segment firms only sell equity.

Note that, in \( SE^q \), there are no capital gains or losses, but in a \( SE \) with synergies, \( H \)-firms that sell assets and \( L \)-firms that sell equity earn capital gains. These gains arise because synergy considerations allow firms to disguise the true motive for selling claims: the camouflage effect. By selling assets, \( H \) pools with some \( L \) firms which choose to sell assets due to operational reasons (dissynergies) rather than overvaluation (since their assets are actually high-quality). Thus, \( H \) does not suffer the full lemons discount and receives a price (28) that is greater than the \( A_H \) received under than \( SE^q \). Conversely, \( L \)-firms that sell equity earn a capital gain as they pool with \( H \)-firms who choose to sell equity, even though it is high-quality, as they do not wish to sell a synergistic

\(^{18}\) \( SE^q \) is also featured in Nanda and Narayanan (1999), who consider a model in which core and non-core assets are always negatively correlated and \( \omega = 0 \). (If these assets are positively correlated, there is no information asymmetry in their model.) Thus, no pooling equilibria are sustainable in the absence of transactions costs. They assume that the transactions costs of asset sales are higher than for equity issuance, which sometimes supports an \( EPE \) but never an \( APE \): the opposite result to our paper.
asset. As with the positive correlation case, the camouflage effect disappears if there are no synergy motives ($\overline{k} = \overline{k} = 0$). In this case, the only sustainable semi-separating equilibrium is $SE$, rather than $SE^q$, and there are no capital gains or losses.

The results of this section are summarized in Proposition 4 below.

**Proposition 4.** (Negative correlation, semi-separating equilibrium.) A semi-separating equilibrium is sustainable in which quality $q$ sells assets if $k \leq k_q^*$ and equity if $k > k_q^*$, where $k_q^*$ is defined by (30). We have $k_H^* > k_L^*$ and $k_H^* > 0$; the sign of $k_L^*$ depends on parameter values. The prices of assets and equity are given by (31) and (32) respectively. A semi-separating equilibrium is sustainable in which all firms of quality $H$ ($L$) sell assets (equity) is sustainable if the following two conditions are satisfied:

\[
\omega \geq \omega^{SE,H} = \frac{F \left( (1 + \overline{k}) - \frac{C_H + A_H + F}{C_L + A_L + F} \right)}{(C_H - C_L - (A_L - A_H)) + F \left( (1 + \overline{k}) - \frac{C_H + A_H + F}{C_L + A_L + F} \right)}
\]
\[
\omega < \omega^{SE,L} = \frac{F \left( \frac{A_L (1 + \overline{k})}{A_H} - 1 \right)}{(C_H - C_L - (A_L - A_H)) + F \left( \frac{A_L (1 + \overline{k})}{A_H} - 1 \right)}
\]

4 **Cash Used For Investment**

This section analyzes the case in which the cash raised ($F$) is used to finance an investment whose expected value exhibits information asymmetry. To make the effects of investment as clear as possible, we will focus on the no-synergies case of $\overline{k} = \overline{k} = 0$. We also assume that:

\[
\frac{A_H}{A_L} < \frac{C_H + A_H}{E[C + A]}. \tag{33}
\]

Equation (33) states that the information asymmetry of assets is not too high compared to core equity. If (33) is not satisfied, then the information asymmetry of assets is so high that, in the core model, $EPE$ is always sustainable regardless of $F$. We discuss the effect of relaxing this assumption at the end of this section.

Since all agents are risk-neutral, only expected values matter. Thus, the model is unchanged if we simply make the investment opportunity volatile, so that its payoff is a random variable with an expected value that is independent of $q$.\footnote{If the expected value of the investment is $F$, all of the expressions remain the same. If the expected value is $Q$, $F$ is simply replaced by $Q$ in all expressions: the relevant variable becomes the (common) expected value of the investment instead of the amount of cash required to finance it.}

For the investment
opportunity to affect the analysis, it must vary with \( q \) so that there is information asymmetry regarding its value: information asymmetry is a critically different concept to volatility. We thus assume that \( F \) is used to finance an investment with expected value \( R_q = F (1 + r_q) \), where \( r_H \geq 0 \) and \( r_L \geq 0 \): since there are no agency problems, the investment will only be undertaken if it is positive-NPV (as in MM). We allow for both \( r_H \geq r_L \) and \( r_H < r_L \). The former is more common as high-quality firms typically have superior investment opportunities, but \( r_H < r_L \) can occur as a firm that is currently performing poorly may have greater room for improvement.

Intuitively, it would seem that, if \( r_H > r_L \), the uncertainty of investment will exacerbate the uncertainty of assets in place, weakening the certainty effect and making equity less desirable. However, we will show that this is not necessarily the case. We consider the case of positive correlation here; the case of negative correlation is very similar to the core model and is in Appendix E. Appendix E also allows for \( r_H < 0 \) and \( r_L < 0 \) and shows that the core intuitions are unchanged.

We first consider \( APE \). The analog of (9), \( H \)'s ND condition, is now:

\[
\frac{A_H}{E[A]} \leq \frac{C_H + A_H + F (1 + r_H)}{C_L + A_L + F (1 + r_L)}.
\] (34)

As is intuitive, \( C_q \) and \( R_q = F (1 + r_q) \) enter symmetrically in all expressions: an equity investor receives a share of \( C \), \( R \) and \( A \), but an asset purchaser receives only a share of \( A \). From (34), \( H \) will not deviate if:

\[
F [A_H (1 + r_L) - E[A] (1 + r_H)] \leq E[A] (C_H + A_H) - A_H (C_L + A_L).
\] (35)

Since (33) implies \( \frac{A_H}{E[A]} < \frac{C_H + A_H}{C_L + A_L} \), the RHS of (35) is positive. We first consider the case of \( \frac{A_H}{E[A]} > \frac{1 + r_H}{1 + r_L} \), i.e. the information asymmetry of investment is not too high. The LHS of (35) is positive, and so we again have an upper bound on \( F \), given by:

\[
F \leq \frac{E[A] (C_H + A_H) - A_H (C_L + A_L)}{A_H (1 + r_L) - E[A] (1 + r_H)}.
\] (36)

In the core model ((8)), and setting synergies to zero, the denominator is \( A_H - E[A] \). If \( r_L > r_H \), the denominator of (36) is greater than in the core model, and so the bound is tighter: it is harder to support \( APE \). This is intuitive: \( L \)'s superior growth options counterbalance its inferior assets in place and reduce the information asymmetry of equity, making asset sales less desirable. One may think that the reverse intuition applies to \( r_H \geq r_L \): the information asymmetry of the investment \( R \) increases the
information asymmetry of the core asset (recall that $C$ and $R$ enter symmetrically),
making equity issuance less attractive. Put differently, it seems that the certainty effect
should weaken since cash is now used for uncertain investment. However, this is not
the case: if $r_H \geq r_L$ but $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the denominator of (36) is higher than in the core
model, making $APE$ harder to sustain.

The above intuition is incomplete because financing investment has two effects.
They can be best seen by the following decomposition of the investment returns:

$$ R_L = F (1 + r_L) $$
$$ R_H = F (1 + r_L) + F (r_H - r_L). $$

The first, intuitive effect is the $F (r_H - r_L)$ term which appears in the $R_H$ equation
only. The value of investment is greater for $H$ and so it suffers a greater capital loss from
selling equity. However, there is a second effect, captured by the $F (1 + r_L)$ term which
is common to both types. This increases the certainty effect: since the investment is
positive-NPV, an equity investor now has a claim to a larger certain value: $F (1 + r_L)$
rather than $F$. Put differently, while investors do not know firm quality, they do know
that the funds they provide will increase in value, regardless of quality. Due to this
second effect, $r_H \geq r_L$ is not sufficient for the upper bound to relax. If $\frac{r_H}{r_L} < \frac{A_H}{E[A]}$, the difference in returns is insufficient to outweigh the first effect, and it is harder to
prevent $H$ from deviating. Only if $\frac{r_H}{r_L} > \frac{A_H}{E[A]}$ does the first effect dominate, loosening
the upper bound. Finally, if $\frac{A_H}{E[A]} \leq \frac{1 + r_H}{1 + r_L}$, i.e. investment exhibits high information
asymmetry, then the LHS of (35) is non-positive and so the ND condition is always satisfied: $APE$ is sustainable for any $F$.

Another way to view the intuition is as follows. Equityholders obtain a portfolio
of the assets in place ($C + A$) and the new investment ($R$); the size of $F$ determines
the weighting of the new investment in this portfolio. The $APE$ is sustainable if $H$’s
capital loss from asset sales, $\frac{A_H}{E[A]}$, is less than the weighted average capital loss on this
overall portfolio. If both the assets in place and the new investment exhibit higher
information asymmetry than non-core assets, i.e. $\frac{C_H + A_H}{C_L + A_L} > \frac{A_H}{E[A]}$ and $\frac{1 + r_H}{1 + r_L} > \frac{A_H}{E[A]}$,
then the weighted average loss on the portfolio is greater regardless of the weights – hence,
$APE$ holds regardless of $F$. Deviation is only possible if the investment is safer than
non-core assets, i.e. $\frac{1 + r_H}{1 + r_L} < \frac{A_H}{E[A]}$. In this case, the weight placed on the new investment
must be low for the weighted average loss to remain higher for the portfolio, and so
deviation to be ruled out. The lower the information asymmetry of investment, the
lower the weight on it required for the overall portfolio to exhibit greater information

asymmetry than non-core assets. Thus, holding \( r_L \) fixed, as \( r_H \) declines towards \( r_L \),
the required \( F \) goes down and \( APE \) is harder to sustain.

Regardless of the specific values of \( r_H \) and \( r_L \), in all cases we require the weight on
the investment to be low. Thus, the result of the core model, that \( F \) must be low for
\( APE \) to be sustainable, continues to hold when cash is used to finance an uncertain
investment, and regardless of the values of \( r_H \) and \( r_L \).

The IC condition is satisfied if:

\[
F \leq E[A](1+r_L) - A_L(1+r_H) \leq A_L(C_H + A_H) - E[A](C_L + A_L). \tag{37}
\]

The contrast with the core model ((10)) is similar as for the ND conditions. If \( \frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L} \), (37) is satisfied for all \( F \). If instead \( \frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L} \leq \frac{r_H}{r_L} \), the upper bound on
\( F \) becomes looser than in the core model since the information asymmetry of the
investment increases \( L \)'s incentives to deviate and be revealed as \( H \), since he will
receive a capital gain on the investment value \( R \) as well as the core asset value \( C \).
However, if \( \frac{E[A]}{A_L} > \frac{r_H}{r_L} \), then the bound becomes tighter. As with the ND condition,
this holds if \( r_L > r_H \) (as is intuitive) but can also hold even if \( r_H \geq r_L \).

The equilibrium is summarized in Lemma 5 below. The proof of the Lemma shows
that the IC condition (37) is always relevant, rather than the ND condition (35).

**Lemma 5.** (Positive correlation, pooling equilibrium, all firms sell assets, cash used
for investment.) Consider a pooling equilibrium in which all firms sell assets \((K_q = A)\)
and a firm that issues equity is inferred as type \( L \). The prices of assets and equity are
\( \pi A_H + (1-\pi)A_L \) and \( C_L + A_L + F \) respectively. The equilibrium is sustainable if

\[
F \leq E[A](1+r_L) - A_L(1+r_H) \leq A_L(C_H + A_H) - E[A](C_L + A_L). \tag{38}
\]

(i) If \( \frac{1+r_H}{1+r_L} \geq \frac{E[A]}{A_L} \), the asset-pooling equilibrium is sustainable for all \( F \).
(ii) If \( \frac{E[A]}{A_L} > \frac{1+r_H}{1+r_L} \), the asset-pooling equilibrium is sustainable if
\( F \leq F^{APE,IC,I} = \frac{A_L(C_H + A_H) - E[A](C_L + A_L)}{E[A](1+r_L) - A_L(1+r_H)} \). Compared to the case where cash remains on the balance sheet
(Lemma 1):

(a) If \( \frac{r_H}{r_L} < \frac{E[A]}{A_L} \), the upper bound on \( F \) is tighter and the asset-pooling equilib-
rium is sustainable across a smaller range of \( F \),
(b) If \( \frac{r_H}{r_L} \geq \frac{E[A]}{A_L} \), the upper bound on \( F \) is weakly looser and the asset-pooling equilib-
rium is sustainable across a larger range of \( F \).

The effect of using cash for investment is similar in \( EPE \), so we defer the analysis
to Appendix E. The comparison of equilibria is summarized in Proposition 5:
Proposition 5. (Positive correlation, cash used for investment, comparison of equilibria.) An asset-pooling equilibrium is sustainable if $F < F_{APE,I}$, and an equity-pooling equilibrium is sustainable if $F \geq F_{EPE,I}$, where $F_{APE,I}$ and $F_{EPE,I}$ are given by:

$$F_{APE,I} = \left\{ \begin{array}{ll}
\frac{A_L(C_H+A_H)-E[A](C_L+A_L)}{E[A](1+r_L)-A_L(1+r_H)} & \text{if } \frac{E[A]}{A_L} > \frac{1+r_H}{1+r_L}, \\
\infty & \text{if } \frac{1+r_H}{1+r_L} > \frac{E[A]}{A_L}.
\end{array} \right.$$

$$F_{EPE,I} = \left\{ \begin{array}{ll}
\frac{A_LE[C+A]-A_H(C_L+A_L)}{A_H(1+r_L)-A_L(1+r_H)} & \text{if } \frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L} \text{ and } \frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}, \\
\frac{A_LE[C+A]-A_H(C_L+A_L)}{A_H(1+E[r_q])-A_L(1+r_H)} & \text{if } \frac{1+r_H}{1+r_L} < \frac{A_H-(1-\pi)A_L}{\pi A_L} \text{ and } \frac{1+r_H}{1+r_L} \geq \frac{C_H+A_H}{C_L+A_L}, \\
\infty & \text{if } \frac{1+r_H}{1+r_L} > \frac{A_H-(1-\pi)A_L}{\pi A_L}.
\end{array} \right.$$

The thresholds $F_{APE,I}$ and $F_{EPE,I}$ are both increasing in $r_H$ and decreasing in $r_L$. If $\frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L}$ we have $F_{EPE,I} < F_{APE,I}$. If $\frac{A_H}{E[A]} < \frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L}$, we have $F_{EPE,I} < F_{APE,I}$ if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H+A_H}{C_L+A_L}$. We also have $\frac{A_H-(1-\pi)A_L}{\pi A_L} > \frac{E[A]}{A_L}$.

Proposition 5 demonstrates the core model’s results continue to hold when cash is used for a purpose about which there is asymmetric information. Regardless of $r_H$ and $r_L$, it remains the case that $APE$ is sustainable for low $F$ and $EPE$ is sustainable for high $F$. As in the core model, the source of financing depends on the amount of financing raised.

In addition to demonstrating the robustness of this idea from the core model, this extension also generates a new result. As $r_H$ rises and $r_L$ falls, the upper bound on $APE$ loosens and the lower bound on $EPE$ tightens. If $r_H$ is sufficiently greater than $r_L$, the bound becomes infinite: $APE$ is sustainable for all $F$ (since the upper bound is now infinity) and $EPE$ is sustainable for no $F$ (since the lower bound is now infinity). This can cause the equilibrium to shift from equity issuance to asset sales.\(^{20}\) Thus, the source of financing also depends on the use of financing: if the funds will be used for investment and the investment may create little value ($r_L$ is low), they are more likely to be raised from asset sales rather than equity issuance. The source of financing can depend on the use of financing in models of moral hazard (uses that are more likely to be subject to agency problems will be financed by debt rather than equity, to avoid the agency costs of outside equity) or bankruptcy costs (purchases of tangible assets are more likely to be financed by debt rather than equity); here we deliver this dependence in a model of pure adverse selection. In addition, our predictions for the use of equity differ from a moral hazard model. With moral hazard, if cash is to remain on the

\(^{20}\) Formally, a given $F$ under which both pooling equilibria were sustainable in the core model may now support only $APE$, when cash is used for investment. A given $F$ under which only $EPE$ was sustainable in the core model may now support both equilibria, or only $APE$. 

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balance sheet, debt will be preferred to equity to avoid the agency costs of free cash
flow (Jensen (1986)). Here, equity is preferred due to the certainty effect.

Appendix E also considers the case in which equation (33) does not hold. In this rare
case, assets exhibit such high information asymmetry that $EPE$ holds in the core model
regardless of $F$. In this case, and if also the information asymmetry of the investment
is sufficiently higher than that of assets, equity issuance is always possible unless the
weight on the investment is sufficiently high that the weighted average information
asymmetry is greater than that of assets. Thus, $EPE$ is now satisfied for low $F$ (and
similarly $APE$ is now satisfied for high $F$). If either one of the above conditions is not
satisfied, we return to the core model’s result that $EPE$ is satisfied for high $F$ and
$APE$ for low $F$.

5 Implications

This section briefly discusses the main implications of the model. Some of the im-
plications are empirically testable; a subset of these have already been tested, while
others are as yet untested and would be interesting to explore in future research. In
addition, the model generates other implications that may not be immediately linkable
to an empirical test. Note that empirical analysis should focus on asset sales that are
primarily financing-motivated.

The first set of empirical implications concerns the determinants of financing choice.
One determinant is the amount of financing required: Proposition 1 shows that equity
issuance is preferred for high financing needs, because the certainty effect reduces the
information asymmetry of equity, while asset sales are preferred for low financing needs.
Thus, equity issuances should be larger on average than financing-motivated asset sales.

A second determinant is the purpose for which funds are being raised: Proposition 5
demonstrates that equity is more likely to be used in two cases. First, it will be used for
purposes with less information asymmetry, such as paying debt or dividends, because
these uses reduce the information asymmetry of the entire firm to which equityholders
have a claim. Thus, a firm that is raising financing due to distress (the need to repay
debt) is more likely to issue equity. Second, it will also be used for uncertain investment
projects, if it is likely that the investment will have a significantly positive-NPV even
for low-quality firms (i.e. $r_L$ is high). This implication generates predictions both along
the cross-section and over the time series. Along the cross-section, growth firms with
good investment opportunities should raise equity. Over the time series, in a strong
macroeconomic environment, even low-quality firms will have good investment projects and so equity is again preferred. In contrast, asset sales will be preferred in mature firms or in poor economic conditions where investment quality is uncertain.

Third, the type of firm will affect financing choice. The analysis of Section 3 shows that asset sales are preferred to equity issuance for firms with negatively-correlated assets due to the correlation effect: even if selling assets implies that they are overvalued, it does not imply overvaluation of the firm as a whole. Thus, conglomerates should be more likely to sell assets, and firms with closely related divisions are more likely to issue equity. Note that the preference for selling non-core assets does not arise simply because they are dissynergistic with the rest of the firm (they can even exhibit positive synergies), but because negative market inferences regarding the value of the assets does not imply negative inferences for the rest of the firm.

A second set of empirical implications concerns the market reaction to financing. If $k^*_H > k^*_L$, which holds always in the negative correlation case, and in the positive correlation case under high $F$, asset sales lead to a positive stock price reaction and equity issuance leads to a negative stock price reaction. Indeed, Jain (1985), Klein (1986), Hite, Owers, and Rogers (1987), and Slovin, Sushka, and Ferraro (1995), among others, find evidence of the former; a long line of empirical research including Asquith and Mullins (1986) documents the latter. The positive correlation model also shows how the market reactions to financing should depend on the amount of financing raised. In particular, the reaction to equity issuance will be more positive for larger issues, since they are more likely to come from high-quality firms; conversely, the reaction to financing-motivated asset sales will be more negative for larger sales. The comparison between the positive and negative correlation models suggests that the market reaction to asset sales should be more positive for assets that are negatively-correlated with the remainder of the firm, i.e. assets that are not in the firm’s core line of business.

We now move to implications for financing choices that may be less readily testable. One advantage of asset sales is that a firm with low-quality assets may be able to sell them for a high price, by pooling with firms that are selling assets, despite them being high-quality, because they are dissynergistic: the camouflage effect. Testing this implication is difficult as it requires the econometrician to have a measure of the “true” value of the asset being traded, that is based on information unobservable to the purchaser at the time. The model’s implications regarding synergies are also harder to test given the difficulty in estimating synergies. For example, equity issuers are likely to have synergistic assets, and asset sellers are likely to be parting with dissynergistic
assets. In addition, high-quality firms are more likely to sell synergistic assets if their financing needs are low, whereas low-quality firms are more likely to do so if their financing needs are high.

6 Conclusion

This paper has studied a firm’s choice between raising financing through asset sales and equity issuance. One relevant consideration is the relative information asymmetry of non-core assets and equity, a natural extension of the MM insight. This paper introduces three important additional effects that drive a firm’s financing decision. First, investors in an equity issue share in the value of the cash raised from the issue, but purchasers of non-core assets do not. Since the value of cash is certain, this mitigates the information asymmetry of equity: the certainty effect. Thus, even if the firm’s equity exhibits more information asymmetry than its non-core assets, an equity issue may be preferred to an asset sale (in contrast to MM) if the financing need is sufficiently high. A firm’s choice of financing thus depends on the amount required – low (high) financing needs are met through asset (equity) sales. This result is robust to allowing the cash to be used to finance an uncertain investment.

Second, the choice of financing may also depend on operational motives (synergies). In a semi-separating equilibrium where a firm’s level of synergy affects its financing choice, a higher financing need pushes high-quality firms towards equity issuance, due to the certainty effect, and reduces the quality and price of assets sold in equilibrium. The synergy motive also allows firms to disguise an asset sale, that is in reality motivated by the asset’s low quality, as instead being motivated by operational reasons (dissynergies) – and thus enjoy a capital gain: the camouflage effect.

Third, a disadvantage of equity issuance is that the market attaches a low valuation not only to the equity being sold, but also to the remainder of the firm, since both are perfectly correlated. This need not be the case for an asset sale, since the asset being sold need not be a carbon copy of the firm. This correlation effect can lead to asset sales being strictly preferred to equity issuance.

In ongoing work, we are allowing firms to choose whether to raise financing. Financing needs are privately known; even if a firm does not need to raise financing, it can choose to do so. This extension captures an additional advantage of asset sales. Typically, the need to raise financing is a negative signal as it implies that the firm’s cash position is weaker than previously thought (e.g. Miller and Rock (1985)). How-
ever, if a firm raises financing by selling assets, it can disguise a financing need as one that is business-motivated, i.e. driven by the desire to dispose of a non-synergistic asset. Since the market does not know the motive for the asset sale, the reaction to raising financing via selling assets is less negative. This is an additional manifestation of the camouflage effect.

An additional extension allows the firm to sell a claim to the core asset alone. This extension demonstrates the robustness of the ideas of the core model. One of the assets (core or non-core) will exhibit more information asymmetry than the other, and so the information asymmetry of equity will lie in between. It may therefore seem (from MM) that the sale of one asset will always dominate equity, since one of the assets will have lower information asymmetry than equity. However, even though equity is never the safest claim, it may still be preferred due to the certainty effect. Allowing the firm to sell the core asset in the negative-correlation model of Section 3 also gives the firm a choice of the asset correlation, while in the current model it is nature that decides whether the asset is positively- or negatively-correlated. In the extension, the firm can either sell the core asset (which is positively-correlated with firm value) or the non-core asset (which is negatively-correlated). We will provide full analyses of these extensions in a future draft.
Table 1: Key variables and abbreviations in the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Firm type</td>
</tr>
<tr>
<td>$q$</td>
<td>Firm quality</td>
</tr>
<tr>
<td>$k$</td>
<td>Firm synergy</td>
</tr>
<tr>
<td>$F$</td>
<td>Level of financing required</td>
</tr>
<tr>
<td>$C_\theta$</td>
<td>Value of core asset in firm of quality $\theta$</td>
</tr>
<tr>
<td>$K$</td>
<td>Claim issued by firm</td>
</tr>
<tr>
<td>$A_\theta$</td>
<td>Value of non-core asset in firm of quality $\theta$</td>
</tr>
<tr>
<td>$APE$</td>
<td>Asset-pooling equilibrium</td>
</tr>
<tr>
<td>$EPE$</td>
<td>Equity-pooling equilibrium</td>
</tr>
<tr>
<td>$SE$</td>
<td>Semi-separating equilibrium</td>
</tr>
<tr>
<td>$SE^q$</td>
<td>Semi-separating equilibrium where all firms of quality $q$ issue the same claim $K$</td>
</tr>
<tr>
<td>$k_q^*$</td>
<td>“Threshold” synergy level. Quality $q$ with $k \leq (&gt; ) k_q^*$ sell assets (equity)</td>
</tr>
</tbody>
</table>

**Positive Correlation Model**

$F_{APE,IC}$: Upper bound on $F$ in $APE$ to satisfy the intuitive criterion condition

$F_{APE,ND,\theta}$: Upper bound on $F$ in $APE$ to satisfy the no-deviation condition for all firms of quality $\theta$

$F_{APE}$: The applicable upper bound on $F$, $= \min \{F_{APE,IC}, F_{APE,ND,H}\}$

$F_{EPE,IC}$: Lower bound on $F$ in $EPE$ to satisfy the intuitive criterion condition

$F_{EPE,ND,\theta}$: Lower bound on $F$ in $EPE$ to satisfy the no-deviation condition for all firms of quality $\theta$

$F_{EPE}$: The applicable lower bound on $F$, $= \max \{F_{EPE,IC}, F_{EPE,ND,H}\}$

$F_{APE,IC,I}$: Upper bound on $F$ in $APE$ to satisfy the intuitive criterion condition in investment model. Other variables with superscript $I$ defined analogously

**Negative Correlation Model**

$\omega$: Manager’s weight on the stock price

$\omega_{APE,IC}$: Upper bound on $\omega$ in $APE$ to satisfy the intuitive criterion condition.

Other $\omega$ variables defined analogously
References


A Proofs

Proof of Lemma 1

The IC condition (10) is stronger than the ND condition (8) if and only if

\[
(C_H + A_H)A_L(1 + \bar{k}) - E[A](C_L + A_L) < (C_H + A_H)E[A] - (C_L + A_L)A_H(1 + \bar{k})
\]

This yields \((1 + \bar{k}) < \frac{E[A]}{\sqrt{A_H A_L}}\).

Proof of Lemma 2

\(F_{EP,E,IC}\) is greater than \(F_{EP,E,ND,H}\) if and only if

\[
\frac{A_L E[C + A](1 + \bar{k}) - A_H(C_L + A_L)}{A_H - A_L(1 + \bar{k})} > \frac{A_L(C_H + A_H) - A_H E[C + A](1 + k)}{A_H(1 + \bar{k}) - A_L}
\]

which becomes:

\[
(1 + \bar{k}) > \frac{A_H A_L}{\pi A_H^2 + (1 - \pi)A_L^2} = \frac{A_H A_L}{E[A^2]}.
\]

Proof of Proposition 1

We only need to prove part (iii). From Lemmas 1 and 2, there are four cases to consider.

(i) If \(1 + \bar{k} \leq \frac{E[A]}{\sqrt{A_H A_L}}\) and \(1 + \bar{k} \geq \frac{A_H A_L}{E[A^2]}\) (i.e. \(\bar{k}\) and \(k\) are small in absolute terms) then \(F_{APE} = F_{APE,IC}\) and \(F_{EPE} = F_{EPE,IC}\). Simple calculations then show that \(F_{APE,IC} > F_{EPE,IC}\). First, the denominator of \(F_{APE,IC}\) is smaller than the denominator of \(F_{EPE,IC}\) since \(E[A] < A_H\) and \(\bar{k} > k\). (The denominator cannot be negative since \(1 + \bar{k} < \frac{E[A]}{\sqrt{A_H A_L}}\)). Second, the numerator of \(F_{APE,IC}\) is larger than the numerator of \(F_{EPE,IC}\) since \((C_H + A_H)(1 + \bar{k}) > E[C + A](1 + k)\) and \(E[A] < A_H\).

(ii) If \(1 + \bar{k} \leq \frac{E[A]}{\sqrt{A_H A_L}}\) and \(1 + \bar{k} < \frac{A_H A_L}{E[A^2]}\), then \(F_{APE} = F_{APE,IC}\) and \(F_{EPE} = F_{EPE,ND,H}\). We have \(F_{EPE,ND,H} < F_{APE,IC}\) if and only if

\[
(1 + \bar{k}) > \frac{1}{\pi \left(\frac{A_H}{A_L}\right) + (1 - \pi)(1 + \bar{k}) \left(\frac{A_H}{E[A]}\right)}.
\]

This is satisfied if \(k\) is sufficiently high, i.e. sufficiently close to zero in absolute terms.

(iii) If \(1 + \bar{k} > \frac{E[A]}{\sqrt{A_H A_L}}\) and \(1 + \bar{k} \geq \frac{A_H A_L}{E[A^2]}\), then \(F_{APE} = F_{APE,ND,H}\) and \(F_{EPE} =
$F^{EPE,IC}$. We have $F^{EPE,IC} < F^{APE,ND,H}$ if and only if:

$$(1 + \bar{k}) < \frac{1 - (1 - \pi) \frac{A_L}{E[A]}(1 + \bar{k})}{\pi \frac{A_L}{E[A]}(1 + \bar{k})}.$$ 

This is satisfied if $\bar{k}$ is sufficiently low, i.e. sufficiently close to zero in absolute terms.

(iv) If $1 + \bar{k} > \frac{E[A]}{\sqrt{A_H A_L}}$ and $1 + k < \frac{A_H A_L}{E[A]}$, then $F^{APE} = F^{APE,ND,H}$ and $F^{EPE} = F^{EPE,ND,H}$. We have $F^{EPE,ND,H} < F^{APE,ND,H}$ if and only if:

$$1 + k > \frac{1}{\pi \left( \frac{A_H}{A_L} \right) + (1 - \pi) \left( \frac{E[A]}{A_L} \right)}.$$ 

This is satisfied if $k$ is sufficiently high, i.e. sufficiently close to zero in absolute terms.

Note that our parametric assumptions (6) and (13) are not sufficient for the three above inequalities to be satisfied. Thus, if $\bar{k}$ and $k$ are sufficiently large in absolute terms, it is possible for $F^{APE} > F^{EPE}$ so there is a range of $F$ in which no pooling equilibrium is sustainable.

**Proof of Proposition 2**

To order the two cutoffs relative to each other, note that the equations defining the cutoffs differ only in the term $\frac{C_q + A_q + F}{A_q}$. This term is greater (smaller) for $H$ when $F$ is above (below) the cutoff given in the proposition, meaning that the cutoff for $H$ must also be higher (lower) than that for $L$. To sign $k^*_H$, cross multiply equation 20 to show that $k^*_q > 0$ if and only if

$$(C_q + A_q + F)E[A|k < k^*_q] > A_q(E[C + A|k > k^*_q] + F)$$

Consider $q = H$, and start with case (iii), so $F > \frac{C_H A_L - A_H C_L}{A_H - A_L}$. Since $k^*_H > k^*_L$, there is a positive price reaction to asset sales, so on the LHS we have $E[A|k < k^*_q] > E[A]$. Similarly, there is a negative price reaction to equity issuance, so $E[C + A|k > k^*_q] < E[C + A]$. Thus, a sufficient condition for this inequality is $(C_H + A_H + F)E[A] > A_H(E[C + A] + F)$. This condition is equivalent to $F > \frac{C_H A_L - A_H C_L}{A_H - A_L}$, the condition required for case (iii) in the first place. On the other hand, if $F < \frac{C_H A_L - A_H C_L}{A_H - A_L}$, then a sufficient condition for $k^*_H < 0$ is to show the opposite inequality. Since the price reactions work on the opposite direction, a sufficient condition for $k^*_H < 0$ is $(C_H + A_H + F)E[A] < A_H(E[C + A] + F)$. This is equivalent to $F < \frac{C_H A_L - A_H C_L}{A_H - A_L}$, the condition required for case (i) in the first place. To show case (ii), observe that

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implies that the two cutoff equations defined by (20) are equal to each other. This implies that the cutoffs are also equal. If both types follow the same cutoff strategies, assets and equity are valued at their unconditional expectations (the market makes no inference about quality based on the claim being sold). Thus, the quantities on the right side of (20) will both be equal to one, implying that both cutoffs are equal to zero.

Proof of Lemma 4
Since $k^*_H > k^*_L$, the LHS of (30) is positive. Thus, the RHS must also be positive when evaluated at $k^*_q$ to achieve equality. Since the RHS is increasing in $k^*_q$ and is negative when $q = H$ and $k^*_H = 0$, this we must have $k^*_H > 0$. This argument does not hold for $k^*_L$, since the RHS evaluated at $q = L$ and $k^*_L = 0$ is already positive. If the LHS is larger than this value, we may have $k^*_L > 0$ as well. Thus, the sign of $k^*_L$ depends on parameter values.

Proof of Proposition 3
Follows from simple calculations.

Proof of Lemma 5
We first consider the case of $\frac{1+r_H}{1+r_L} > \frac{E[A]}{A_L}$. Since $\frac{E[A]}{A_L} > \frac{A_H}{E[A]}$, this case implies that the LHS of both (35) and (38) are negative so they are trivially satisfied, and so the upper bound on $F$ is $\infty$). If $\frac{A_H}{E[A]} < \frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L}$, the ND upper bound is $\infty$ (the LHS of (35) is negative) but the IC upper bound is finite and as stated in the Lemma. Finally, if $\frac{1+r_H}{1+r_L} < \frac{A_H}{E[A]}$, both upper bounds are nontrivial (less than $\infty$). From (33), $\frac{A_H}{E[A]} < \frac{C_H + A_H}{C_L + A_L}$ and so $\frac{1+r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}$. This is a sufficient condition for the IC condition to be stronger, and so it appears in the Lemma.

Proof of Proposition 5
It only remains to show that $F^{EPE,I} < F^{APE,I}$ (i.e. the equilibria overlap) if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}$. When the APE bound is not trivial ($\frac{1+r_H}{1+r_L} < \frac{E[A]}{A_L}$), the relevant bound is always given $F^{APE,IC}$, as explained in the proof of Lemma 5. We first wish to show that, when the IC bound is also the relevant bound for $EPE$, $F^{APE,IC,I} > F^{EPE,IC,I}$. This inequality is equivalent to:

$$
(C_H + A_H)(1 + r_L)[\pi A_L E[A] - A_H A_L + (1 - \pi)A_L^2] \\
> (C_L + A_L)(1 + r_H)[\pi A_L E[A] - A_H A_L + (1 - \pi)A_L^2]
$$

The bracketed term is positive, so the above inequality is equivalent to $\frac{1+r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}$, which holds since the IC bound is the relevant one for $EPE$. 44
When ND is the relevant bound for $EPE$, we need to compare $F_{APE,IC,I}$ and $F_{EPE,ND,I}$. Lemma 6 showed that ND is the relevant bound ($F_{EPE,ND,I} > F_{EPE,IC,I}$) when $1 + \frac{r_H}{1+r_L} \geq \frac{C_H + A_H}{C_L + A_L}$, and the paragraph above showed that, if $1 + \frac{r_H}{1+r_L} \geq \frac{C_H + A_H}{C_L + A_L}$, $F_{EPE,IC,I} > F_{APE,IC,I}$. Thus, $F_{EPE,I} < F_{APE,I}$ if and only if $\frac{1+r_H}{1+r_L} < \frac{C_H + A_H}{C_L + A_L}$. 

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